KALMAN SMOOTHING BASED CHANNEL ESTIMATION FOR SPACE-TIME BLOCK CODING

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ABSTRACT

This paper addresses the problem of channel estimation and tracking for space-time block coding. The coding/decoding technique allows using Kalman smoothing in estimating time-selective fading channels. In the simulations it is shown that this technique can preserve the orthogonality property of the decoding matrix even for time-selective channels. A realistic channel model described in COST 207 project is used in our examples.

1 Introduction

Time-varying multipath fading is a fundamental phenomenon which makes the wireless transmission difficult. In most scattering environments, antenna diversity is a practical technique for reducing the effect of multipath fading [5]. Space-Time block coding (STBC) is a simple diversity technique which improves the signal quality at the receiver by simple processing across two antennas at the transmitter. This type of coding/decoding scheme introduced by Alamouti [1], requires channel estimates at the receiver in order to decode the transmitted symbols.

Smoothing is defined as a more general estimation problem in which the state at some or all times in the total sampling interval is estimated from data obtained at time that both precede and succeed the time being considered [7]. From the point of view of the channel tracking algorithm, which operates at each time index, the structure of STBC codes creates a delay of one symbol. Taking advantage of this, single stage smoothing can be applied in order to estimate and track the channel. This smoothing technique leads to improved estimates compared to regular Kalman filtering.

The problem of channel tracking for STBC was also investigated in [6] where regular Kalman filter is used. In this paper we develop a novel decoding scheme for Alamouti's space time (ST) code in time-selective channels. Kalman smoothing is used to estimate and track the channel. A detailed description of how to link the symbol decoding stage with channel tracking stage is also presented. Extensive simulations show the ability of the smoothing technique to give good estimates of the channel in scenarios where time-selective fading channels were generated based on the channel model derived in COST 207 project [3].

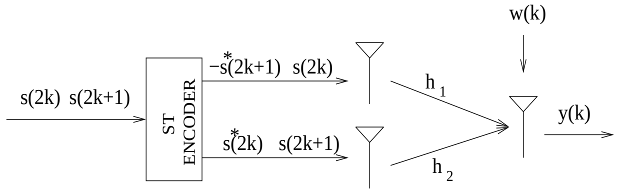

Figure 1: ST transmission diagram

The rest of the paper is organized as follows. In the next section we present the system model and the coding/decoding technique. In section three we show how Kalman smoothing can be used for channel estimation and tracking and we present how to link the symbol decoding stage with the channel estimation stage. Simulation results are presented in section four. In section five some conclusions are drawn.

2 System model

2.1 Coding

The coding scheme suggested by Alamouti works in the following manner. The original symbol stream is divided in two separate symbol streams which are then transmitted using two antennas. We receive the following signal y:

$$\begin{aligned} y_{2k} &= s_{2k}h_{1,2k} + s_{2k+1}h_{2,2k} + w_{2k} \\ y_{2k+1} &= -s^*_{2k+1}h_{1,2k+1} + s^*_{2k}h_{2,2k+1} + w_{2k+1} \end{aligned} \quad (1)$$

where $h_{1,2k}$ and $h_{2,2k}$ denotes the channel taps h_1 and h_2 at moment $2k$, s is the transmitted symbol, w is additive white Gaussian noise (AWGN) and $*$ denotes complex conjugate. The transmission model using STBC is presented in Figure 1.

After applying complex conjugation operation on the received signal at time index $2k+1$ we have the following vector model:

$$\mathbf{y} = \mathcal{H}\mathbf{s} + \mathbf{w} \quad (2)$$

where $\mathbf{y} = \left[y_{2k} \;\; y^*_{2k+1}\right]^T$ contains the received signal, $\mathbf{s} = [s_{2k} \;\; s_{2k+1}]^T$ represents the transmitted information, $\mathbf{w} = \left[w_{2k} \;\; w^*_{2k+1}\right]^T$ represents the additive Gaussian noise and the matrix $\mathcal{H}$ can be written in the following form:

$$\mathcal{H} = \begin{bmatrix} h_{1,2k} & h_{2,2k} \\ h^*_{2,2k+1} & -h^*_{1,2k+1} \end{bmatrix} \tag{3}$$

