# PROGRAMMABLE MODIFIED FRACTIONAL COMB DECIMATION FILTER

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## ABSTRACT

In multistandard radio receivers, the hardware should be configurable or programmable for the reception of different types of signals having different symbol rates. The decimation by a non-integer factor becomes a critical functionality of the multistandard radio. The Cascaded Integrator-Comb (CIC) filters are commonly used for decimation by an integer. By using polynomial interpolation filter between integrator and comb stages of CIC, and non-integer delay in the feed-forward branch of the comb stage, we achieve improved attenuation for the aliasing frequency components, and we make possible to use this type of structure for decimation by a non-integer factor. We name this structure as a programmable fractional CIC filter structure. This paper presents an efficient fractional structure for flexible decimation in the multistandard radio receivers. This structure is based on the so-called modified comb filter using the programmable fractional CIC principle. The main advantages of the proposed structure are flexibility and programmability with increased attenuation for aliasing frequency components.

## 1. INTRODUCTION

In multistandard receivers the hardware should be configurable or programmable for the reception of different types of signals having different symbol rates. After the AD conversion, utilizing commonly the delta-sigma AD-conversion principle and high oversampling ratio, the sampling rate is reduced to be an integer multiple of the symbol rate. The problem is in that the needed decimation factor can be a difficult fractional number, or even an irrational number. The frequency bands, which cause aliasing in decimation, should have good attenuation. Since the basic idea of the software radio is to support different system standards by common hardware platform, it is desired to have programmable decimator structure.

Furthermore, the overall implementation should be simple in order to have economic implementation and low power consumption, because the decimation filter is used in the digital front-end of mobile receivers where the sampling rate is high [1].

Cascaded integrator-comb (CIC) filters are commonly used for decimation by an integer factor providing efficient anti-imaging and anti-alas filtering [2]-[5]. The CIC filter has a simple, regular structure without multipliers. However, when the decimation factor is a non-integer number, the CIC filter cannot be directly used. One solution is to use CIC filters in a combination with an interpolation filter [3], [4]. The role of the interpolation filter is to perform fine tuning of sampling rate and to provide better attenuation for aliasing bands. The decimator structure presented in [4] consists of CIC filter having an interpolation filter between integrator and comb stages. We named this structure as a programmable fractional CIC filter. As it has been shown, it is possible to adjust the position of zeros in frequency response and attenuation of aliasing bands of the overall structure.

The role of this paper is twofold. First, we give generalized overview of the programmable fractional CIC filter with non-integer delay in the comb stage, and interpolation filter between integrator and comb stages. After that, we present an efficient decimator structure that is based on the mentioned method implemented within a modified comb filter structure. The modified comb filter has been presented by Saramäki and Ritoniemi in [6]. The CIC filters are useful for a narrowband desired signal, where the nulls of the filter are wide enough to protect the desired passband from aliasing distortion. The modified comb filter structure has wider nulls, thus they are useful for wider desired signal passband. Though the several multipliers may be required in the realization of the modified comb filter, the computational complexity is not increased. Under certain conditions those multiplications are realized as shift operations.

## 2. PROGRAMMABLE FRACTIONAL CIC FILTER

The CIC filters are commonly used for decimation and interpolation by an integer ratio providing efficient anti-imaging and anti-alias filtering [2]. The CIC architecture has main advantage in its simplicity, as CIC filter does not require any multiplier. The $N^{th}$ order CIC decimation filter
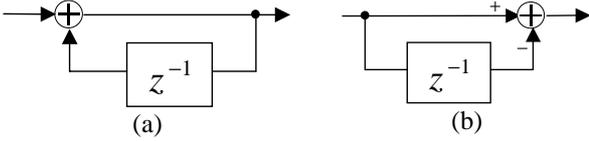
Fig. 1.(a) Integrator filter. (b) Comb filter.

consists of $N$ cascaded digital integrator stages operating at high input rate $F_{in}$, followed by $N$ cascaded comb or differentiator stages operating at low output sampling rate $F_{out}$, see Fig. 1. The frequency response of the $N^{th}$ order CIC filter of length $K$ is given by

$$H(e^{j2\pi f/F_{in}}) = e^{-jN\pi f(K-1)/F_{in}} \left( \frac{\sin\left(\pi f K / F_{in}\right)}{K\sin\left(\pi f / F_{in}\right)} \right)^N . \qquad (1)$$

