USING A MODEL OF SCATTERING IN A LOW-INTERSYMBOL-INTERFERENCE CHANNEL FOR ARRAY BEAMFORMING

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ABSTRACT

We propose a new model of scattering for use in evaluating the degradation of a signal impinging on a base station's antenna after passing through a Nakagami-Rice fading channel. We have proposed a scheme to reduce degradation of the signal by selecting the direction-of-arrival (DOA) of the signal for use in forming the antenna beam. In this scheme, the degradation is considered to be caused by the delay spread of scattered waves arriving from the vicinity of the nominal DOA. However, the degradation observed in a low-ISI environment cannot be explained by this consideration. To explain degradation in a low-ISI environment, we propose a new model of scattering in which we consider signal degradation to be caused by Nakagami-Rice fading. We then apply the method to determine the DOA that provides the best quality for the signal. Some numerical results are given to verify that the proposed method is effective.

1 INTRODUCTION

In the mobile communications environment, multipath fading degrades the quality of arriving signals. Many schemes for using an array antenna to overcome this problem have been proposed.

Of these, we focus on beamforming methods that are based on direction-of-arrival (DOA). Since the DOA is a parameter that is common to both up- and downlinks and is independent of the signal's waveform, DOA can be used for the beamforming. Moreover, DOA-based methods are easy to apply to frequency-divided duplex systems and to high-speed communications systems. However, it is generally difficult to selecting the DOA for use in beamforming because the degree of degradation will differ for each arriving signals. When the DOA-based method is applied in an environment that subjects the signal to multipath fading, the least degraded signal should be selected for use in beamforming.

In our previous paper [1], we considered each of the DOAs to be spread around the nominal DOA according to a local scattering model [2], and that this spread is caused by the scattering of waves such that they are distributed around the strong wave which is arriving from the nominal DOA.

We also assumed that the delay spread of these waves caused the degradation of the received signals. If the distances from the base station to the sources of the scattering of the wave to each actual DOA to be nearly uniform, it is possible to estimate the delay spread from the angular spread of directions of arrival. On the basis of these assumptions, we proposed a method for selecting a DOA such that the BER of the received signal is improved.

These assumptions are proper when we consider an environment where the signal is subjected to serious ISI. However, our experience is that the signals received at a base station have a worse BER than would be expected even when the level of ISI is low [3]. The above assumptions do not account for this effect.

In this paper, we explain the above phenomenon by assuming that Nakagami-Rice fading degrades the received signals and considering the fading as being caused by the scattered waves. When a signal that has an angular spread is received at an array antenna, the observed array response vector is different from conventional array-response vector. Therefore, we estimate the degree of error in the conventional array-response vector use an approximation derived from a first Taylor expansion [2]. Furthermore, we propose a scheme for selecting the DOA by using a parameter that is related to the estimated error.

This paper is organized as follows. In Section 2, we describe the signal model for multipath propagation. The new parameter is described in Section 3. Section 4 describes the method used to estimate the parameter. Finally, demonstration of the validity of the proposed by simulation is described in Section 5.

2 SIGNAL MODEL

In this section, we explain our model of signals. Firstly, the model of scattering is described. Next, we apply this model to a multipath environment.

2.1 Model of Scattering

We use the model of scattering shown in Fig. 1(a) to associate the received signal $\boldsymbol{x}_i(t)$ with Nakagami-Rice fading. In an environment where signals are subject to

Nakagami-Rice fading, a strong main wave and many weak scattered waves are arriving at the base station. We define a virtual scattering disc and assume that all sources of scattered signals are distributed over the scattering disc. The main wave comes from the center of the disc, θ_i. When the number of scattered waves is defined as N_i, the kth scattered wave will be coming from an angle of $\theta_i + \tilde{\theta}_{i,k}$ $(k = 1, ..., N_i)$. Probability density functions (pdfs) have been proposed for $\tilde{\theta}_{i,k}$ in several papers. We only define the expected value and variance of $\tilde{\theta}_{i,k}$:

$$E\{\tilde{\theta}_{i,k}\} = 0, \quad V\{\tilde{\theta}_{i,k}\} = E\{\tilde{\theta}_{i,k}^2\} = \delta_i^2, \tag{1}$$

where $E\{\cdot\}$ is the expected value of $\{\cdot\}$ and $V\{\cdot\}$ is the variance of $\{\cdot\}$.