2.2 Decoding

Based on the model (2) we start by forming the matrix $\mathcal{H}$ as shown in (3). To estimate the transmitted symbols $\mathbf{s}$ we compute the soft estimate $\mathbf{z} = [z_{2k} \;\; z_{2k+1}]^T$ as follows:

$$\begin{bmatrix} z_{2k} \\ z_{2k+1} \end{bmatrix} = \mathcal{H}^H \begin{bmatrix} y_{2k} \\ y^*_{2k+1} \end{bmatrix} \tag{4}$$

The independent estimation of the transmitted symbols $\hat{s}_{2k}$ and $\hat{s}_{2k+1}$ is possible due to the fact that the outputs z_{2k} and z_{2k+1} are decoupled. This is due to the orthogonality property of the matrix $\mathcal{H}$, thus $\mathcal{H}^H\mathcal{H}$ is diagonal. The symbols $[\hat{s}_{2k} \; \hat{s}_{2k+1}]^T$ are estimated from the vector $[z_{2k} \;\; z_{2k+1}]^T$ using a decision device, actually this is the ML receiver in the case of flat fading channels. The orthogonality condition of $\mathcal{H}$ is the key property that must be preserved both at the transmitter and receiver in order to achieve error free decoding. To attain this goal, good estimates of the channel must be available at the receiver, which is our concern in this paper. The decoding technique can be summarized as follows:

1. Predict $\hat{\mathbf{h}}_{2k|2k-2}$ $\hat{\mathbf{h}}_{2k+1|2k-1}$
2. Form the matrix $\mathcal{H}$ as described in (3)
3. Decode $\hat{s}_{2k}$ and $\hat{s}_{2k+1}$ as shown in (4)

where $\hat{\mathbf{h}}_{2k|2k-2}$ represents the channel estimate at time $2k$ based on the observations at time $2k-2$. A prediction of the channels may not be possible, in this case one can use the previous estimated values, i.e. $\hat{\mathbf{h}}_{2k|2k-2} = \hat{\mathbf{h}}_{2k-2|2k-2}$ and $\hat{\mathbf{h}}_{2k+1|2k-1} = \hat{\mathbf{h}}_{2k-1|2k-1}$.

3 Channel Estimation and Tracking

Let us start this section by making the following clarification: the coding/decoding technique works over two consecutive symbols, $2k$ and $2k+1$, whereas the channel tracking algorithm has to operate at each time step $2k$. In this section we view the sytem from the tracking algorithm perspective. The set of equations (1) can be written in the following vector form:

$$y_{2k} = \bar{\mathbf{s}}^T\mathbf{h}_{2k} + w_{2k} \tag{5}$$

where $\mathbf{h}_{2k} = [h_{1,2k} \;\; h_{2,2k}]^T$ is a vector of channel taps, and the vector $\bar{\mathbf{s}}$ is given by the following rule: at even time indices we have $\bar{\mathbf{s}} = \mathbf{s}^{even} = [s_{2k} \;\;\; s_{2k+1}]^T$, at odd time indices we have $\bar{\mathbf{s}} = \mathbf{s}^{odd} = \left[-s^*_{2k+1} \;\;\; s^*_{2k+1}\right]^T$. Let us also assume that the channel taps evolve over time according to the following state equation:

$$\mathbf{h}_{2k} = A\mathbf{h}_{2k-1} + \mathbf{v}_{2k} \tag{6}$$

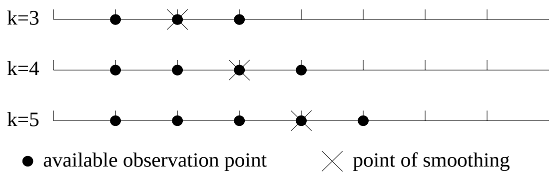

Figure 2: Fixed lag smoothing scheme for L=1

where $\mathbf{v}_{2k}$ is considered to be AWGN with covariance Q and A is the state transition matrix which is an identity matrix in this case. Equations (5) and (6) form the state-space equations.

