Symbol by symbol reduced complexity highly selective OFDM channel estimation

Emmanuel Jaffrot[1], Van K. Nguyen[2], Marcella Soamiadana[1,2]*, Langford B. White[2]†, Inbar Fijalkow[3]‡
[1] ENSTA, France
[2] Internet Technology Center, Adelaide University, Australia
[3] ETIS, ENSEA - Univeristé de Cergy-Pontoise
e-mail: jaffrot@ensta.fr, lang.white@adelaide.edu.au, fijalkow@ensea.fr

ABSTRACT

In OFDM systems, receiving techniques are mainly sensitive to the employed channel estimation strategy. Channel modelling is a crucial point in deriving a channel estimation method. We address here a simple channel model taking into account physical characteristics of the channel. We also show that this simple model can lead to good performance in symbol-by-symbol reception schemes.

1 Introduction

In recent years, many studies have been lead on OFDM receiving techniques and specially about channel estimation methods [3, 1, 4, 5]. Modelling the channel is a decisive step in channel estimation, and the complexity of the model confers the complexity of the estimation technique. Hence it is very important to have simple models. Simple models usually don't take into account the physical parameters of the channel.
In this paper, we define a new channel model from the channel model defined in [3], based on an auto-regressive (AR) representation, and taking into account the physical parameters of the channel.
We will show that from this new model, it is possible to obtain good performance with symbol by symbol processing receivers.

2 OFDM Signal model

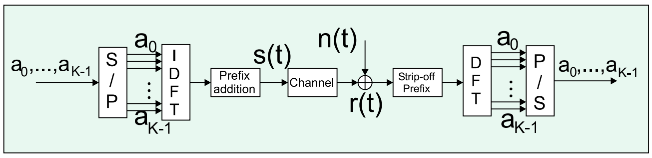

Figure 1: Baseband OFDM system

*M. Soamiadana is funded by the French RNRT project Festival

†L. B. White was partially supported by ETIS on a French RNRT project

‡This work was conducted for the French RNRT project FESTIVAL

We consider the base-band OFDM system shown in Figure 1. The subcarriers modulation is performed via an inverse discrete Fourier transform (IDFT). The time-varying impulse response, $c(\tau;t)$, of the channel seen by the receiver is described by

$$c(\tau;t) = \sum_{i=0}^{I-1} \alpha_i(t)\delta(\tau - \tau_i(t)), \qquad (1)$$

where $\alpha_i(t)$ and $\tau_i(t)$ are respectively the complex-valued attenuation and delay of the i^{th} path, I is the number of paths and $\delta(.)$ is the Dirac function. Let $n(t)$ represent the complex additive white Gaussian noise corrupting the reception. The demodulation is done by a discrete Fourier transform (DFT) at each diversity branch of the receiver.
The OFDM signal can be represented by a data block [2]. Each block is with dimension $M \times N$, M being the OFDM symbols numbers and N the sub-carrier number in the time-frequency block, and is composed by $M \times N$ symbols $\{a_{mn}\}$, belonging to a constellation Ω with two-dimensional position (mF, nT) where F and T are respectively the frequency and time spacing between two adjacent symbols. The symbols a_{mn} (m being

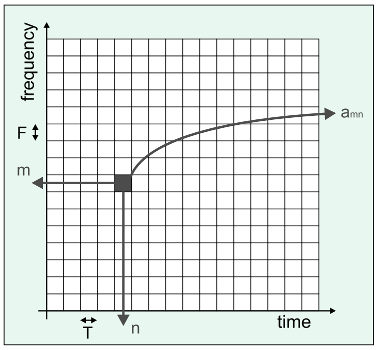

Figure 2: Time-frequency block structure

the frequency index and n the time index in the time-frequency block) are transmitted. After removal of the cyclic prefix, the received signal is first demodulated by a N-point FFT algorithm [6]. Considering perfect time and frequency synchronization at the receiver, we get the signal R_{mn} at the output of the FFT-demodulator associated to the symbol a_{mn} :

$$R_{mn} = c_{mn}a_{mn} + N_{mn} \quad (2)$$

where c_{mn} is the discrete channel gain factor associated to the symbol a_{mn} and N_{mn} is an complex additive gaussian white noise with variance N_0. The channel gain factors are correlated in time and frequency.