The programmable fractional CIC filter, shown in Fig. 2, consists of $N$ integrator stages operating at input rate $F_{in}$, polynomial interpolation filter $h_a(t)$ (PIF in Fig. 2), resampler and $N$ comb stages operating at output sampling rate $F_{out}$. The role of polynomial interpolation filter is to provide the corresponding sample to the input of the comb stage. The interpolation filter provides also better attenuation of aliasing bands. It should be noted that the interpolation filter does not work all the time, as it prepares the samples for the output stages that work at the output sampling rate. The control logic of the interpolation filter is very similar to the one presented in [3]. In this way the workload is reduced as the interpolation filter in practical realization work at the output sampling rate. In the case of non-integer decimation factor $R = {}^{Fin}/_{Fout}$, we can realize the frequency response (1) by placing a non-integer delay $D$ in the feed-forward branches of the comb stages. $D$ is determined by desired length of moving average (CIC) filter $K$ and overall decimation ratio $R$ as $D = {}^K/_R$. The moving average filter length $K$ has influence on the frequency response of the overall structure. The value of $K$ determines the positions of the zeros in the overall frequency response. The frequency response of the overall system is a product of two frequency responses of the systems in cascade, that is

$$H_{ov}(e^{j2\pi f/F_{in}}) = H(e^{j2\pi f/F_{in}})H_a(jf), \qquad (2)$$

where $H_a(jf)$ is the frequency response of the interpolation filter. The attenuation of the aliasing bands can be improved and programmed either by increasing the order of the CIC filter $N$, or by adjusting the parameter $K$, that is $D$ in actual implementation.

## 2.1. The time domain conditions

We assume that the CIC filter uses the special modulo arithmetic, in which case it does not have stability or overflow problems. The CIC filter operates correctly only if it is implemented using a 'wrap around' number system, like the two's complement number system. With this kind
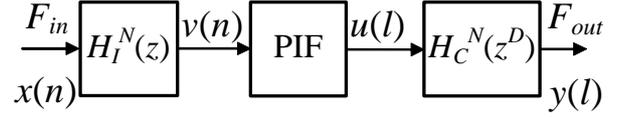


Fig. 2. The programmable fractional CIC filter.

of number system, overflow is modulo operation, i.e., what is actually stored in the register is the residue of the true value, see [5]. Therefore, all components of the proposed structure must support this type of number system. In the programmable fractional CIC filter, the critical issue is the polynomial interpolation filter because it does not give the correct sample value in case of overflow. The problem is avoided by using an interpolation method that is based on finite differences. For example, in the case of linear interpolation instead of using

$$y(l) = x(n_l) + \left[x(n_l + 1) - x(n_l)\right]\mu_l, \qquad (3)$$

the following expression is used

$$y(l) = x(n_l) + \Delta_l \mu_l, \qquad (4)$$

where $x(n_l)$ and $x(n_l+1)$ are the two adjacent input samples, $\Delta_l$ is the first finite difference, $y(l)$ is the resulting output sample, and all operations are modulo operations. If two's complement overflows occur in an integrator, the large difference in absolute value is of no consequence, using modulo arithmetic is still correct value. The necessary and sufficient condition for the filter to work for any input is that the number range must be the same within each stage of the structure.

## 3. MODIFIED CIC FILTER

The advantage of the proposed decimator structure is in that the integrator part does not require any multipliers thus it is computationally efficient first stage solution in VLSI multistage decimators. However, the main drawback of the CIC structure is in that the filter order $N$ increases fast when required stopband attenuation for the overall decimator is increased. This problem can be overcome by the structure presented by Saramäki and Ritoniemi in [6], so-called the modified comb filter structure. In this structure the CIC filter order $N$ is decreased by using a few additional interconnections. Combining the modified CIC filter idea and the programmable fractional CIC decimation idea we have obtained the efficient proposed decimator structure of Fig. 3. The optimization procedure that has been derived in [6] can also be applied to this case.

