Nonstationary Methods and application to Noisy Ultrasound Doppler

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Abstract— **Real time flow velocity measurement using ultrasound Doppler is a problem of considerable practical interest in industrial and biomedical applications. The principle consists in accessing flow velocity through estimation of Doppler spectrum or frequency. In presence of colored noise, the commonly used time frequency distributions and parametric Autoregressive methods become highly inaccurate. We propose in this paper to show via colored noisy Doppler simulation the suitability of modified parametric and parametric Wigner-ville techniques in this problem.**

I. Introduction

DOPPLER ultrasound is a noninvasive technique which has been widely explored to access blood flow in vessels or pipe fluid flow in industrial applications, through the analysis of Doppler frequency or spectrum. In practice, the precise determination of the Doppler frequency may sometimes be cumbersome because of various physical phenomena which introduce uncertainties in the velocity estimation [1].

Here we will investigate the case of fluid flow measurement in industrial pipes. The purpose of this study is to access the Doppler frequency in order to estimate the flow velocity profile. This problem may be very tricky since, due to the industrial environment, many noise sources such as engine noise, machine vibrations,... are present together with the ultrasound source. In particular, during the measurement process, very strong colored noise with a flat low frequency spectrum is often added to the time varying Doppler signal . Therefore the conventional approaches are no longer suitable, since the readability of the spectrum is compromised. Here we propose an alternative to overcome these limitations.

The paper presents as alternative modified instrumental variable and autoregressive Wigner Ville methods to overcome these limitations. Computer simulations of Doppler signal with colored noise is used to perform comparisons.

II. TIME FREQUENCY DISTRIBUTIONS FOR DOPPLER FREQUENCY ASSESSMENT

A flow velocity can be estimated by using ultrasound(US) Doppler spectrum. An ultrasound beam at frequency f_0 is emitted in the media and the backscattered beam with a frequency shift f_d is recorded. The flow velocity V and the Doppler frequency are linked through the emitting frequency f_0 and angle α between the directions of flow and US beam (fig.1):

$$f_d = \frac{2V f_0 \cos\alpha}{c} \tag{1}$$

where c is the wave velocity in the media.

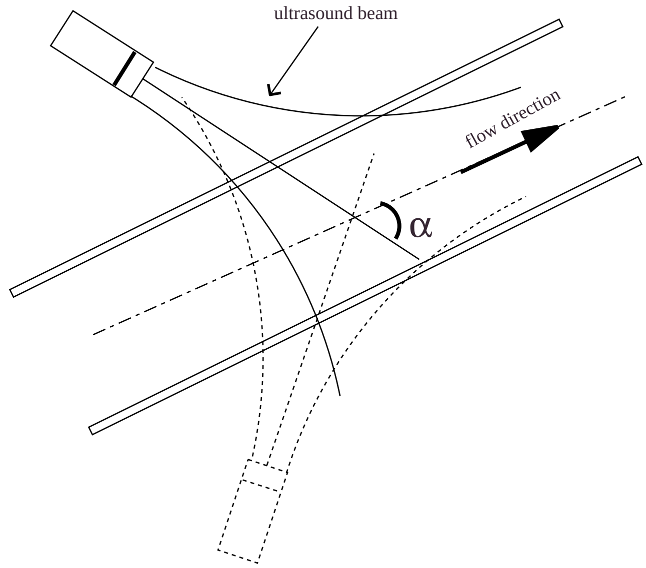

Fig. 1.
Doppler signal acquisition principle

The problem of concern here can be stated as follows : How can f_d be accessed via the observed time varying signal x which consists of x_d the actual Doppler signal and x_c a colored noise with a band limited spectrum, that is :

$$x(t) = x_d(t) + x_c(t) \tag{2}$$

Moreover the spectrum of x_c is close to x_d. In fig.(2) is shown a typical colored noisy Doppler signal obtained by introducing colored noise effect in Doppler simulation algorithm [?].