The Kalman filter provides an estimate of the state based on all observations, the smoothing technique estimates the state using the observations up to and after the current state. Thus, a delay is introduced between the available data and the smoothed estimate. Meditch [7] introduced a classification of smoothing into *fixed-interval*, *fixed-point*, and *fixed-lag* problems. The last technique fits to our problem, meaning that $\hat{\mathbf{h}}_{2k|2k+L}$ an estimate of $\mathbf{h}_{2k}$ based on the observations up to $2k+L$, where L is the lag, as shown in Figure 2.

We shall proceed by investigating the available information in Alamouti's scheme for two consecutive time indices, $2k$ and $2k+1$. Note that some information is already available at time $2k$, (i.e. the observation y_{2k+1}). Let us recall regular Kalman filter computes the prediction and correction error covariance matrices and also the Kalman gain:

- at time $2k$ we have/compute:
  - receive the data y_{2k} and y_{2k+1}
  - based on the previous channel estimates we compute $\hat{s}_{2k}$ and $\hat{s}_{2k+1}$
  - using Kalman filtering we compute: $P_{2k|2k-1}$, $P_{2k|2k}$, K_{2k}, $\hat{\mathbf{h}}_{2k|2k-1}$ and $\hat{\mathbf{h}}_{2k|2k}$

- at time $2k+1$:
  - using Kalman filtering we compute: $P_{2k+1|2k}$, $P_{2k+1|2k+1}$, K_{2k+1}, $\hat{\mathbf{h}}_{2k+1|2k}$ and $\hat{\mathbf{h}}_{2k+1|2k+1}$

Consequently, by inspecting the processing steps above we observe that at time $2k$ we have all the Kalman filter parameters available and also *future* information: y_{2k+1} and $\hat{s}_{2k+1}$. Smoothing can be applied. *Single-stage smoothing*, which is equivalent with fixed-lag smoothing for delay one. This means that before applying the Kalman filter at step $2k+1$ we can compute a smoothed estimate of $\hat{\mathbf{h}}_{2k|2k}$ which is given by $\hat{\mathbf{h}}_{2k|2k+1}$. Unfortunately, single-stage smoothing can be applied only at even time $2k$ since at the time $2k+1$ we do not have available *future* information needed by the smoother, i.e. y_{2k+2} and $\hat{s}_{2k+2}$. Hence, smoothed channel taps will be available only at even time indices and at odd time indices we rely on filtered estimates. However, the prediction of the channel taps at even time indices is based on the smoothed estimate from the previous odd time step.

The change in the decoding scheme is the following: when building the matrix $\mathcal{H}$, smoothed channel taps $\hat{h}^s_{1,2k|2k-1}$ are used instead of filtered taps $\hat{h}_{1,2k|2k-2}$. The smoothed taps are computed as follows: first the prediction error covariance matrix for time index $2k+1$ is computed:

$$P_{2k+1|2k} = AP_{2k|2k}A^T + Q \tag{7}$$

Then, we calculate the smoothing filter gain matrix $K^s_{2k|2k+1}$:

$$K^s_{2k|2k+1} = P_{2k|2k}A^T\hat{\mathbf{s}}^{odd}\left[\left(\hat{\mathbf{s}}^{odd}\right)^T P_{2k+1|2k}\hat{\mathbf{s}}^{odd} + \sigma_w^2\right]^{-1} \tag{8}$$

The smoothed estimate is given by:

$$\hat{\mathbf{h}}^s_{2k|2k+1} = \hat{\mathbf{h}}_{2k|2k} + K^s_{2k|2k+1}\left[y_{2k+1} - \left(\hat{\mathbf{s}}^{odd}\right)^T A\hat{\mathbf{h}}_{2k|2k}\right] \tag{9}$$

4 Simulations

The channel impulse response used in simulations is a time-selective fading channel, i.e. time-selective but frequency flat. It is obtained as a superposition of $N_p = 100$ paths and can be described using the Gaussian distributed *Wide-Sense Stationary with Uncorrelated Scattering* (WSSUS) model [4]: $h_i(t) = 1/\sqrt{N_p}\sum_{p=1}^{N_p} e^{j(2\pi f_{d,p}t+\theta_p)}h_{RF}(t)$, where $f_{d,p}$ is the Doppler spread, θ_p is the angular spread and $h_{RF}(t)$ is the impulse response of the receive filter. This model is suitable for many channels of practical interest in mobile wireless communications [3]. Four propagation environments are widely used: Typical Urban (TU), Bad Urban (BU), Hilly Terrain (HT) and Rural Area (RA).

