# Coarse First Arriving Path Detection for Subscriber Location in Mobile Communication Systems 

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#### Abstract

The objective of this paper is to determine the potentialities of a detection scheme within the framework of subscriber location. Usually, it is the first arrival that bears the necessary information for user location. In NLOS situations, this first arrival is very much attenuated with respect to the RAKE synchronisation time instant, and it is placed well before this point. The determination of a window comprising this point is a must for a later use of a high resolution technique. Using a Generalized Likelihood Ratio Test (GLRT), an improved coarse first arriving path detector from propagation channel estimates is derived. Furthermore, an expression for false alarm probability is provided and detector performance is evaluated for different receiver configurations and signal propagation conditions.


## I. INTRODUCTION

From the viewpoint of subscriber location, accurate estimates of Time of Arrival (TOA) from received signal are required, and in order to use angular information, a proper first path detection acquires special relevance. See [1], [2] for details. However, first path arrived to the receiver may not necessarily be the one bearing the highest power. In the NLOS case, for i.e. the first arrival may suffer attenuation higher than other later arrivals, receiver is usually synchronized to the highest power path and therefore will provide a wrong TOA information. For the case of Code Division Multiple Access (CDMA) Spread Spectrum Systems such as W-CDMA a pilot channel or training sequences are provided, allowing channel estimation. These estimates are used to demodulate data channels and feed RAKE receivers. However RAKE receivers rely most powerful signal arrivals. As mentioned, these do not necessarily include first arrival path, in particular for NLOS situations.

The scheme proposed in this paper consists in searching the first arriving path from a set of vector channel estimates obtained from multiple receiving antennas, and computed from correlation measurements over CPICH downlink channel (suitable for OTDOA positioning) or DPCH in uplink or downlink (for RTT measurements) [4]. For this purpose, a lag window before the first RAKE finger component is studied, and
an statistical test is performed to discriminate properly between noise and signal. See [6] for an approach similar to this contribution but based on the maximum power arrival. The finding of a first path allows the determination of a window over which high resolution techniques could be used to obtain better accuracies [] (see figure 1). It will be assumed that the receiver has previously obtained slot, frame and frequency synchronisation.


Figure 1: First arrival path detection block diagram.

## II. ESTIMATED CHANNEL MODEL

With these premises, the vector of observations will be a collection of channel impulse response vectors infected with noise (when there is a signal arrival) or simply noise (when no signal is present at a certain lag).

After matched filter, the estimated channel sampled at chip rate may be modeled as:

$$
\begin{align*}
& \hat{\mathbf{h}}\left(\tau_{o} ; t\right)=\mathbf{h}\left(\tau_{o} ; t\right)+\mathbf{n}(t)  \tag{1}\\
& \mathbf{h}\left(\tau_{o} ; t\right)=\alpha \exp \left(j \omega_{d} t\right) \mathbf{a}+\mathbf{w}(t)
\end{align*}
$$

where the first term in the summation accounts for possible LOS component, $\omega_{\mathrm{d}}$ is the Doppler frequency, a is the array steering vector and the $k$ th element is expressed as follows:

$$
\begin{equation*}
a_{k}=\exp \left[-j \frac{2 \pi d_{k}}{\lambda} \sin \left(\theta_{o}\right)\right] \tag{2}
\end{equation*}
$$

and $\mathbf{w}$ models the scattered signal component. The term $\mathbf{n}$ accounts for the estimation noise and is assumed to be a temporally stationary, complex Gaussian random process, temporally uncorrelated and independent of the channel vectors.