A. Parametric Methods

Let x, as above, be the nonstationary analyzed (Doppler) signal. Following parametric modeling, signal x can written as :

$$x(n) = \sum_{i=1}^{p} a_i(n)x(n-i) + \sum_{j=0}^{q} b_j(n)w(n-j) + \eta(n) \tag{3}$$

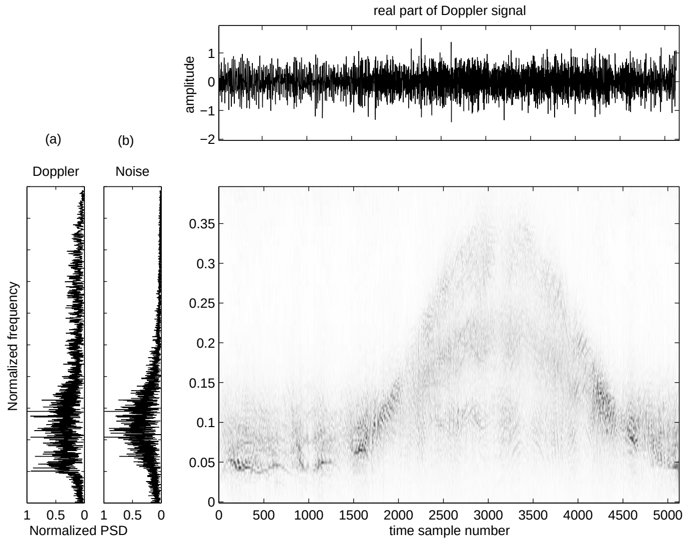

Fig. 2.
Power Spectrum Density(PSD)of Doppler simulated signal(a), PSD of colored Noise (b), Real part of simulated colored noisy Doppler signal, and its WVD (d)

where $w(n)$ is a white noise and $\eta(n)$ a noise which will be discussed below. This is a general well-known ARMA (AutoRegressive Moving Average) model. If the parameters of the ARMA model are correctly estimated, the model is assumed to fit the physical system better than a simpler model such as AR. However, most of studies dealing with Doppler signals use AR model. The main reasons for that are :

- An ARMA model has its equivalent AR model. Unfortunately this equivalent AR model is not always stable, and its order is not a finite number.
- ARMA parameters estimation leads to computations which are at somewhat cumbersome.

However, the AR model can allow to access to the conventional Doppler spectrum or frequency provided the estimation is made under correct hypotheses. Let us recall the AR model.

$$x(n) = -\sum_{i=1}^{p} a_i(n)x(n-i) + \eta(n) \quad (4)$$

$\eta(k)$ which represents in fact the "ignorance" one has on the measurement process.

By defining $\phi^T(n) = [x(n-p), ..., x(n-1)]$ and $\theta^T(n) = [-a_p(n), ..., -a_1(n)]$. Eq(4) can then be written as :

$$x(n) = \phi^T(n)\theta(n) + \eta(n) \quad (5)$$

An important point is that : commonly $\eta(k)$ is assumed to be white noise, particularly for classical Doppler frequency estimation. From eq.(4), the time-varying Power Spectrum Density (PSD) (distribution) is given by :

$$PSD_x(n,f) = \frac{\sigma^2}{|1+\sum_{k=1}^{p} a_n(k)\exp(-2j\pi kf)|^2} \quad (6)$$

where $-1/2 \leq f \leq 1/2$ and σ^2 is the energy of the noise η.

In order to estimate the parameters to be used in eq.(6), for example , Instrumental Variable (IV) algorithm can be used [3],[4], [5],[9].