We assume time invariance of the distribution on the disc during the period of observation. With the model of scattering, the incident signal on an antenna array of M elements can be modeled as the combination of $N_i + 1$ waves in the following way.

$$\boldsymbol{x}_i(t) = \sum_{k=0}^{N_i} \beta_{i,k} \boldsymbol{a}(\theta_i + \tilde{\theta}_{i,k}) s(t - \tau_{i,k}), \tag{2}$$

where $\beta_{i,0} = A_i$ is the amplitude of the main wave and $\beta_{i,k}(k = 0, ..., N_i)$ are the complex amplitudes of the scattered waves. The DOA of the main wave is θ_i, so $\tilde{\theta}_{i,0} = 0$. $\boldsymbol{a}(\theta)$ is a conventional array-response vector. $s(t - \tau_{i,k})$ is the signal from the kth source. For low-bit-rate signals, $\Delta\tau_{i,k} = \tau_{i,k} - \tau_{i,0}$ is very small, $\Delta\tau_{i,k} \ll T_s$, and $s(t - \tau_{i,k})$ can thus be approximated as $s(t) \exp(-j2\pi f_c \tau_{i,k})$, where f_c is the carrier frequency and T_s is the period of one symbol. Equation (2) may then be approximated as

$$\boldsymbol{x}_i(t) \approx \sum_{k=0}^{N_i} \alpha_{i,k} \boldsymbol{a}(\theta_i + \tilde{\theta}_{i,k}) s_i(t) = \boldsymbol{v}_i s_i(t), \tag{3}$$

where $\alpha_{i,k} = \beta_{i,k} \exp(-j2\pi f_c \tau_{i,k})$, $s_i(t) = s(t - \tau_{i,0})$, and $\boldsymbol{v}_i$ is the approximated spatial signature, $\boldsymbol{v}_i = \sum_{k=0}^{N_i} \alpha_{i,k} \boldsymbol{a}(\theta_i + \tilde{\theta}_{i,k})$.

2.2 Model of a NLOS Multipath Environment

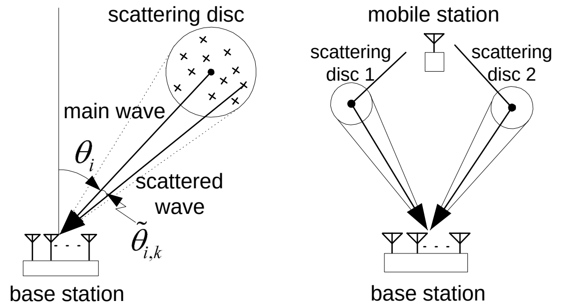
Fig. 1(b) shows a non-line-of-sight (NLOS) multipath environment. We here consider a scenario with D reflectors and a single mobile station. In this case, the outputs of the array antenna elements $\boldsymbol{y}(t)$ are expressed as

$$\boldsymbol{y}(t) = \sum_{i=1}^{D} \boldsymbol{x}_i(t) + \boldsymbol{n}(t), \tag{4}$$

where $\boldsymbol{n}(t)$ is an additive-noise vector.

3 SCATTERING PARAMETERS

Firstly, we propose a new parameter for use in evaluating the quality of signals that arrive. This parameter is



(a) The model of scattering.

(b) NLOS Multipath environment ($D = 2$).

Fig. 1: The model of scattering.

derived from $\boldsymbol{v}_i$, the approximation of the spatial signature. We also clarify the reflection in this parameter of the quality of the received signal.