Stacking the vector impulse response in time for each lag $\tau_{0}$ :

$$
\begin{align*}
& \mathbf{h}\left(\tau_{o} ; n\right)= \\
& =\left[\begin{array}{llll}
\hat{\mathbf{h}}\left(\tau_{o} ; n T_{s}\right)^{T} & \hat{\mathbf{h}}\left(\tau_{o} ;(n-1) T_{s}\right)^{T} & \cdots & \hat{\mathbf{h}}\left(\tau_{o} ;(n-p+1) T_{s}\right)^{T}
\end{array}\right]^{T} \\
& \mathbf{n} \rightarrow C N\left(\mathbf{0}, \sigma_{n}^{2} \mathbf{I}\right) \\
& \mathbf{h} \rightarrow C N\left(\mathbf{0}, \mathbf{R}_{n}+\sigma_{n}^{2} \mathbf{I}\right) \tag{3}
\end{align*}
$$

where $T_{\mathrm{s}}$ corresponds to the time interval between two consecutive estimations, (if using the CPICH in UMTS, it is the slot time) and $K$ is the number of estimates (or a given number of slots). $p$ is the duration in number of slots of the temporal correlation. of impulse responses.
$\mathbf{R}_{\mathbf{h}}$ is the channel vector correlation matrix from estimates, expressed in more general form by (4), and $\sigma_{\mathrm{n}}{ }^{2}$ is the noise variance:

$$
\begin{equation*}
\mathbf{R}_{\mathbf{h}}=\mathbf{R}_{\phi} \otimes \mathbf{T}_{\mathbf{k}}+\sigma_{n}^{2} \mathbf{I} \tag{4}
\end{equation*}
$$

$\mathbf{T}_{\mathbf{k}}$ is the temporal correlation matrix among the channels estimated in different slots, $\mathbf{R}_{\phi}$ contains the correlation coefficients between sensors, and $\otimes$ is the Kronecker product operator. Note that we are implicitly including the LOS component as a rank-one term in the correlation matrix.

Note that we are implicitely assuming that the TOA values have long coherence window times, much longer than the channel amplitudes coherence time.

## III. GENERALIZED LIKELIHOOD RATIO TEST

With the goal of determining the first signal arrival, we compute and arrange all estimated channels within a temporal window of size $K T_{s}$, on $\mathbf{X}\left(\tau_{\mathrm{o}}\right)$ matrix. The two possible conjectures are that observed data is just noise (hypothesis $H_{o}$ ) or that signal plus noise is present (alternative $H_{1}$ ). See [3]:

$$
\begin{aligned}
& H_{o}: \mathbf{X}\left(\tau_{o}\right)=\mathbf{N} \\
& H_{1}: \mathbf{X}\left(\tau_{o}\right)=\mathbf{H}\left(\tau_{o}\right)
\end{aligned}
$$

where $\mathbf{X}, \mathbf{N}$ and $\mathbf{H}$ matrices have $K$ rows and $p N_{s}$ columns, and results of rearranging data, noise and impulse response estimates vectors respectively as follows:

$$
\mathbf{X}\left(\tau_{o}\right)=\left[\begin{array}{c}
\mathbf{h}^{T}\left(\tau_{o} ; n\right) \\
\mathbf{h}^{T}\left(\tau_{o} ; n+p\right) \\
\vdots \\
\mathbf{h}^{T}\left(\tau_{o} ; n+K-p\right)
\end{array}\right]
$$

A Generalized Likelihood Ratio Test (GLRT) Detector decides $H_{1}$ if the likelihood ratio $\mathrm{L}\left(\mathbf{X}\left(\tau_{0}\right)\right)$ exceeds a threshold $\gamma$. It maximizes detection probability for a given false alarm probability, and may be expressed as shown in (5):

$$
\begin{equation*}
L\left(\mathbf{X}\left(\tau_{o}\right)\right)=\frac{p d f\left(\mathbf{X}\left(\tau_{o}\right) / \hat{\mathbf{R}}_{\mathbf{h}} ; H_{1}\right)}{p d f\left(\mathbf{X}\left(\tau_{o}\right) / \hat{\sigma}_{n}^{2} ; H_{o}\right)}>\gamma \tag{5}
\end{equation*}
$$

This test is performed by first estimating ML signal parameters, as if signal were present, and then comparing likelihood of $H_{1}$ with the true parameters replaced for their estimates to that of $H_{0}$.