An error source in parametric Doppler spectrum or frequency estimation is linked to the choice of model order (number of parameters). For quite complicated Doppler signal like colored Doppler signal the number of parameters may change during the observation duration. On one hand, if the number of chosen parameters very low (undermodelling), the estimated parameters are biased. On the other hand, if the number of chosen parameters is very high (overmodelling), extra parameters generate extra frequency components in the spectrum which yields inaccurate frequency estimation. This problem has been solved using test on standard criteria such as Akaike information or Final prediction error [6]. The principle of this test consists in computing, for a set of model orders, the values of the criterion. The order for which the criterion achieves an optimum is assumed to be the model order. Unfortunately the test is usually performed only once (often at the beginning of the parameters estimation process) regardless of the possible changes of the model order. In order to accounted for the possible model order changes,we propose to use a factored form [7], [13] of the covariance matrix UDV^H method. The following method which was introduced in [8], is a complex-data form of the recursive real-data method developed by Niu et Fisher [10]. Consider the model represented by the expression (5).
One can define an *augmented* regression vector $\varphi^T(n) = [x(n-M), ..., x(n-1), x(n)] = [\phi^T(n), x(n)]$ and an *augmented* instrumental vector $\zeta^T(n) = [z(n-M), ..., z(n-1), z(n)] = [\psi^T(n), z(n)]$ where $z(n)$ is defined as by $\psi^T(n) = [z(n-p), ..., z(n-1)]$. The dimension of the regression (and instrumental) vector is $d = M+1$. Note that the choice of M is arbitrary. Then one can define :

$$C(n) = \left[\sum_{j=1}^{n} \zeta(j)\varphi^T(j)\right]^{-1}_{d\times d} \quad (7)$$

$C(n)$ can be decomposed in the form of UDV^H, that is :

$$C(n) = U(n)D(n)V^H(n) \quad (8)$$

where H denotes the hermitian transpose, U is an upper triangular matrix with all diagonal elements equal to unity.

$$U = \begin{bmatrix} 1 & \hat{\theta}_1^2(n) & \ldots & \hat{\theta}_1^p(n) \\ & 1 & \ldots & \hat{\theta}_2^p(n) \\ & & \ddots & \\ & & 1 & \vdots \\ & & \ddots & \\ & \mathbf{0} & & \hat{\theta}_{d-1}^p(n) \\ & & & 1 \end{bmatrix} \quad (9)$$

V is also a upper triangular matrix with all diagonal elements equal to unity.

$$V = \begin{bmatrix} 1 & \hat{\alpha}_1^2(n) & \dots & \hat{\alpha}_1^p(n) \\ & 1 & \dots & \hat{\alpha}_2^p(n) \\ & & \ddots & \\ & & 1 & \vdots \\ & & \ddots & \\ & \mathbf{0} & & \hat{\alpha}_{d-1}^p(n) \\ & & & 1 \end{bmatrix} \tag{10}$$

Matrix D is a diagonal matrix with the form

$$D^{-1}(n) = diag[J(0), J(1), ..., J(d)] \tag{11}$$

It can be shown [10] that

$$J(n) = \sum_{j=1}^{n} [y(j) - \phi^T \hat{\theta}(j)][z(j) - \psi^T \hat{\alpha}(j)] \tag{12}$$

The minimum diagonal element of D^{-1} gives the model order, since the *a-priori* order M can be chosen as high as possible. The above decomposition can be performed recursively (at each time).

From eq.(7) it follows that

$$C(n) = [C^{-1}(n-1) + \zeta(n)\varphi^T(n)]^{-1} \tag{13}$$

One can then define the variables $f_M = U^T(n-1)\varphi(n)$ and $g_M = D(n-1)f_M^*$ related to the regression vector; and $f_I = V^T(n-1)\varphi(n)$ and $g_I = D(n-1)f_I^*$ related to the instrumental vector. The asterisk denotes the complex conjugate. Finally one can define $\beta(n) = 1 + f_M^T g_I$

Thus, at the rank one update relation, the matrix $C(n)$ can be expressed:

$$C(n) = U(n)D(n)V^H(n) \tag{14}$$

$$= U(n-1)\left[D(n-1) - \frac{g_I g_M^H}{\beta(n)}\right] V^H(n-1)$$

The bracket part of $C(n)$ is decomposed trough a series of orthogonal transformations to obtain:

$$D(n-1) - \frac{g_I g_M^H}{\beta(n)} = \bar{U}(n)\bar{D}(n)\bar{V}^H(n) \tag{15}$$

$$C(n) = U(n-1)\bar{U}(n)\bar{D}(n)\left((V(n-1)\bar{V}(n)\right)^H$$

This relation recursively update the $U(n-1)$ and $V(n-1)$ matrices to $U(n)$ and $V(n)$. With this augmented structure, a multiple model structure is produced. The model parameters are calculated in the above diagonal of the columns in $U(n)$. The parameters related to the instrumental model (say $\alpha(n)$) are available in $V(n)$. The inverse of the generalized loss function $J(n)$ is available in the diagonal of the matrix —D(n)—. In expression (13), a forgetting factor λ can be introduced to follow the non stationarity of the signal.