The proposed decimator structure can be further simplified by quantizing the coefficients $a_r$ to be integers possibly powers of two. When $a_r$ is a power of two, then shifters are used instead of multipliers. Even though overflows occur in the feedback loops of these structures, the output is correct provided that modulo arithmetic is used. The transfer function of an integrator shown in Fig. 3(b) is
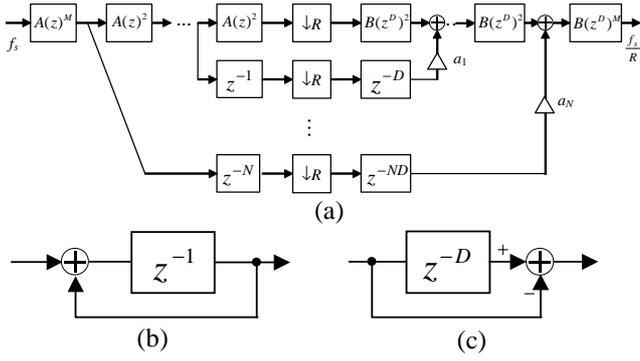
Fig. 3. (a) Modified CIC with non-integer delay. (b) Integrator filter with transfer function $A(z)$. (c) Comb filter with transfer function $B(z)$.

$$A(z) = \frac{z^{-1}}{z^{-1} - 1}, \qquad (5)$$

whereas the transfer function of a comb filter from Fig. 3(c) is given as

$$B(z) = z^{-D} - 1. \qquad (6)$$

However, the non-integer delay filter is not ideal and we have to take into consideration the actual approximation. Various methods for designing fractional delay filter are presented in [7]. As the non-integer delay approximation we use the FIR fractional delay filter designed using the Lagrange interpolation method [7], [8]. In this method the delay $z^{-D}$ is approximated by

$$H_D(z) = z^{-D} \approx \sum_{n=0}^{L} h_D(n) z^{-n}, \qquad (7)$$

where $L$ is the FIR filter length, and filter coefficients $h_D(n)$ have the explicit form as

$$h_D(n) = \prod_{k=0, k \neq n}^{L} \frac{D-k}{n-k} \quad n=0, 1,\ldots, L. \qquad (8)$$

The transfer function of the comb stage can be expressed as the transfer function of an FIR filter as follows

$$\hat{B}(z) = z^{-D} - 1 \approx h_D(0) + \sum_{n=1}^{L} h_D(n) z^{-n} - 1.. \qquad (9)$$

The transfer function of the overall structure that takes into account the FIR fractional delay filter is given as

$$E(z) = E_1(z) E_2(z), \qquad (10)$$

with

$$E_1(z) = \left[ z^{-1} \frac{\hat{B}(z^R)}{K(1-z^{-1})} \right]^M, \qquad (11)$$

and

$$E_2(z) = \left[ z^{-1} \frac{\hat{B}(z^R)}{K(1-z^{-1})} \right]^{2N}$$
$$+ \sum_{r=1}^{N} a_r z^{-r} H_D^r(z^R) \left[ z^{-1} \frac{\hat{B}(z^R)}{K(1-z^{-1})} \right]^{2(N-r)}. \qquad (12)$$

We start the derivation of the frequency response of the overall decimator structure by expressing the frequency responses of the building blocks as

$$E_1(e^{j2\pi f/F_{in}}) = e^{-j\left(M\pi f/F_{in} + \arg\left\{\hat{B}(e^{j2\pi f/F_{in}})\right\}\right)} \left( \frac{\left|\hat{B}(e^{j2\pi f/F_{in}})\right|}{K \sin\left(\pi f / F_{in}\right)} \right)^M, \qquad (13)$$

and

$$E_2(e^{j2\pi f/F_{in}}) = e^{j\arg\left\{E_2(e^{j2\pi f/F_{in}})\right\}} \left| E_2(e^{j2\pi f/F_{in}}) \right|. \qquad (14)$$

Frequency response of the overall structure is given by

$$H_{ov}(e^{j2\pi f/F_{in}}) = E(e^{j2\pi f/F_{in}}) H_a(jf), \qquad (15)$$

where

$$E(e^{j2\pi f/F_{in}}) = e^{-j\left(M\pi f/F_{in} + \arg\left\{\hat{B}(e^{j2\pi f/F_{in}})\right\} + \arg\left\{E_2(e^{j2\pi f/F_{in}})\right\}\right)} e(f), \qquad (16)$$

with

$$e(f) = \left( \frac{\left|\hat{B}(e^{j2\pi f/F_{in}})\right|}{K \sin\left(\pi f / F_{in}\right)} \right)^M \left| E_2(e^{j2\pi f/F_{in}}) \right| H_a(f). \qquad (17)$$

## 4. DESIGN EXAMPLE

In the sequel we give an example and we examine how the non-ideal fractional delay may change the frequency response of the overall system. We try to satisfy the same requirements used in [3], and [4], using the proposed decimator structure. The requirements are as follows: sampling rate change factor $R=34^1/_{34}$, desired signal bandwidth $f_p=0.001F_{in}$, attenuation in the frequency bands that cause aliasing at least 80dB, and passband distortion less than 0.1dB. These requirements are met by the proposed decimation structure with $M=1$, $N=1$, $a_1=8$, $K=69$, thus $D=2.027658$, using the linear interpolation filter. This has the same complexity as the third order programmable fractional CIC filter in [4]. The non-integer delay filter has been designed as an FIR filter of length $L=6$, using the Lagrange interpolation method.