3 Channel model and rank-reduction strategy

In this paper, we assume the channel to be a multipath doppler fading channel. The multipath channel is a propagation environment in which the signal is received at the receiver from multiple paths generated by multiple reflection and spreading effects. These effects may lead to fast phase and amplitude variations. Each path is characterized by its mean power, its propagation delay and its Doppler power spectrum depending on the environment, the mobile speed and the carrier on which the signal is transmitted. In general, the propagation channel is represented as :

$$g(\tau;t) = \sum_q \alpha_q(t)e^{-2j\pi f_c\tau_q(t)}\delta(\tau - \tau_q(t)) \quad (3)$$

where $\alpha_q(t)$ is the time-varying attenuation factor, f_c the carrier frequency and $\tau_q(t)$ the q^{th} path delay. The propagation channel is assumed non-varying on one carrier during a symbol period. On a given carrier f_m and for the n^{th} received OFDM symbol, the channel gain factor becomes:

$$c_{mn} = \sum_q \alpha_q e^{-j2\pi f_m\tau_q}\delta(\tau - \tau_q). \quad (4)$$

3.1 A physical model for delay-Doppler Spread channels

In [3], a physical model is used to characterize the correlation between the time and frequency gains factors c_{mn} in (2). This model is using the statistics of the channel and uses a block representation of the channel based on its covariance matrix. For a classical Doppler power spectrum and exponential multipath intensity, the correlation between two symbols spaced in time and frequency respectively with Δt and Δf is given by:

$$\phi(\Delta f, \Delta t) = \phi(0)\phi_t(\Delta t)\phi_f(\Delta f), \quad (5)$$

with $\phi(0) > 0$,

$$\phi_t(\Delta t) = J_0(\pi B_d\Delta t), \quad (6)$$

and

$$\phi_f(\Delta f) = \frac{1}{1 + j2\pi T_m\Delta f}. \quad (7)$$

Here J_0 is the zero-order Bessel function of the first kind, with B_d and T_m being the Doppler and delay spreads respectively of the propagation channel.

Let C be the vector corresponding to all the gain factors observed of a time-frequency block,

$$C = (c_{0,0}, \dots, c_{0,N-1}, \dots, c_{M-1,0}, \dots, c_{M-1,N-1})^T \quad (8)$$

$(.)^T$ being the transpose operator. The $(p,q)^{th}$ element of the $NM \times NM$ hermitian covariance matrix $\Phi = \mathbb{E}[CC^T]$ of the channel corresponding to the correlation between $c_{m_pn_p}$ and $c_{m_qn_q}$:

$$\Phi_{p,q} = \phi((m_p - m_q)F, (n_p - n_q)T). \quad (9)$$

3.1.1 *Bi-dimensional form*

In [3], a model is derived for the covariance matrix of the channel using the Karhunen-Loève expansion theorem. The channel is represented by:

$$C = \sum_{k=0}^{NM-1} G_kV_k \quad (10)$$

where $\{\mathbf{V}_k\}_{k=0}^{NM-1}$ are the normalized eigenvectors of the hermitian covariance matrix Φ of the equivalent channel vector $\mathbf{C}$ and the $\{G_k\}_{k=0}^{NM-1}$ are zero-mean independent complex Gaussian random variables. The variances of these random variables are equal to the eigenvalues $\{\Gamma_k\}_{k=0}^{NM-1}$ of the hermitian matrix Φ.

