A TIME-FREQUENCY SYNCHRONIZATION ALGORITHM FOR MC-CDMA SYSTEMS IN LMDS APPLICATIONS

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ABSTRACT

We propose in this paper a new algorithm for estimating the synchronization parameters (frequency offsets and delays) of multiple users in a MC-CDMA system targeted for broadband LMDS applications. The proposed algorithm exploits the code properties in the frequency domain to perform delay estimation, whereas it is based on the insertion of an appropriate header sequence to estimate the frequency offset in the time domain. Simulation results show that the algorithm obtains accurate estimates with modest computational complexity, and is robust to the presence of multiple-access interference.

1 INTRODUCTION

Broadband fixed wireless access systems techniques are gaining worldwide interest in the academic and industrial community [1]. Indeed, in a deregulated telecommunications market, wireless access techniques offer to new operators a cost-effective path for provisioning broadband multimedia services to subscribers, without the need to deploy expensive wired infrastructures. In particular, due to crowding of the existing frequency bands, wireless access system operating between 24 and 48 GHz, often referred to as *local multipoint distribution systems (LMDS)* [2], are the object of intense standardization activities (see e.g. the ETSI HIPERACCESS project in Europe and the IEEE 802.16 working group in the United States). LMDS systems are intended to offer integrated broadband services to residential and business subscribers, with a cellular-like network architecture, where a base station (BS), called *hub*, serves several subscribers within a cell radius of 2 to 5 km.

Since the subscriber antenna in a LMDS system is a narrowbeam directional antenna pointed toward the serving BS, LMDS networks are essentially free of multipath propagation [3]. The major impairments are instead frequency-flat rain fading, which limits the cell coverage, and intercell interference, which limits the frequency reuse. In such multipath-free scenario, the combination of *multicarrier* modulation and code-division multiple-access (CDMA) systems offer three distinct advantages: (i) reduced symbol rate, which eases time synchronization especially in high-speed applications; (ii) higher flexibility and finer resource partitioning, which allows handling of heterogeneous multimedia traffic; (iii) increased system capacity. Among the drawbacks, besides sensitivity to nonlinear distortion, multicarrier techniques are extremely prone to frequency offset errors, which arise because of unavoidable mismatches between transmitter and receivers RF sections, which motivate the need for the synthesis of fast and reliable synchronization techniques.

When MC-CDMA techniques are adopted in the uplink (subscriber to BS) direction, due to different propagation paths, the subscriber data frames are not synchronized at the BS. Thus, the adoption of orthogonal codes does not assure perfect separation of the users at the BS, which might entail severe performance degradation. A viable strategy [4] is to have the BS continuously track the synchronization parameters of the users, sending feedback information to subscriber units; each subscriber unit can then adjust its parameters, in order to be in alignment with other users at the BS. However, in this asynchronous scenario, acquisition and tracking of synchronization parameters for all the users is complicated, and often needs insertion of pilot symbols across both time and frequency [5].

In this paper, we propose a simple and accurate synchronization algorithm for tracking synchronization parameters (time delay and frequency offset) in asynchronous MC-CDMA systems. The proposed algorithm requires training symbols for frequency offset estimation, but no additional information is needed, besides spreading code, for delay estimation.

2 THE MC-CDMA SYSTEM MODEL

Let us consider the uplink of a MC-CDMA system with N subcarriers and J users all transmitting asynchronously at the same symbol rate $1/T$. The symbol d_{jm}, transmitted by the jth user in the $m$the symbol interval, is first copied N times, then is spread in the frequency domain by the code $\boldsymbol{c}_j = [c_j(0), c_j(1), \ldots, c_j(N-1)]$, and finally transformed by N-point IDFT, obtaining $s_{jm}(q) = d_{jm}\,\mathrm{IDFT}[c_j(k)]$, with $q = 0, 1, \ldots, N-1$. After insertion of a cyclic prefix (CP) of length L, one obtains, accounting for the periodic properties of IDFT, the *extended* sequence

$$\tilde{s}_{jm}(q) = \frac{1}{N}\, d_{jm} \sum_{k=0}^{N-1} c_j(k)\, e^{j\frac{2\pi}{N}kq} \,, \qquad (1)$$