Asztély and Ottersten [2] assumed a small angular spread to obtain the following approximation.

$$\boldsymbol{v}_i = \gamma_i \boldsymbol{a}(\theta_i) + \phi_i \boldsymbol{d}(\theta_i), \tag{5}$$

where $\boldsymbol{d}(\theta) = \partial \boldsymbol{a}(\theta)/\partial\theta$,

$$\gamma_i = \sum_{k=0}^{N_i} \alpha_{i,k}, \quad \phi_i = \sum_{k=1}^{N_i} \tilde{\theta}_{i,k} \alpha_{i,k}. \tag{6}$$

This approximation is derived from the first Taylor expansion of the spatial signature. When the ith wave is impinging upon the array, γ_i can be assumed to be non-zero. Therefore, we can obtain a new parameter $\tilde{\boldsymbol{v}}_i$:

$$\frac{\boldsymbol{v}_i}{\gamma_i} = \tilde{\boldsymbol{v}}_i = \boldsymbol{a}(\theta_i) + \rho_i \boldsymbol{d}(\theta_i), \tag{7}$$

where $\rho_i = \phi_i/\gamma_i$. From (3) and (7), $\boldsymbol{x}(t)$ is expressed as

$$\boldsymbol{x}(t) \approx \sum_{i=1}^{D} \gamma_i \tilde{\boldsymbol{v}}_i s_i(t) = \sum_{i=1}^{D} \tilde{\boldsymbol{v}}_i \tilde{s}_i(t), \tag{8}$$

where $\tilde{s}_i(t) = \gamma_i s_i(t)$.

If $\tilde{\boldsymbol{v}}_i$ is assumed to be an actual spatial signature, $\rho_i \boldsymbol{d}(\theta_i)$ stands for the error in the conventional array-response vector $\boldsymbol{a}(\theta_i)$. With the models in Section 2, this error varies with time. In our previous paper [1], we use the average $|\rho_i|$ to estimate the signal's angular spread. The theoretical expression for the average $|\rho_i|$ was not shown on account of the derivation the complex of the processing involved. Here, we evaluate a new parameter η_i as

$$\eta_i = E\{|\phi_i|\}/E\{|\gamma_i|\}. \tag{9}$$

Where the theoretical expression for $E\{|\gamma_i|\}$ and $E\{|\phi_i|\}$ can be derived independently.

Next, we clarify the relation between the parameter η_i and the quality of received signal.

A theoretical expression for $E\{|\gamma_i|\}$ may be derived in the following way. When the power of $s_i(t)$ is normalized as $E\{s_i(t)s_i^*(t)\} = 1$, the following expression for $|\gamma_i|^2$ is derived from the power of $\tilde{s}(t)$.

$$E\{\tilde{s}_i(t)\tilde{s}_i^*(t)\} = E\{\gamma_i s_i(t)s_i^*(t)\gamma_i^*\} = |\gamma_i|^2. \quad (10)$$

Here, $|\gamma_i|$ means the average amplitude envelope of the signal received at the base station and varies according to Nakagami-Rice fading. Since the phase of $\alpha_{i,k}$ changes randomly, the expected values and variances of $\text{Re}\{\alpha_{i,k}\}$ and $\text{Im}\{\alpha_{i,k}\}$ can be expressed as $E\{\alpha_{\text{Re}}\} = E\{\alpha_{\text{Im}}\} = 0$, and we obtain

$$V\{\alpha_{\text{Re}}\} = E\{\alpha_{\text{Re}}^2\} = \frac{B_i^2}{2}, \quad V\{\alpha_{\text{Im}}\} = V\{\alpha_{\text{Re}}\}, \quad (11)$$

where B_i is the amplitude of $\alpha_{i,k}$. Let z be an arbitrary complex value and z_{Re} and z_{Im} be the real and imaginary parts of z, respectively. The pdf of $|\gamma_i|$ can be approximated as a Rice distribution. Rice factor $K_i = A_i^2/(2\mu_i^2)$, is the ratio of the power level of the main wave, $A_i^2/2$, to that of the scattered waves, $\mu_i^2 = N_i V\{\alpha_{\text{Re}}\} = N_i V\{\alpha_{\text{Im}}\}$. The scattered waves cause the fading, so we can consider K_i to represent the ratio between the power levels of the main wave and the over all fading component. Since K_i increases as the fading component decreases, the signal quality improves with increasing K_i.