If temporal correlation between consecutive estimates is different to zero and below to one, we are treating the most general case of a Partially Coherent Distributed (PCD) source, and above expression leads to (6):

$$
\begin{equation*}
L(\mathbf{X})=\frac{1}{\hat{\sigma}_{n}} \operatorname{tr}\left(\mathbf{X}\left(\tau_{o}\right)^{H} \mathbf{X}\left(\tau_{o}\right)\right)-\log \left[\operatorname{det}\left(\mathbf{X}\left(\tau_{o}\right)^{H} \mathbf{X}\left(\tau_{o}\right)\right)\right]>\gamma^{\prime} \tag{6}
\end{equation*}
$$

Note that $\mathbf{X}\left(\tau_{\mathrm{o}}\right)^{\mathrm{H}} \mathbf{X}\left(\tau_{\mathrm{o}}\right) / N$ is an estimation of the vector channel correlation matrix and, combined with the trace operator, becomes an incoherent accumulation. This test has to be applied over different $\tau_{o}$ values within a certain window so as to assess a coarse instant of the first arrival. In the sequel the term $\tau_{o}$ will be removed from the equations.

## IV. FALSE ALARM PROBABILITY

When the noise level is available (as it is usually the case at the receiver), and it is used for noise variance ( $\sigma_{\mathrm{n}}{ }^{2}$ ) estimation, Constant False Alarm Rate (CFAR) detectors may be built [5]. Operating with expression (6) in the null hypothesis, it may be shown that false alarm probability $\mathrm{P}_{\mathrm{fa}}$ is described approximately by:

$$
\begin{equation*}
P_{f a}=\mathbf{Q}_{\chi_{2 p N_{s}}^{2}}\left\{\frac{2}{N-1}\left[\gamma^{\prime}+p N_{s}\left\{\log \left(N \sigma_{n}^{2}\right)-1\right\}\right]\right\} \tag{7}
\end{equation*}
$$

where $\mathbf{Q}_{\chi}($.$) defines the right tail cumulative function$ for a chi squared distributed variable with $2 p N_{s}$ degrees of freedom, when $K$ channel vectors and $N_{s}$ sensors are used, and provided that the number of secondary data is high enough. The proof is omited due to lack of space but may be provided under request. Expression (7) is of utmost importance since it allows the definition of a threshold in equation (6) for the verification of the hypothesis.

On the other hand, the probability of detection is given by:
$P_{D}=\sum_{n=0}^{p N_{s}-1} C_{n} \exp \left(-\frac{\gamma^{\prime}-p N_{S}+p N_{S} \log (N)+G}{\beta_{n}}\right)$
where,

$$
\begin{aligned}
& C_{n}=\prod_{i=0 ; i \neq n}^{N-1} \frac{1}{1-\beta_{i} / \beta_{n}} \\
& G=\sum_{i=0}^{p N_{S}-1} \log \left(\lambda_{i}\right) \\
& \beta_{n}=N \lambda_{n} / \sigma_{n}^{2}-1 \\
& \lambda_{n}=\lambda_{\phi_{i}} \lambda_{\tau_{j}} \quad \forall i=1: N_{s} ; j=1: p
\end{aligned}
$$

with $\lambda_{\phi}$ and $\lambda_{\tau}$ being the eigenvalues of $\mathbf{R}_{\phi}$ and $\mathbf{T}_{\mathbf{k}}$ respectively. When an adequate probability of false alarm has been defined by selecting a threshold from (7), probability of detection will be given by channel characteristics. Note that $P_{D}$ in (8) is a function of SNR, $K, N_{s}$, and temporal and spatial correlation.

## V. EVALUATING FIRST ARRIVAL DETECTABILITY

In order to evaluate the performance of this detector, the first arrived path is supposed to be confined within a temporal window of length $L$ samples before the first significant path available at RAKE receiver. Sampling is supposed to be at chip time. A first arrival component is generated and placed randomly within this window.


Figure 2: First arrival RMS detection error as a function of $P_{F A}$ and for different SNR. One sensor, and a high temporal correlation.