for $q = -L, \ldots, 0, 1, \ldots, N-1$, where $\tilde{s}_{jm}(q) = s_{jm}(q)$ for $q = 0, 1, \ldots, N-1$; such a sequence of $Q \triangleq L + N$ samples is parallel-to-serial converted, and filtered by the D/A device, which operates at rate $1/T_c = Q/T$, obtaining hence the complex baseband signal $u_j(t)$ transmitted by the jth user:

$$u_j(t) = \sum_{m=-\infty}^{\infty} \sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, \psi_{D/A}(t - q\,T_c - m\,T) \,, \qquad (2)$$

with $\psi_{D/A}(t)$ denoting the impulse response of the D/A converter. It should be noted that, after D/A conversion, the subcarrier separation is given by $\Delta f = 1/T_u$, with $T_u \triangleq N\,T_c$ representing the "useful" symbol portion. The baseband signal is then upconverted to RF and transmitted through the channel.

We will assume in the following that the channel is nonselective both in time and in frequency: both assumptions are reasonably true in the considered LMDS operation scenario. Moreover, to keep notation (reasonably) simple, we will not explicitly account for noise effects in our derivations. Then, the received complex envelope is given by

$$r(t) = \sum_{j=1}^{J} h_j\, u_j(t - \tau_j)\, e^{j2\pi\theta_j t} \,, \qquad (3)$$

where, with reference to the jth user, h_j is the complex gain (amplitude plus phase) of the channel (which is assumed to be constant within the observation interval), $|\tau_j| \le T/2$ accounts for the combined effect of transmission delay and asynchronism between users, and θ_j denotes the carrier frequency offset. Our aim is to estimate τ_j and θ_j of each user, without requiring knowledge of the channel gains h_j; moreover, our estimates must be tolerant to multiple-access interference.

2.1 Digital pre-processing at the receiver

In the A/D device, the received baseband signal $r(t)$ is first filtered by $\psi_{A/D}(t)$, which yields $y(t) = r(t) * \psi_{A/D}(t)$, with $*$ denoting linear convolution, and then sampled at rate $f_c = 1/T_c$, obtaining hence $r_n(k) = y(n\,T + k\,T_c)$, with $n \in \mathbb{Z}$ and $k = -L, \ldots, 0, 1, \ldots, N-1$, which can be expressed as

$$r_n(k) = \sum_{j=1}^{J} h_j \sum_{m=-\infty}^{\infty} \sum_{q=-L}^{N-1} \tilde{s}_{jm}(q) \cdot f_{\theta_j}\left[(k-q)T_c + (n-m)\,T - \tau_j\right] e^{j2\pi\theta_j(nT + kT_c)} \,, \qquad (4)$$

where $f_{\theta_j}(t) \triangleq \psi_{D/A}(t) * [\psi_{A/D}(t)\, e^{-j2\pi\theta_j t}]$ is the overall A/D and D/A impulse response, which depends also on the frequency offset θ_j. It should be noted that since the D/A and A/D filters have bandwidth of the order of $1/T_c$, when $|\theta_j| \ll \frac{1}{T_c}$, we can neglect the frequency shift in the definition of $f_{\theta_j}(t)$, writing $f_{\theta_j}(t) \approx f(t) \triangleq \psi_{D/A}(t) * \psi_{A/D}(t)$. Moreover, if we assume that $f(t)$ has finite memory, that is, $f(t) \neq 0$ for $t \in [0, L_f\,T_c)$, and the CP length L is such that

$$L > L_f + \frac{1}{T_c} \max_j(\tau_j) \,, \qquad (5)$$

then the intersymbol interference (ISI) can be perfectly suppressed by removing the first L samples (CP removal), i.e., those with $k = -L, \ldots, -1$. Note that this assumption requires only a coarse time synchronization between the users, which is reasonable since we are continuously transmitting feedback information for alignment. The remaining samples, for $k = 0, 1, \ldots, N-1$, can be expressed as

$$r_n(k) = \sum_{j=1}^{J} h_j\, e^{j2\pi(\theta_j T)n} \sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, g_{\tau_j}(k-q)\, e^{j2\pi(\theta_j T_c)k} \qquad (6)$$