When the condition $K_i \gg 1$ applies to K_i, the pdf of $|\gamma_i|$ is an approximately Gaussian distribution. The expected value of $|\gamma_i|$ can be approximated as

$$E\{|\gamma_i|\} \approx A_i. \quad (12)$$

On the other hand, when there are almost equal power levels in the main wave and all scattered waves, the pdf of $|\gamma_i|$ is approximated by a Rayleigh distribution. Therefore the expected value of $|\gamma_i|$ is

$$E\{|\gamma_i|\} \approx \sqrt{\frac{\pi}{2}}\sqrt{\frac{A_i^2}{2} + \mu_i^2} = \sqrt{\frac{\pi}{2}}\mu_i\sqrt{K_i + 1}. (13)$$

From (6), the real and imaginary parts of ϕ_i are

$$\phi_{\text{Re}} = \sum_{k=1}^{N_i} \alpha_{\text{Re},i,k}\tilde{\theta}_{i,k}, \quad \phi_{\text{Im}} = \sum_{k=1}^{N_i} \alpha_{\text{Im},i,k}\tilde{\theta}_{i,k}. \quad (14)$$

According to the law of averages, the pdfs of ϕ_{Re} and ϕ_{Im} may be approximated as Gaussian distributions. From (1) and (11), their expected values are zero. Their variances are expressed in the following forms.

$$V\{\phi_{\text{Re}}\} = NE\{\tilde{\theta}^2\}E\{\alpha_{\text{Re}}^2\} = \mu_i^2\delta_i^2 \quad (15)$$

$$V\{\phi_{\text{Im}}\} = NE\{\tilde{\theta}^2\}E\{\alpha_{\text{Im}}^2\} = \mu_i^2\delta_i^2 \quad (16)$$

A theoretical expression is derived for $E\{|\phi_i|\}$ in the following way. Since the distributions of ϕ_{Re} and ϕ_{Im} are Gaussian, the pdf of $|\phi| = \sqrt{\phi_{\text{Re}}^2 + \phi_{\text{Im}}^2}$ becomes a Rayleigh distribution. From (15) and (16), the expected value of $|\phi|$ is

$$E\{|\phi|\} = \sqrt{\frac{\pi}{2}}\mu_i\delta_i. \quad (17)$$

Consequently, two approximations of η_i are used. Under the condition $K_i \gg 1$, we define a parameter $\eta_{R,i}$ by approximating η_i in the following way.

$$\eta_i = \frac{E\{|\phi_i|\}}{E\{|\gamma_i|\}} \approx \sqrt{\frac{\pi}{2}}\frac{\mu_i\delta_i}{A_i} = \sqrt{\frac{\pi}{4}}\frac{\delta_i}{\sqrt{K_i}} = \eta_{R,i}. \quad (18)$$

The result of the approximation is that $\eta_{R,i}$ is proportional to δ_i and inversely proportional to $\sqrt{K_i}$. Furthermore, we use the approximation of $A_i \approx B_i$ to define $\eta_{NR,i}$ as

$$\eta_i = \frac{E\{|\phi_i|\}}{E\{|\gamma_i|\}} \approx \frac{\delta_i}{\sqrt{K_i + 1}} = \eta_{NR,i}. \quad (19)$$

Here, $\eta_{NR,i}$ is proportional to δ_i, but inversely proportional to $\sqrt{K_i + 1}$.

Equations (18) and (19) make it clear that the parameter is η_i dependent on the Rice factor K_i and the angular spread of the directions of arrival. If the angular spread of each DOA is same, the value of η_i depends on the Rice factor K_i. Since the approximated η_i is inversely proportional to $\sqrt{K_i}$ or $\sqrt{K_i + 1}$, we can consider the signal quality to be improved by reducing η_i.