B. Hybrid Methods

This method uses both parametric AR and WVD computations. It will be referred to as AR Wigner Ville Distribution or AWV. In fact, the WVD and it variants are Fourier transforms of an adequate kernel. So for frequency modulated signal such as a Doppler signal, an improvement in accuracy is expected by replacing the Fourier transform of the bilinear kernel by high resolution methods [12] such AR methods presented above. The principle of the computation is following:

- compute the bilinear kernel from the analytical signal.
- estimate, each time, the AR parameter fitting the real part of the kernel (since the kernel has Hermitian symmetry, the imaginary part does not contain additional information) similarly to eq.6.
- compute the cross-section of the WVD using eq.(6)

III. Frequency estimation for velocity measurement

The principle of the Doppler velocity measurement is to access the frequency components of the Doppler signal. Many frequency estimators have been widely explored. These estimators can be classified into two groups : the fast ones, which are directly based on the temporal signal such as the derivative of the signal phase [11] or the autocorrelation of the signal [14],[15],[16]. These estimators are known to be highly sensitive to (even little level of) noise. The other ones, more accurate but computationally heavy are based on a function of the signal. Here we consider estimation based on time frequency distributions since they have been shown to be more accurate [17],[18]. The frequency estimation commonly used to access Doppler flow velocity is the peak frequency) of the Time Frequency Distribution (TFD), which is defined by:

$$f_m(t) = arg \max_f (W(t,f)) \tag{16}$$

where $W(t,f)$ is the TFD. This estimator is very sensitive to the nature of the noise.

It is the value of f so that $W(t,f)$ is maximum. This estimator may be sensitive to local maximum in the TFD. So care must to taken to use it. When parametric TFD are used, a pole frequency estimator can be used to access the velocity. Considering expression(6), PDS_x can be rewritten as :

$$PSD_x(n,f) = \frac{\sigma^2}{|1 + \sum_{k=1}^{p} a_n(k) z^k|^2_{z=\exp(-2j\pi f)}} \tag{17}$$

or by using the poles z_k (that is the roots of the denominator) of eq.(17)

$$PSD_x(n,f) = \frac{K}{\prod_{k=1}^{p} |z - z_k|^2_{z=\exp(-2j\pi f)}} \tag{18}$$

where K is a constant related to σ^2. Thus PSD_x is the sum of first order elementary TFD, the peak frequency of

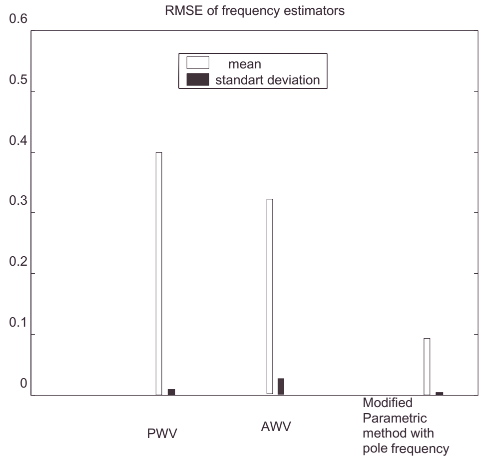

Fig. 3.

Relative Mean Square Error of the frequency estimates using 50 realizations. The x-axis show the distribution names.

which is given by :

$$f_k(t) = \frac{fe}{2\pi} \arg(z_k) \qquad (19)$$

These frequencies $f_k$must be sorted to access flow velocity

IV. Results and Discussion

In fig.(3), we show the root mean square error (rmse) for standard pseudo wigner ville (PWV) method and the two proposed methods. As it can be seen the new techniques are better as far as rmse is concerned. More simulation and industrial signals results confirming this preliminary observations will be shown during presentation.