where $g_{\tau_j}(k) \triangleq f(kT_c - \tau_j)$. We observe that, since $\tilde{s}_{jm}(q)$ is the periodic extension of $s_{jm}(q)$, then $\sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, g_{\theta_j \tau_j}(k-q) = s_{jm}(k) \underset{N}{\odot} g_{\tau_j}(k)$, where the symbol $\underset{N}{\odot}$ denotes *circular convolution* over N points. Thus, by taking the DFT of $r_n(k)$, and applying straightforward DFT properties, one obtains:

$$v_n(i) = \mathrm{DFT}[r_n(k)] = \sum_{j=1}^{J} h_j\, e^{j2\pi(\theta_j T)n} \cdot \frac{1}{N}\, \mathrm{DFT}[s_{jm}(k) \underset{N}{\odot} g_{\tau_j}(k)] \underset{N}{\odot} \mathrm{DFT}[e^{j2\pi(\theta_j T_c)k}] \,. \qquad (7)$$

The last DFT in (7) is given by:

$$\mathrm{DFT}[e^{j2\pi\theta_j T_c k}] = \mathcal{D}_N\left(\frac{i}{N} - \theta_j\,T_c\right) \qquad (8)$$

where $\mathcal{D}_N(\nu) \triangleq \frac{\sin(\pi N \nu)}{\sin(\pi\nu)}\, e^{-j\pi(N-1)\nu}$ is the Dirichlet function. If we assume that the frequency offset is such that $|\theta_j| N T_c \ll 1$, i.e., $|\theta_j| \ll \Delta f$, then (8) can be approximated by $N\,\delta(i)$, and, therefore, equation (7) simplifies to:

$$v_n(i) \approx \sum_{j=1}^{J} h_j\, G_{\tau_j}(i)\, c_j(i)\, d_{jm}\, e^{j2\pi(\theta_j T)n} \,, \qquad (9)$$

where $G_{\tau_j}(i)$ is the N-point DFT of $g_{\tau_j}(k)$. Finally, by defining an *equivalent channel* $\tilde{h}_{\tau_j}(i) \triangleq h_j\, G_{\tau_j}(i)$, we can rewrite (9) as:

$$v_n(i) \approx \sum_{j=1}^{J} \tilde{h}_{\tau_j}(i)\, c_j(i)\, d_{jm}\, e^{j2\pi(\theta_j T)n} \,, \qquad (10)$$

which will be the model whereupon our synchronization algorithm is based.

2.2 The equivalent channel structure

Before tackling the algorithm derivation, it is necessary to investigate how $\tilde{h}_{\tau_j}(i)$ depends on τ_j. The Fourier transform of $g_{\tau_j}(k)$ is given by

$$G_{\tau_j}(\nu) = f_c \sum_{k=-\infty}^{\infty} F[(\nu - k) f_c] \, e^{-j2\pi(\nu-k) f_c \tau_j} \,. \quad (11)$$

with $F(f)$ denoting the Fourier transform of $f(t)$. If $F(f) \neq 0$ for $|f| \leq f_p$ and $F(f) \approx 0$ for $|f| \geq f_s$, and if $f_c > f_p + f_s$, we can approximately write, for $|\nu| \leq \nu_p \triangleq f_p/(2f_c)$

$$G_{\tau_j}(\nu) \approx f_c \, F(\nu \, f_c) \, e^{-j2\pi\nu \, f_c \, \tau_j} \,, \quad (12)$$

and, therefore, $\tilde{h}_{\tau_j}(i) \approx f_c \, F(i\Delta f) \, e^{-j2\pi i \Delta f \, \tau_j}$, for $|i| \leq N \, \nu_p$. By substituting back in (10), and rearranging indexes, we have simply:

$$v_n[i] \approx f_c \sum_{j=1}^{J} h_j \, F(i\Delta f) \, c_j[i] \, d_{jm} e^{-j2\pi\mu_j \, [i]} e^{j2\pi\lambda_j \, n} \,. \quad (13)$$

where $[\cdot]$ denotes modulo-N operation, and we have introduced the *normalized* synchronization parameters $\lambda_j \triangleq \theta_j T$ and $\mu_j \triangleq \tau_j \Delta f = \frac{\tau_j}{T_u}$.

