INSTANTANEOUS FREQUENCY ESTIMATION OF FREQUENCY MODULATED SIGNALS IN THE PRESENCE OF MULTIPLICATIVE AND ADDITIVE NOISE: APPLICATION TO MOBILE COMMUNICATION SYSTEMS

Ghasem Azemi, Bouchra Senadji and Boualem Boashash

Signal Processing Research Centre
Queensland University of Technology
GPO Box 2434, Brisbane, QLD 4001, Australia
g.azemi@student.qut.edu.au

1. ABSTRACT

In this paper, we address the problem of estimating the instantaneous frequency (IF) of an FM modulated signal in the presence of multiplicative and additive noise processes. In particular, we consider the estimation of the IF of the received signal at the mobile unit antenna. We show that the existing non-parametric methods based on the Wigner-Ville spectrum (WVS) fail to extract the desired signal. Instead, the first moment of the Wigner-Ville distribution (WVD) of the signal can be used to estimate the IF of the signal under consideration. The estimator is unbiased and analytical expressions for the variance of the estimator in the absence and presence of additive noise are given.

2. INTRODUCTION

In many applications such as sonar, radar, and telecommunications, the instantaneous frequency (IF) carries the useful information [1, 2]. It is, therefore of great importance to have accurate and effective IF estimation methods. Most of the existing IF estimation techniques assume that the signal under consideration has a constant amplitude. While this is an acceptable assumption in many cases, in some situations the signal is subjected to a random amplitude modulation which can be modelled as multiplicative noise. One of such situations is fading in wireless communication systems where the information bearing signals are faded by the time-varying propagation medium [3, chapter 6].

Existing methods for estimating the IF of a given signal in the presence of multiplicative noise can be classified into two classes. The first class consists of parametric methods which use a model for the IF of the signal and try to estimate the parameters of the model [4, 5]. The second class consists of non-parametric methods which do not require the full knowledge of the signal. A well-known subclass of non-parametric methods for IF estimation, is based on the time-frequency distribution (TFD) of the signal [1].

In [6, 7, 8, 9] the peak of the Wigner-Ville spectrum (WVS) is proposed as an estimator for the IF of frequency modulated signals in the presence of multiplicative noise. Although the estimators based on the time-frequency spectrum are useful in some applications, we show that they fail to estimate the IF of the received signal at the mobile unit antenna. Instead, in this paper, we propose another non-parametric estimator based on the first-order moment of the Wigner-Ville distribution (WVD) of the signal. Statistical performance of the estimator is evaluated in the presence of additive and multiplicative noise.

The remainder of this paper is organised as follows. Section 3 describes the model of the signal received by the mobile unit antenna. In section 4, the IF estimator based on the WVS of the signal is reviewed briefly. We show that this estimator would have a significant error in estimating the IF of the received signal by the mobile unit. In section 5, we show that the first-order moment of the Wigner-Ville distribution (WVD) of the received signal can extract the desired signal. Expressions for the variance of the WVD-based estimator are given in section 6. Section 7 concludes the paper.

3. SIGNAL MODEL

The received signal at the mobile unit antenna can be modelled as an FM modulated signal with time-varying amplitude as [3, page: 272]

$$y(t) = m(t)x(t) + n(t) \tag{1}$$

where $m(t)$ is the result of a constructive and destructive superposition of plane waves at the receiver input, referred to as multi-path component, $n(t)$ represents the additive bandpass Gaussian noise centred at the carrier frequency, f_c, with two-sided power spectral density (PSD) $S_{nn}(f) =$

$N_0/2$, and

$$x(t) = \exp\{j2\pi(f_c t + f_\Delta \int_0^t s(\tau)d\tau)\} \; . \qquad (2)$$

In (2), f_Δ is the frequency deviation, and $s(t)$ is the actual message. The multi-path component, $m(t)$, can also be written as [3, page: 241]

$$m(t) = r(t)\exp\{j\psi_r(t)\} \qquad (3)$$

where $r(t)$ is Rayleigh distributed, and $\psi_r(t)$ is uniformly distributed over $[0, 2\pi)$. The aim is to recover the message signal $s(t)$, given one realisation of the received signal $y(t)$.

The IF of the signal $x(t)$ is given by [10]

$$f_{i,x}(t) = f_c + f_\Delta s(t) \; . \qquad (4)$$

Since the message signal is clearly in the IF of the signal $x(t)$, to be able to to recover the message signal $s(t)$ from the received signal $y(t)$, we need to solve the problem of estimating the IF of the signal $x(t)$ in the presence of additive and multiplicative noise. In the next section, we review the currently existing solution to this problem, which is based on the WVS.