3.1.2 *Two-1D form*

In matrix form, we can define the $M \times M$ real symmetric Toeplitz matrix R by:

$$[R]_{t,s} = J_0(\pi B_d|t-s|), \quad (11)$$

and the $N \times N$ complex hermitian Toeplitz matrix S by:

$$[S]_{k,l} = \frac{1}{1 + j2\pi T_m(k-l)}. \quad (12)$$

Hence,

$$\mathbb{E}\left[c_{Nt+k}\, c^*_{Ns+l}\right] = [R]_{t,s}\,[S]_{k,l}\,. \quad (13)$$

We can the see that the $NM \times NM$ matrix Φ can be redefined by (13), which consists in the tensor product $\Phi = R \otimes S$. Thus, in particular, the eigenvalues of Φ consists precisely of the set of all the products of the eigenvalues of R and S. In addition , the eigenvectors of Φ can be written in terms of the eigenvectors of R and S. Thus to perform an eigen-decomposition of Φ, only the eigen-decompositions of R and S are needed. More specifically, if

$$R = \sum_{k=0}^{M-1} \lambda_k\, u_k\, u_k^T \quad (14)$$

$$S = \sum_{k=0}^{N-1} \mu_k\, v_k\, v_k^T, \quad (15)$$

then

$$\Phi = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} \lambda_k\mu_l(u_k \otimes v_l)(u_k \otimes v_l)^T \quad (16)$$

3.1.3 Rank reduction strategy

As observed in [3], since the $\{G_k\}_{k=0}^{k=N-1}$ are gaussian variables with variance equal to the $\{\Gamma_k\}_{k=0}^{k=N-1}$, the G_k with a very small associated eigenvalue is insignificant. Thus, for a given channel, we can define a small number of eigenvectors representing the channel, see figure 3. This fact represented here in the case defined in 3.1.1

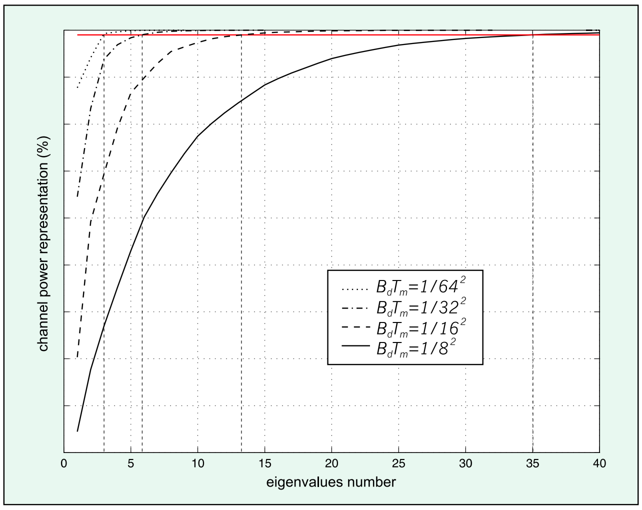

Figure 3: Cumulated eigenvalues

but is also valuable for the tensor-based representation defined above in 3.1.2.

Using this channel model may lead to quite complex computation since size of time-frequency blocks may be important. The main idea in this paper is to use a more simple type of channel model, but also taking into account physical characteristics of the channel.

3.2 Time-correlated Rayleigh flat fading channel model

3.2.1 Model description

In this section, the channel gains are modelled as a first-order vector Gauss-Markov process expressed in complex form by

$$c_{n+1} = A\ c_n + v_n\ , \tag{17}$$

where $A \in \mathbb{C}^{N\times N}$ is a known, stable matrix representing the state transition matrix of the Gauss-Markov process, c_n is the vector containing the gain factors observed on the n^{th} OFDM symbol and v_n is a complex Gaussian white noise process with mean μ and covariance Q. We assume that at the initial time $n = 0$ that c_0 is chosen to be Gaussian with the steady state statistics

$$E\{c_0\} = (I - A)^{-1}\ \mu\ , \tag{18}$$

and $\mathrm{Cov}\{c_0\} = P$ where P satisfies the Lyapunov equation $APA^H + Q = P$. Thus c_n will be a stationary process with mean given by (18) and with autocorrelation matrix

$$E\{c_n\ c_{n-l}^H\} = \begin{cases} A^l\ P & l \geq 0 \\ P\ \left(A^H\right)^{-l} & l < 0\ . \end{cases} \tag{19}$$