3 THE PROPOSED ALGORITHM

Observe that $F(f)$ in (13) is known at the receiver, hence we can compensate its effects by defining

$$y_n[i] \triangleq \frac{v_n[i]}{f_c F(i\Delta f)} = \sum_{j=1}^{J} h_j \, c_j[i] \, d_{jn} \, e^{-j2\pi\mu_j [i]} \, e^{j2\pi\lambda_j n} \,. \quad (14)$$

Thus, based on this model, a suitable approach to estimate λ_j and μ_j could be to perform 2D-FFT over $y_n[i]$ followed by $2D$ search in order to simultaneously extract the frequency-offset and time-delay estimates. Nevertheless, such an algorithm would be exceedingly complex; then, although suboptimum, we will separate frequency-offset estimation from time-delay estimation.

3.1 Frequency-offset estimation

We perform frequency-offset estimation by observing a single subcarrier i. Let us consider, without restriction, the contribution of the first user in (14), which is

$$y_n[i] = h_1 \, c_1[i] \, d_{1n} \, e^{-j2\pi\mu_1 [i]} \, e^{j2\pi\lambda_1 n} \,. \quad (15)$$

Thus, we can estimate λ_1, and hence θ_1, by cross-correlating (in n) $y_n[i]$ with $d_{1n} \, e^{j2\pi\nu n}$, and finding the maximum with ν of the correlation magnitude. This requires sending a training sequence d_{1n} of length M, which can be accommodated in a suitable header of the frame. The proposed estimator for θ_1 is then:

$$\hat{\theta}_1 = \frac{1}{T} \, \arg \max_{|\nu| \leq 0.5} \left| \sum_{n=0}^{M-1} y_n[i] \, d_{1n}^* \, e^{-j2\pi\nu n} \right| \,, \quad (16)$$

which coincides with the maximum-likelihood estimator for θ_1 in white Gaussian noise, and can be efficiently implemented by means of 1D-FFT. The estimate range of the proposed estimator is $|\theta_1| \leq \frac{1}{2T}$, which is more than adequate for tracking purposes. Note that the contribution of the MAI to the estimator is

$$\sum_{j=2}^{J} h_j \, c_j[i] \, e^{-j2\pi\mu_j [i]} \left\{ \sum_{n=0}^{M-1} d_{jn} \, d_{1n}^* \, e^{j2\pi(\lambda_j - \nu)n} \right\} \,, \quad (17)$$

which essentially depends on the term in brackets, which represents the zero-delay frequency-shifted cross-correlation between the training sequences. Thus, the MAI can be reduced by an appropriate choice of the training sequences.

3.2 Time-delay estimation

A dual reasoning is followed to carry out delay estimation, which is performed by looking at a single symbol n in the frame. Indeed, by considering (15) for a fixed value of n, it can be seen that we can estimate μ_1 by cross-correlating (in i) $y_n[i]$ with $c_1[i] \, e^{-j2\pi\mu[i]}$ and finding the maximum with ν of the correlation magnitude. Note that in this case training sequences are not needed, but we exploit the spreading codes to this aim. The proposed estimator is:

$$\hat{\tau}_1 = T_u \, \arg \max_{|\nu| \leq 0.5} \left| \sum_{|i| \leq N\nu_p} y_n[i] \, c_1[i]^* \, e^{j2\pi\nu[i]} \right| \,, \quad (18)$$

which can be efficiently implemented by means of 1D-FFT. The estimate range is $|\tau_1| \leq \frac{T_u}{2}$, which again is more than adequate for tracking (it is approximately equal to the symbol interval). In this case, the contribution of the MAI to the estimator is

$$\sum_{j=2}^{J} h_j \, e^{j2\pi\lambda_j n} d_{jn} \left\{ \sum_{|i| \leq N\nu_p} c_j[i] \, c_1[i]^* \, e^{-j2\pi(\mu_j - \nu)[i]} \right\} \,, \quad (19)$$

which can be reduced by choosing the spreading codes of the users such that the zero-delay frequency-shifted cross-correlation (in brackets) is negligible.

4 NUMERICAL RESULTS

The performance of the proposed synchronization algorithm in the uplink channel have been investigated by computer simulations. We considered the following parameters in all the simulations: BPSK signaling for all users, $N = 256$ subcarriers, $L = 32$ (CP length), $f_c = 16$MHz, signal-to-noise ratio $\mathrm{SNR} = 20$dB. Moreover, length-31 concatenated Gold sequences were employed both as training sequences and spreading codes. We considered $J = 4$ users with delays (normalized to T_c) $\boldsymbol{\tau} = [-54, -23, 13, 35]$ and frequency offsets (normalized to $1/T$) $\boldsymbol{\theta} = [-0.47, -0.27, 0.20, 0.48]$.