The frequency response of the comb stage in the case of the ideal delay $D$ in feed-forward branch together with the frequency response in the case of the non-integer delay filter in the feed-forward branch, are given in Fig. 4. We see that there is a small degradation for the higher frequencies as a result of the selected method for the fractional delay filter realization.
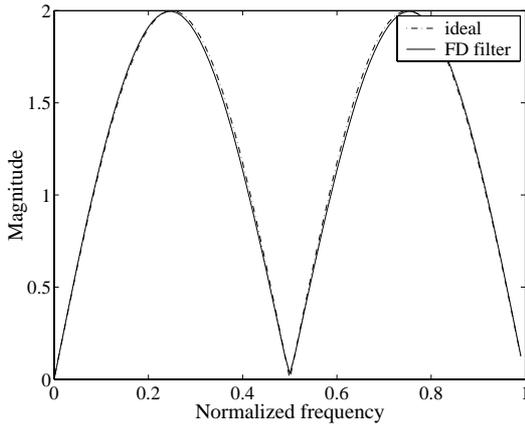
Fig. 4. The frequency response of the comb stage with the ideal delay, and non-integer delay filter.
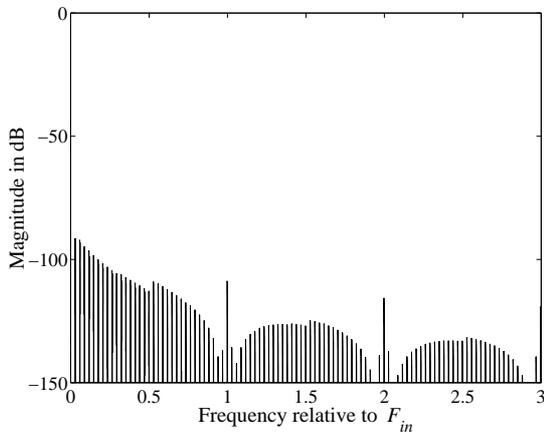


Fig. 5. Aliasing bands of the overall structure.

The aliasing bands of the overall structure are shown in Fig. 5. We see that the design requirements are met using this type of the fractional delay filter. However, there is a slight degradation of the frequency response at very high frequencies (2-3 dBs). Possible improvements using some other fractional delay filter type and design method will be a topic of our future work.

## 5.  PROGRAMMABILITY

As it was said earlier, the proposed decimator structure with non-integer delay filter in the comb part is easier to program than the decimator structure proposed earlier in [3]. The main reason is that the aliasing band attenuation in the structure of [3] depends heavily on the fractional part of the sampling rate conversion factor. The position of zeros in frequency response of the new structure can be changed by proper selection of the parameter $K$. This means different value of the non-integer delay $D$, which implies a different set of coefficients of the non-integer delay filter. A change of the decimator factor also implies a change of the

coefficients. Therefore, in order to program the proposed decimator structure, that is to change the CIC filter length or decimation factor, it is enough to change the coefficients of the non-integer delay filter at the comb part of the CIC filter. In our example, there are $L+1$ coefficients, where $L$ is order of the non-integer delay FIR filter. The FIR filter can have relatively low order, thus the number of coefficients is small. Therefore, the proposed principle can be efficiently exploited in the multistandard receiver's decimation chain as the first stage. For each standard there is a unique decimation factor, and the corresponding set of coefficients.

## 6.  CONCLUSIONS

In this paper we have presented a novel efficient decimator structure intended to be the first stage in the decimation chain of multistandard radio receivers. The proposed decimator is based on the modified comb filter structure proposed by Saramäki and Ritoniemi in [6], using the programmable fractional CIC building blocks. The proposed structure has good anti-aliasing and anti-imaging properties, further it is simple and power efficient. A very important task is the selection of the best non-integer delay approximation, i.e., selection of the appropriate non-integer delay filter type and design. The proposed decimator structure has very high flexibility and programmability. The filter length and decimation factor can be easily programmed by changing the coefficients of the non-integer delay filter in the comb stage.

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