4 ESTIMATING THE PARAMETER η_i

The parameter η_i is generally unknown, and must be estimated on the basis of observed signals. Since η_i is the ratio of the average of $|\gamma_i|$ to that of $|\phi_i|$, the problem is how to estimate these parameters. In our previous paper [1], we proposed a method for estimating the scattering parameter $|\rho_i|$ and signal waveform $\tilde{s}_i(t)$. This method consist of five steps which are repeated until the parameters have converged. Although the parameter $|\phi_i|$ is represented as $|\phi_i| = |\gamma_i||\rho_i|$, it is not possible to use the method in our earlier paper to estimate $|\gamma_i|$. We add a new step to the earlier method for estimating $|\gamma_i|$. This step is derived from (10). The new method is then as follows.

Step 1. The DOAs $\hat{\theta}_i$ are given; $\hat{\theta}_i$ means an estimated value of θ_i.

Step 2. $\hat{\rho}_i$ and $\hat{\tilde{\boldsymbol{v}}}_i$ are initialized. $\hat{\rho}_i = 0$, $\hat{\tilde{\boldsymbol{v}}}_i = \boldsymbol{a}(\theta_i)$.

Step 3. $\hat{\tilde{\boldsymbol{s}}}(t) = [\hat{\tilde{s}}_1(t), ..., \hat{\tilde{s}}_D(t)]^T$ is estimated by using a maximum likelihood estimator. The signal waveform is estimated by the following formula

$$\hat{\tilde{\boldsymbol{s}}}(t) = (\hat{\boldsymbol{V}}^H\hat{\boldsymbol{V}})^{-1}\hat{\boldsymbol{V}}^H\boldsymbol{y}(t) = \hat{\boldsymbol{W}}\boldsymbol{y}(t), \quad (20)$$

where $\hat{\boldsymbol{V}} = [\hat{\tilde{\boldsymbol{v}}}_1, ..., \hat{\tilde{\boldsymbol{v}}}_D]$, and $\hat{\boldsymbol{W}} = (\hat{\boldsymbol{V}}^H\hat{\boldsymbol{V}})^{-1}\hat{\boldsymbol{V}}^H$.

Step 4. Values for $\hat{\rho}_i$ are estimated by using minimum mean-square error estimation. The evaluation function is

$$J = E\{|\hat{\boldsymbol{x}}(t) - \boldsymbol{y}(t)|^2\}, \quad (21)$$

where $\hat{\boldsymbol{x}}(t) = \sum_{i=1}^{D}\{\boldsymbol{a}(\hat{\theta}_i) + \hat{\rho}_i \boldsymbol{d}(\hat{\theta}_i)\}\hat{\hat{s}}_i(t) = \hat{\boldsymbol{V}}_i \hat{\hat{\boldsymbol{s}}}(t)$.

Step 5. Step 3 and Step 4 are repeated until the parameters have converged.

Step 6. From (10), $|\hat{\gamma}_i|$ is estimated and $|\hat{\phi}_i| = |\hat{\rho}_i||\hat{\gamma}_i|$.

In a practice, DOAs will generally be unknown. They must then be estimated by using an algorithm such as MUSIC or ESPRIT.

5 RESULTS OF SIMULATION

Simulations were carried out to examine the performance of our proposed method for estimating η_i and of the scheme for using η_i to select the DOA.

The Simulation parameters will now be described. The mobile station is stationary and the fading is flat during the period of observation. Each scattering disc has $N_i = 50$ scattered sources. The amplitude of each scattered wave is $B_i = 1/\sqrt{50}$. A uniform linear array of eight elements is used and each element is separated from next by half a wavelength. The modulation method is $\pi/4$-shift DQPSK. 100 bits of data are transmitted in one iteration of estimating η_i. The expected value $E\{\cdot\}$ is approximated as the average of the values obtained in 10000 iterations. The rate of low transmission means that ISI will have very little effect on the BER. We define the avarage CNR as $(K_i + 1)\mu_i^2/P_n$, where P_n is the level of noise power of each element at the array antenna, and set it at 10 dB.