Figures 1 and 2 show the cost functions for frequency offset estimation (16) and time-delay estimation (18) evaluated for the first user (similar results were obtained for the other

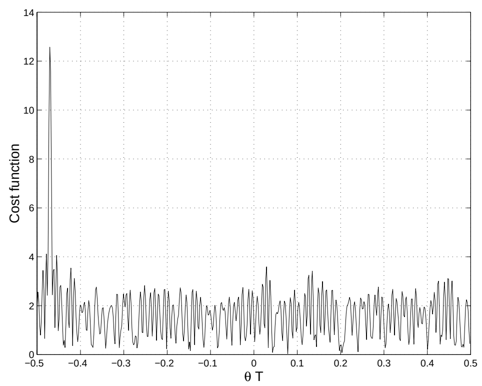

Figure 1: Cost function for frequency offset estimation of the first user (true value $\theta_1 T = -0.47$).

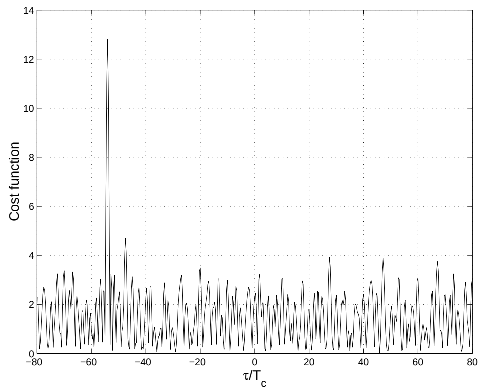

Figure 2: Cost function for time delay estimation of the first user (true value $\tau_1/T_c = -54$).

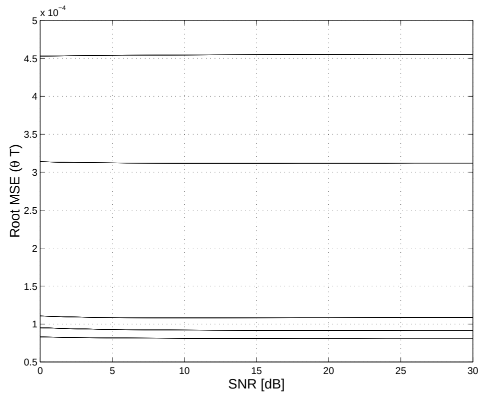

Figure 3: Root mean-square error (RMSE) of frequency offset estimates versus SNR.

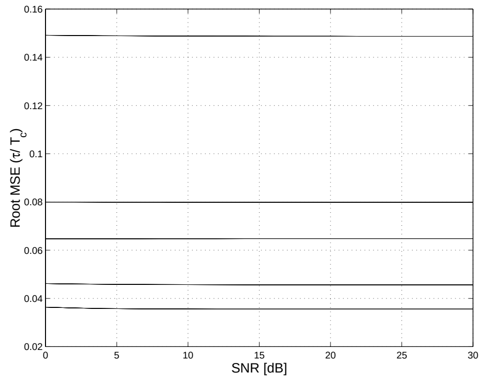

Figure 4: Root mean-square error (RMSE) of time delay estimates versus SNR.

users). It can be seen that the MAI contribution is effectively canceled out by the good frequency-shifted cross-correlation properties of the Gold sequences adopted, both for training and spreading.

In the second experiment, we investigated the performance sensitivity to thermal noise level. In particular, Figs. 3 and 4 report the root mean-square error (RMSE) of the offset and delay estimates for all the users as a function of SNR, varying from 0 to 30 dB. The curves were obtained by Monte Carlo simulation over 500 trials. The insensitivity to SNR is due to the fact that the bias depends on time or frequency discretization step, and therefore in nearly insensitive to SNR and predominates over the variance, which moreover decreases as SNR^{-1}.

5 ACKNOWLEDGEMENTS

The authors are grateful to Prof. L. Paura for his careful reading of the manuscript.

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