4. WVS-BASED IF ESTIMATOR

The WVS of an analytic signal $z(t)$ is defined as [1]

$$\mathcal{W}_z(t,f) = E\{W_z(t,f)\} \qquad (5)$$

where $E\{\,\}$ stands for the expectation operator and $W_z(t,f)$ is the WVD of $z(t)$ which is defined as

$$W_z(t,f) = \int_{-\infty}^{\infty} z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau \; . \qquad (6)$$

It follows that for a non-stationary random signal $z(t)$ which is the product of a complex-valued stationary random process $a(t)$ multiplied by a deterministic non-stationary signal $h(t)$, namely,

$$z(t) = a(t)h(t) \qquad (7)$$

the WVS is given by

$$\mathcal{W}_z(t,f) = S_{aa}(f) *_f W_h(t,f) \qquad (8)$$

where $S_{aa}(f)$ is the PSD of $a(t)$, and $*_f$ stands for the convolution operator with respect to f. It has been shown that the peak of the $\mathcal{W}_z(t,f)$ is an optimal estimator for the IF when $z(t)$ is a linear FM signal. In the case of polynomial phase signals, the use of the polynomial WVS has been shown to give optimal estimation of the IF [7, 11]. We show that this claim is true only when $S_{aa}(f)$ has its maximum at $f = 0$. Suppose that the signal $h(t)$ in (7) is a linear FM signal, $h(t) = \exp\{j2\pi(\beta t + \alpha t^2/2)\}$. The IF of $h(t)$ is given by [10]

$$f_i(t) = \beta + \alpha t \qquad (9)$$

and because $W_h(t,f) = \delta(f - f_i(t))$ ([12]), we obtain

$$\mathcal{W}_z(t,f) = S_{aa}(f - f_i(t)) \; . \qquad (10)$$

From (10), it is clear that $\mathcal{W}_z(t,f)$ has a peak along the IF of $h(t)$, if and only if $S_{aa}(f)$ has its maximum at $f = 0$. Otherwise the peak of the WVS can not be used to estimate the IF of a linear FM signal.

Now consider the received signal at the mobile unit antenna. The PSD of the multiplicative noise $m(t)$ in (3) is given by [3, page: 246]

$$S_{mm}(f) = \frac{3f^2}{2\pi f_m^2\sqrt{f_m^2 - f^2}} \qquad (11)$$

where $f_m = \frac{v}{\lambda}$ (where v is the velocity of the mobile unit and λ is the wavelength of the received signal) is the maximum Doppler frequency shift. The PSD of $m(t)$ is shown in Fig. 1. Clearly, the maximum of $S_{mm}(f)$ occurs at $|f| = f_m$.

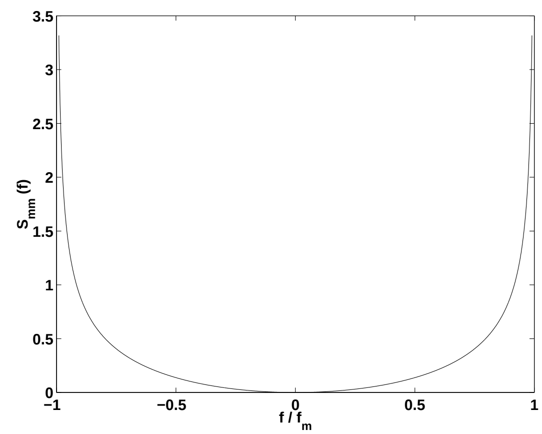

Fig. 1. Power spectral density of the multiplicative noise $m(t)$ in (1).

It follows that, even in the absence of additive noise, the peak of the WVS can not be used to recover the message signal $s(t)$ from the received signal $y(t)$.

5. WVD-BASED IF ESTIMATOR

The first-order moment of the WVD of the signal $z(t)$ is defined as [1]

$$\Omega_z(t) = \frac{\int_{-\infty}^{\infty} fW_z(t,f)df}{\int_{-\infty}^{\infty} W_z(t,f)df} \; . \qquad (12)$$

Based on the above definition, we can derive the first-order moment of the WVD of $z(t) = a(t)\exp\{j\phi(t)\}$ as [12]

$$\Omega_z(t) = \frac{1}{2\pi}\dot{\phi}(t) \qquad (13)$$

which is exactly the IF of the signal $\exp\{j\phi(t)\}$. This suggests that the first-order moment of the WVD can be used to estimate the IF of a signal in the presence of multiplicative noise. In the next section, based on the first-order moment of the WVD of $y(t)$ in (1), we propose an estimator for the message signal as

$$\hat{s}(t) = \frac{\Omega_y(t) - f_c}{f_\Delta} \qquad (14)$$

and evaluate its performance in the absence and presence of additive white Gaussian noise (AWGN).