The simulations are performed according to ETSI [8] recommendation for Enhanced GPRS (EGPRS). Regarding the channels types to be used, the following scenarios are suggested: *1st scenario* Indoor/Low range outdoor where a TU channel can be used with a receiver speed of 3 km/h, up to 10 km/h. *2nd scenario* is about urban environment characterized by TU at 50 km/h up to 150 km/h and suburban outdoor where we use HT channel at 100 km/h. The *3rd scenario* defines the Rural outdoor characterized by the RA-type of channel with the receiver speed in the range 130-250 km/h and HT at 100 km/h. Linearized GMSK [2] modulation is used in our simulations. A training sequence of 15 symbols is used for initialization, after that the algorithm runs in decision directed mode.

In the first simulation a RA-type of channel is used, with a receiver speed of 200 km/h. The channel estimation is performed using Kalman filter and also smoothing is considered at even time indices. The results are averaged over 100 realizations. The SER is depicted in Figure 3.

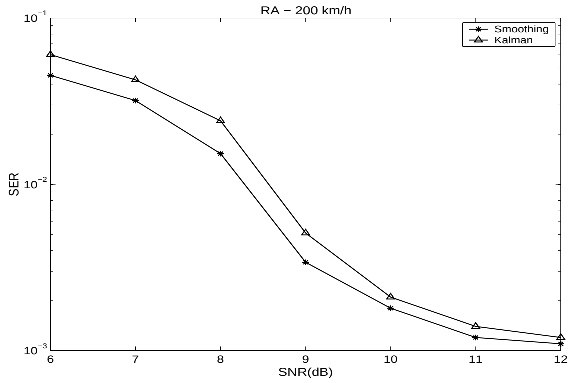

Figure 3: Comparison of Kalman and smoothed Kalman tracking

In the following example we compare the channel obtained using Kalman filtering and smoothing. The estimation error is computed by subtracting the real part of the estimated channel from the real part of the true channel. The same operation is done for computing the smoother's estimation error. For making the comparison more simple, the variance of the error is computed. The results are averaged over 100 realizations and they are depicted in Figure 4.

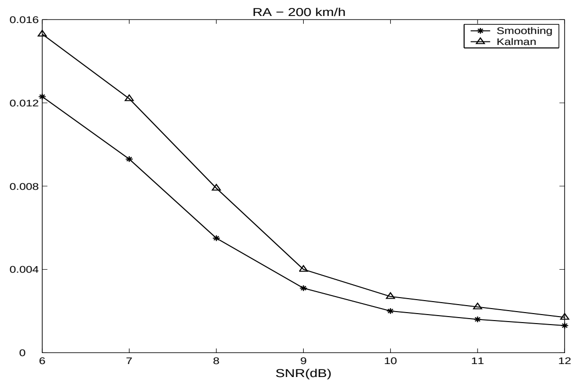

Figure 4: Variance of the error between the real part of the true channel and the real part of Kalman and smoothed Kalman, respectively.

Alamouti's coding/decoding scheme is based on the property of the channel matrix $\mathcal{H}$, orthogonality, i.e. $\mathcal{H}^{\mathcal{H}}\mathcal{H}$ should be diagonal. At each time step $2k$ we compute the product $\mathcal{P} = \mathcal{H}^{\mathcal{H}}\mathcal{H}$. The off-diagonal complex terms of $\mathcal{P}$ are squared and added. Let us call the absolute value of this sum the *performance index*. In the ideal case of perfect channel estimation the performance index must be zero for any time index $2k$. The most important goal of channel tracking is to preserve the orthogonality property of $\mathcal{H}$.