V. Conclusion

In this paper the specific problem of Doppler spectrum and frequency estimation in presence of colored noise with flat spectrum has been addressed. Due to the specific nature of noise, two relevant techniques are proposed as alternative to unsuitable classically used techniques.These techniques outperform the classical ones.

References

[1] J.L.Garbini, F.K.Forster, and J.E. Jorgensen, "Measurement of fluid turbulence based on pulsed ultrasound techniques part 1 and 2 analysis and experimental investigation," *J.Fluid.Mech.*, vol. 118, pp. 445–505, 1982.

[2] F. Wendling, S.A. Jones, and D.P. Giddens, "Simulation of doppler ultrasound signals for laminar, pulsatil, nonuniform flow," *Ultrasound in Med. & Biol.*, vol. 18, pp. 179–193, 1992.

[3] T. Soderstrom and P. Stoica, "Comparison of some instrumental variable methods - consistency and accuracy aspects," *Automatica*, vol. 17, pp. 101–115.

[4] L Ljung, T T. Soderstrom, and P. Stoica, *Practice of Recursive Identification*, MIT Press, Cambridge, MA,, 1983.

[5] T T. Soderstrom and P. Stoica, *Instrumental Variable Methods for System Identification*, Springer, New York, 1983.

[6] H. Akaike, "A new look at statistical model identification," *IEEE Trans. Aut. Contr.*, vol. 19, pp. 716–723, 1974.

[7] G.J.Bierman and C.L.Thornton, *Covariance Factorisation for Kalman filtering*, Control and Dynamic Systems, 1980.

[8] JP. Reménieras, D. Kouame, JM. Grégoire, and F. Patat, "Multiphase pipe flow velocity measurement in strong colored noisy doppler ultrasound," in *EEE Ultrasonic Symp.*, 1996.

[9] D. Kouamé, J.M. Girault, V. Labat, and A. Ouahabi, "Delta high order cumulant-based recursive instrumental variable algorithm," *IEEE Sig. Proces. Let.*, vol. 7, pp. 262–265.

[10] S. Niu, D.G. Fisher, and D. Xiao, "A factored form of instrumental variable algorithm," *Int. Journ. of . Adapt. Cont. and Sig. Proc*, vol. 7, pp. 261–273.

[11] S.M. Kay, "Statistically / computationally efficient frequency estimation," *Proc. ICASSP'88, New-York*, pp. 2292–2295, 1988.

[12] B.Boashash, *Signals Analysis Processing,*, Advanced In Spectrum Analysis and Array Time-Frequency Simon Hakyn Editor Prentice Hall NJ, 1991.

[13] G.J. Bierman, *Factorization Methods for Discrete Sequentiel Estimation*, New York Academic, 1977.

[14] C. Kasai, K. Namekawa, A. Koyane, and R. Omoto, "Real-time two-dimensional blood flow imaging using an autocorrelator technique," *IEEE Trans. Sonics Ultrasonics*, vol. 32, pp. 458–463, 1985.

[15] B.A.J Angelsen and K. Kristoffersen, "Discrete time estimation of the mean doppler frequency in ultrasonic blood velocity measurements," *IEEE Trans. Biomed. Eng.*, vol. 30, pp. 207–217, 1983.

[16] K. Kristoffersen, "Time-domain estimation of the center frequency and spread spectra in diagnostic ultrasound," *IEEE Trans. Ultra. Ferro. Freq. Contr.*, vol. 35, pp. 484–497, 1988.

[17] D. Kouamé, J.P. Remenieras, J.F. Roux, A. Ouahabi, and M. Lethiecq, "A modified parametric estimate applied to ultrasound doppler velocity measurement," in *Proc. IEEE IMT.Conf., Bruxelles,*, 1996.

[18] J.M. Girault, D. Kouamé, A. Ouahabi, and F. Patat, "Estimation of the blood doppler frequency shift by a time-varying parametric approach," *Ultrasonics*, vol. 38, pp. 682–687, 2000.