Path searching process defined by (6) is repeated along the window until alternative $\mathrm{H}_{1}$ is verified. If a new path is not detected, finger path is chosen as the earliest. $L$ has been set to $5 T_{c}$ for figures shown in this paper. Under the defined channel setup, mean square error is related to false alarm and detection probabilities through equation (9); where $p(n / m)$ corresponds to the probability of detecting an arrival at lag $n$ when arrival is located at lag $m$, and $\varepsilon_{\mathrm{q}}$ is an error term included due to the temporal quantization of the delay axis, as 1 sample per chip:

$$
\begin{align*}
E\left\{e^{2}\right\}=\frac{1}{L} & \sum_{m=1}^{L} \sum_{n=1}^{L} p(n / m) \cdot(m-n)^{2}+ \\
& +\left(1-P_{f a}\right)^{L-1}\left(1-P_{D}\right) \sum_{n=1}^{L} n^{2}+E\left\{\varepsilon_{q}^{2}\right\} \tag{9}
\end{align*}
$$

$p(n / m)= \begin{cases}P_{f a}\left(1-P_{f a}\right)^{n-1} & n<m \\ P_{D}\left(1-P_{f a}\right)^{n-1} & n=m \\ P_{f a}\left(1-P_{f a}\right)^{n-2}\left(1-P_{D}\right) & n>m\end{cases}$
$p(n / m)=p\left(\hat{\tau}_{o}=n T_{c} / \tau_{o}=m T_{c}+e_{q}\right)$
$\varepsilon_{q} \in\left[-1 / 2 T_{c}, 1 / 2 T_{c}\right]$
$\mathbf{R}_{\phi}$ has been built using the distributed source model proposed in [7], and for Monte Carlo Simulations $10^{5}$ realization were used to evaluate Detection Probability and $10^{6}$ to evaluate false alarm. For a highly correlated spatial distribution (as it would be the case in the
uplink), the case of correlated sensors an angular spread of 5 degrees and a mean direction of arrival of zero degrees is supposed. Temporal correlation of the scattering was simulated using a first order AR process. Correlation factors of 0.1 and 0.999 are used, accounting for moving ans statis terminals respectively. Direct path arrives from broadside and sensors are linearly and uniformly spaced. Doppler frequency for direct path corresponds to a mobile speed of $50 \mathrm{~km} / \mathrm{h}$, $N=100$ and $p=15$ for figures 2-3 and 6 .


Figure 3: First arrival RMS detection error as a function of $P_{F A}$ and for different powers of the direct path. One sensor, and high temporal correlation.

Figure 2 shows the RMS error in determination of the first arrival for different values of the false alarm probability. Even with just one sensor and a poor SNR of 0 dB (after correlation by the pilot sequence) for the first arrival a good accuracy is achieved when a false alarm around $10^{-3}$ is chosen. At higher values of SNR the error is not very sensitive to the value of the threshold. Figure 3 shows how detection improves in a highly correlated scenario when a direct path is present, compared to raw scattering.

Figure 6 shows that temporal uncorrelation of the scattering term is a beneficial factor in all cases.


Figure 4: First arrival RMS detection error as a function of $P_{F A}$ and the number of sensors, for spatially correlated and uncorrelated sources.

Increasing the number of sensors or the data record length leads in some cases to important detection gains as it can be seen in figures 4-5, and weaker signals may be detected.


Figure 5: First arrival RMS detection error as a function of $P_{F A}$ and data record length. Different values of the assumed duration of the temporal correlation are used.

## VI. SUMMARY AND CONCLUSSIONS

A method for improving subscriber location accuracy has been described. A CFAR detector for PCD sources has been derived in (6), and some results have been shown for different environments and detector configurations. It has been shown that given a low SNR and a false alarm probability, by enhancing data records or increasing the number of sensors, better results are observed for temporally uncorrelated sources and uncorrelated sensors.

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Figure 6: First arrival RMS detection error as a function of $P_{F A}$ and for different temporal correlation values and direct path powers.

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