6. STATISTICAL PERFORMANCE EVALUATION

6.1. In the Absence of Additive Noise

In the absence of additive noise ($n(t) \equiv 0$), the first-order moment of the WVD of the received signal at the mobile unit is

$$\Omega_y(t) = f_c + f_\Delta s(t) + \frac{1}{2\pi}\dot{\psi}_r(t)\ . \qquad (15)$$

The second term of eq. (15) is proportional to the message signal, but the third term is the IF of the multi-path component, $m(t)$, which referred to as random FM. Therefore eq. (14) can be used to estimate the message signal $s(t)$. The error in estimating $s(t)$ using the estimator in (14) is

$$e_1(t) = \frac{1}{2\pi f_\Delta}\dot{\psi}_r(t)\ . \qquad (16)$$

Lee [3, page: 254] has derived the probability density function of $\dot{\psi}_r$ as

$$p(\dot{\psi}_r) = \frac{1}{\sqrt{2}\beta v}[1 + \frac{2}{(\beta v)^2}\dot{\psi}_r^2] \qquad (17)$$

where $\beta = \frac{2\pi}{\lambda}$. It follows that the random FM signal $\dot{\psi}_r(t)$ is zero mean and its power in a frequency band (f_1, f_2) is [3, page: 256]:

$$E\{\dot{\psi}_r^2(t)\} = \frac{(\beta v)^2}{2}\ln\frac{f_2}{f_1}\ . \qquad (18)$$

Using (16) and the fact that $\dot{\psi}_r(t)$ is zero mean, we obtain

$$E\{e_1(t)\} = 0\ . \qquad (19)$$

Therefore the estimator in (14) is unbiased and its variance is

$$\begin{aligned} Var(\hat{s}(t)) &= \frac{1}{4\pi^2 f_\Delta^2}E\{\dot{\psi}_r^2(t)\} \\ &= \frac{1}{2}(\frac{f_m}{f_\Delta})^2\ln\frac{f_2}{f_1}\ . \end{aligned} \qquad (20)$$

For typical values: $f_c = 900$ MHz, $f_1 = 300$ Hz, $f_2 = 3000$ Hz, and $f_\Delta = 18$ KHz, the variance of the estimator as a function of the mobile unit velocity has been shown in Fig. 2.

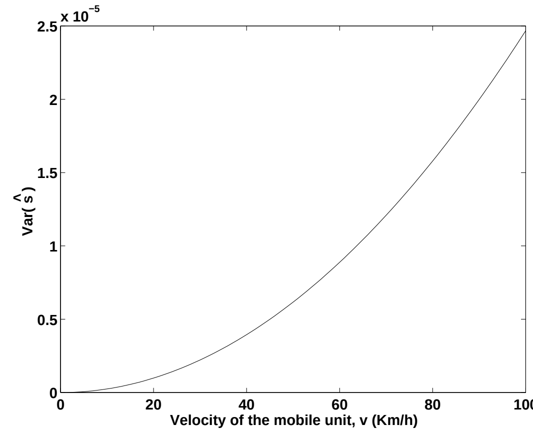

Fig. 2. Estimator variance as a function of the mobile unit velocity, in the absence of additive noise.

When the mobile unit velocity is zero, the random FM component in (15) disappears and the variance of the estimator becomes zero. Since the power in random FM is proportional to v^2 (eq. (18)), the variance of the estimator increases as the mobile unit velocity increases.

6.2. In the Presence of AWGN

Using the envelope-phase description, $n(t)$ in (1) can be written as

$$n(t) = R_n(t)\exp\{j2\pi(f_c t + \psi_n(t))\}\ . \qquad (21)$$

Based on the results in [13, section 7.5], when the signal-to-noise ratio at the receiver input is more than a threshold level (10 dB), the first-order moment of $W_y(t, f)$ in (1) can be written as

$$\Omega_y(t) = f_c + f_\Delta s(t) + \frac{1}{2\pi}\dot{\psi}_r(t) + \xi(t) \qquad (22)$$

where $\xi(t)$ contains both message and noise, with $E\{\xi(t)\} = 0$ and the power [13, page: 278]

$$E\{\xi^2(t)\} = \frac{N_0}{6S_x}(f_2^3 - f_1^3) \qquad (23)$$

in a frequency band (f_1, f_2), where S_x is the power of the signal $x(t)$. On the other hand, in the presence of $n(t)$, the error in estimating $s(t)$ using (14) is

$$e_2(t) = \frac{1}{2\pi f_\Delta}\dot{\psi}_r(t) + \frac{1}{f_\Delta}\xi(t)\ . \qquad (24)$$

Using the fact that both $\dot{\psi}_r(t)$ and $\xi(t)$ are zero mean, we obtain

$$E\{e_2(t)\} = 0\,. \qquad (25)$$

It follows that in the presence of AWGN the estimator in (14) is unbiased and its variance is

$$Var(\hat{s}(t)) = \frac{1}{2}(\frac{f_m}{f_\Delta})^2 \ln\frac{f_2}{f_1} + \frac{N_0}{6f_\Delta^2 S_x}(f_2^3 - f_1^3)\,. \qquad (26)$$

Fig. 3 shows the variance of the estimator as a function of the mobile unit velocity in the presence of AWGN with $(\frac{S}{N})_R = 10$ dB. The values for f_c, f_1, f_2, and f_Δ are the same as in Fig. 2.