In the next example we want to emphasize the benefits of using the smoothing technique in a demanding scenario [1]. RA-type of channel with a receiver speed of 200 km/h is used at SNR 8 dB. This simulation is in the third category suggested by ETSI. Three algorithms are compared in channel estimation and tracking: Kalman smoothing, regular Kalman and RLS. The performance index is calculated for every simulation and averaged over 100 realizations. The boxplot of the averaged performance indices is depicted in Figure 5. The box has lines at the lower quartile, median, and upper quartile values. The lines extending from each end show the extent of the rest of the results with outliers presented as crosses. It can be observed that the smoothing technique has the most stable performance.

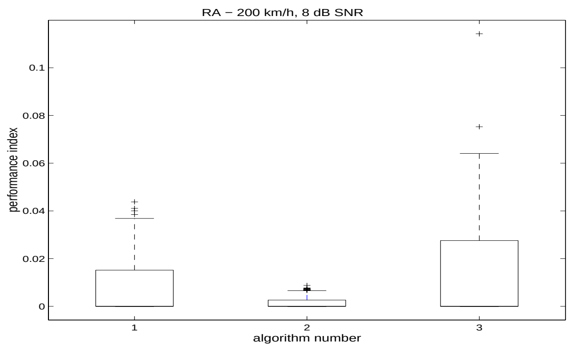

Figure 5: Performance index based on the orthogonality of $\mathcal{H}$. Algorithm numbers correspond (in order): Kalman, Kalman smoothing, RLS

In the next simulation we focus on the second ETSI scenario. We use regular Kalman, smoothing Kalman and RLS. The channel type is HT with a receiver speed of 100 km/h and SNR 8 dB. The performance index is shown in Figure 6. We observe that RLS has a satisfactory performance. Thus, it could be used in not so demanding scenarios. Finally, in the most easy scenario proposed by ETSI, a TU channel is used with a receiver speed of 30 km/h and SNR 5 dB. The tracking

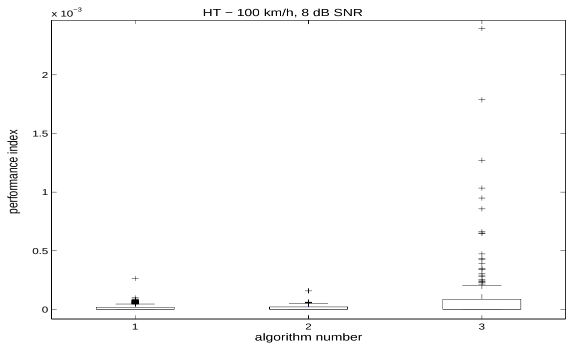

Figure 6: Performance index based on the orthogonality of $\mathcal{H}$. Algorithm numbers correspond (in order): Kalman, Kalman smoothing, RLS

[1] By *demanding scenario* we understand a receiver with high speed in a difficult environment at low SNR.

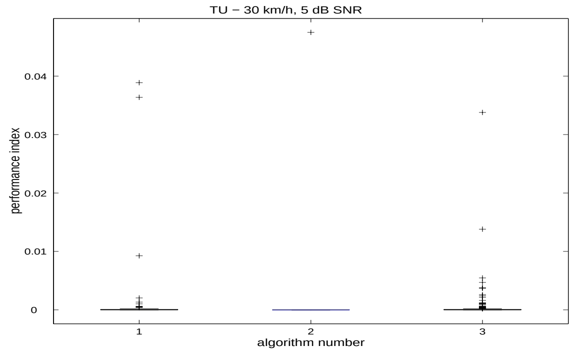

Figure 7: Performance index based on the orthogonality of $\mathcal{H}$. Algorithm numbers correspond (in order): Kalman, Kalman smoothing, RLS

algorithms are regular Kalman, smoothing Kalman, and RLS. The performance index is shown in Figure 7.

5 Conclusions

In this paper a new approach on channel estimation and tracking for space-time block coding is introduced. The delay in coding/decoding technique introduced by Alamouti is exploited by us in Kalman smoothing for channel estimation and tracking. We have shown by simulations that the orthogonality property of the channel matrix $\mathcal{H}$ is preserved in high demanding scenarios by using the smoothing technique.

References

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