In the first simulation, one scattering disc is derived from a single mobile station. The nominal DOA is 0°. Fig. 2 shows the parameters η_i obtained for different angular spreads and Rice factors. Estimate 1 indicates the parameter η_i as estimated using the known signal waveform $s(t)$ and the Estimate 2 is the estimate based on the estimated signal waveform $\hat{\hat{s}}(t)$. When the angular spread is $\delta_i = 1°$, the approximated and estimated values are in good agreement with the calculated value. When δ_i is larger than 1°, the approximated value agrees with the calculated value while the estimated value does not. These differences appear to depend on the error introduced by the approximation of (7).

In the second simulation, two scattering discs are derived from a single mobile station and the angular spreads for both DOAs are set to $\delta_1 = \delta_2 = 3°$. The nominal DOAs are $\theta_1 = -15°$ and $\theta_2 = 15°$. Fig. 3(a) shows the estimated and true parameters, η_1 and η_2, when K_1 varies from -15 to 20 dB and K_2 is set at 5 dB.

Fig. 3(b) shows the BER of the signals obtained by two Chebychev beams whose sidelobe level is -25 dB. The mainlobe of Beam 1 is directed at $\theta_1 = -15°$ and that of Beam 2 is directed at $\theta_2 = 15°$.

Fig. 3(a) and Fig. 3(b) indicate that the BER is dependent on the Rice factor K_i and on η_i. We are able to improve the BER of a received signal by selecting smaller value for η_i.

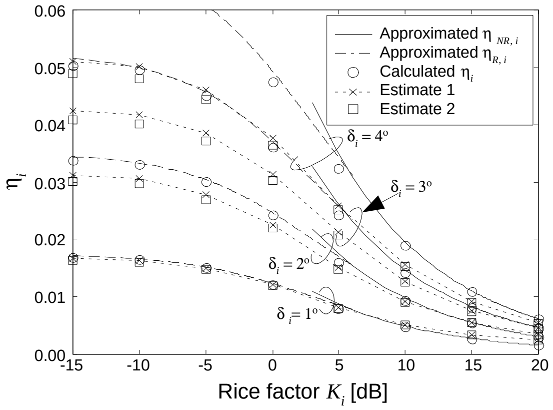
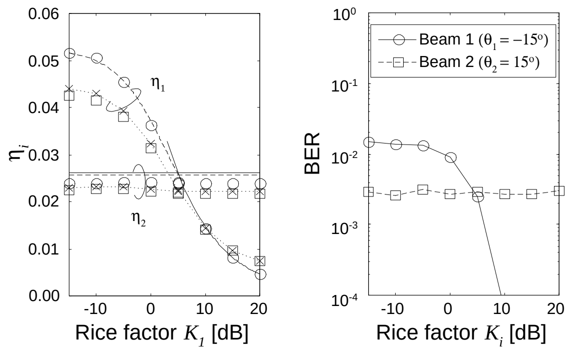


Fig. 2: η_i versus Rice factor ($D = 1$).



(a) η_i versus Rice factor.

(b) BER performances of Chebychev beamforming.

Fig. 3: Two scattering discs are derived from a single mobile station ($D = 2$).

6 CONCLUSION

We proposed a new beam-forming scheme to select a DOA in an environment that is subject to low ISI and to Nakagami-Rice fading. The result of simulation using a scattering disc, when the angular spread of each DOA δ_i is approximately equal, the method produced improved BER performance.

References

[1] H. Tsuji, K. Yamada, and H. Ogawa, "A new approach to array beamforming using local scattering modeling," in *Proc. IEEE 54th VTC*, Sep. 2001, pp. 1284-1288.

[2] D. Asztély and B. Ottersten, "The effects of local scattering on direction of arrival estimation with MUSIC ," *IEEE Trans. Signal Proccesing*, vol. 47 , no. 12, pp 3220-3234, Dec. 1999.

[3] A. Kanazawa, H. Tsuji, H. Ogawa, Y. Nakagawa, and T. Fukagawa, "An experimental study of DOA estimation in multipath environment using an adaptive array antenna equipment", in *Proc. APMC*, Dec. 2000, pp. 804-807.