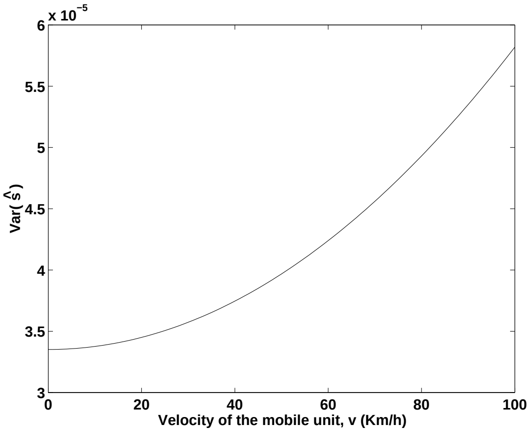

Fig. 3. Estimator variance as a function of the velocity of the mobile unit, in the presence of AWGN with $(\frac{S}{N})_R = 10$ dB .

Since the contribution of the additive noise to the variance of the estimator is independent of the mobile unit velocity, Fig. 3 and 2 are the same except for a vertical shift.

7. CONCLUSION

This paper considered the estimation of the IF of the received signal at the mobile unit antenna using using the first moment of the Wigner-Ville distribution (WVD) of the signal. This signal could be modelled as a frequency modulated signal affected by complex-valued multiplicative and additive noise. We showed that the estimator was unbiased and expressions for the variance of the estimator were derived.

8. REFERENCES

[1] B. Boashash, *Time-Frequency Signal Analysis*, vol. 1, chapter 9, In S. Haykin, editor: Advances in Spectrum Analysis and Array Processing, pp. 418–517, Prentice-Hall, 1991.

[2] B. Boashash, “Estimating and interpreting the instantaneous frequency of a signal- part 2: Algorithms and applications,” *Proceedings of the IEEE*, vol. 80, no. 4, pp. 539–568, Apr. 1992.

[3] W. C. Y. Lee, *Mobile Communication Engineering: Theory and Applications*, McGraw-Hill, 1998.

[4] G. Zhou, G. B. Giannakis, and A. Swami, “On polynomial phase signals with time-varying amplitudes,” *IEEE Trans. on Signal Processing*, vol. 44, no. 4, 1996.

[5] O. Besson, M. Ghogho, and A. Swami, “On estimating random amplitude chirp signals,” *in the proc. of International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 3, pp. 1561–1564, 1999.

[6] T. L. Nguyen and B. Senadji, “Detection of frequency modulated signals in rayleigh fading channels based on time-frequency distributions,” *in Proc. ICASSP*, vol. 2, pp. 729–732, June 2000.

[7] B. Barkat, “Analysis of frequency modulated signals in multiplicative noise,” *in Proc. IEEE Sixth International Symposium on Signal Processing and its Applications (ISSPA)*, vol. 2, pp. 753–756, Aug. 2001.

[8] B. Barkat and B. Boashash, “If estimation of linear fm signals corrupted by multiplicative and additive noise: A time-frequency approach,” *in Proc. ICASSP*, vol. 2, pp. 661–664, June 2000.

[9] B. Boashash and B. Ristich, “Analysis of fm signals affected by gaussian am using the reduced wigner-ville trispectrum,” *in proc. ICASSP*, vol. 4, pp. 408–411, Apr. 1993.

[10] B. Boashash, “Estimating and interpreting the instantaneous frequency of a signal- part 1: Fundamentals,” *Proceedings of the IEEE*, vol. 80, no. 4, pp. 519–538, April 1992.

[11] B. Boashash and P. J. O’shea, “Polynomial wigner-ville distributions and their relationship to time-varying higher order spectra,” *IEEE Trans. on Signal Processing*, vol. 42, no. 1, pp. 216–220, Jan. 1994.

[12] T. A. C. M. Claasen and W. F. G. Mecklenbrauker, “The wigner distribution- a tool for time-frequency signal analysis,” *Philips Journal of Research*, vol. 35, no. 3, pp. 217–250, 1980.

[13] A. B. Carlson, *Communication systems : an introduction to signals and noise in electrical communication*, McGraw-Hill, 1981.