3.2.2 Taking into account physical parameters

From (19) and (5), we can see that the P matrix represents by itself the frequency correlation and A^l is the time correlation between two symbols separated by a time shift equal to l. Thus immediately,

$$P_{k,l} = \frac{\phi(0)}{1 + 2j\pi T_m(k-l)F}. \tag{20}$$

Due to a separable nature of the physical channel correlation function (5), we take our state transition matrix A to be of the form $A = a\ I$, where a is constant complex. Comparing the time correlation between two OFDM symbols separated by l symbol periods, we need to have $a^l = J_0(\pi B_d l T)$ for every l. This is theoretically impossible, so an approximation has to be made for the $\{a^l\}_{l=0}^{M-1}$ to fit the $\{J_0(\pi B_d l T)\}_{l=0}^{M-1}$, M denoting the number of OFDM symbols corresponding to the time duration after which the correlation is considered to be insignificant. This approximation is given by solving the minimum mean-square error following problem:

$$\min_a \sum_{l=0}^{M-1} |J_0(\pi B_d l T) - a^l|^2\ . \tag{21}$$

4 Simulation results

In this paper, we don't aim at defining the most accurate representation of the channel. We define a simple AR-based model taking into account the frequency and time selectivity physical parameters of the channel. We are mainly interested in symbol by symbol processing in a recursive fashion, including past-states forgetting . This is leading to online channel estimate computation, which is a very important point for broadcasting systems.

To show that with this kind of simple channel model, it is possible to get receivers with good performance, we made some simulation. We considered a single user OFDM system with 16 carriers. The modulation scheme employed is BPSK.

The data sequence is arranged into a block of 16 OFDM symbols. Each symbol is transmitted over the 16 orthogonal frequency channels using the DFT. We assumed that the fading channels are independent from block to block. However, within each block, the time-frequency covariance of the fading channels between any two symbols is given by the model in section 3.1. In this simulation, we used $B_dT = 0.25$ and $T_mF = 0.5$. Fig. 4 shows the time-frequency correlation of the fading channels. With these values of B_dT and T_mF, the fading channels are highly decorrelated in the frequency coordinate. Fig. 4.a shows the time correlation of the fading channels modelled by the zeroth-order Bessel function of

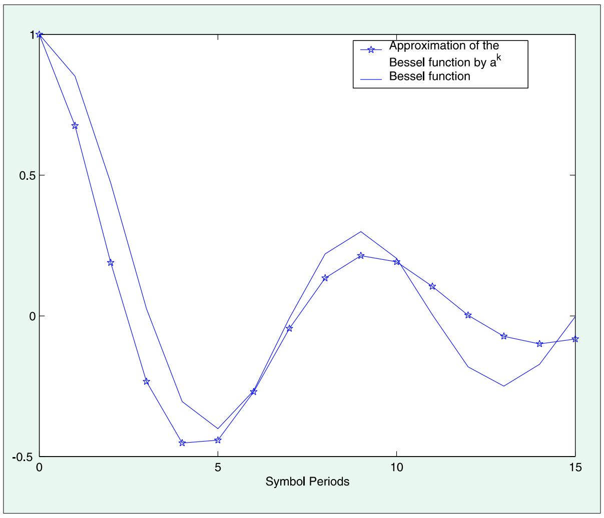

Figure 4: Time and frequency correlation of the channel

the first kind and its approximation as detailed above. For $B_d T = 0.25$, the channels are reasonably decorrelated in the temporal dimension. The correlation of the fading coefficients between any two consecutive symbols is 0.852 and the first zero of the Bessel function is at the third symbol. In this simulation we evaluated the performance of two Kalman-based receiving techniques. The first Kalman-based method is an *a posteriori* probability technique (APP) and the second one uses the EM algorithm for an iterative estimation of the channel. In addition, as a benchmark, we evaluated the performance of the coherent Maximum Likelihood receiver which has the full knowledge of the fading coefficients at every symbol interval. Both APP and EM techniques perform symbol by symbol detection. Simulation results are given in figure 5. This figure shows that using the

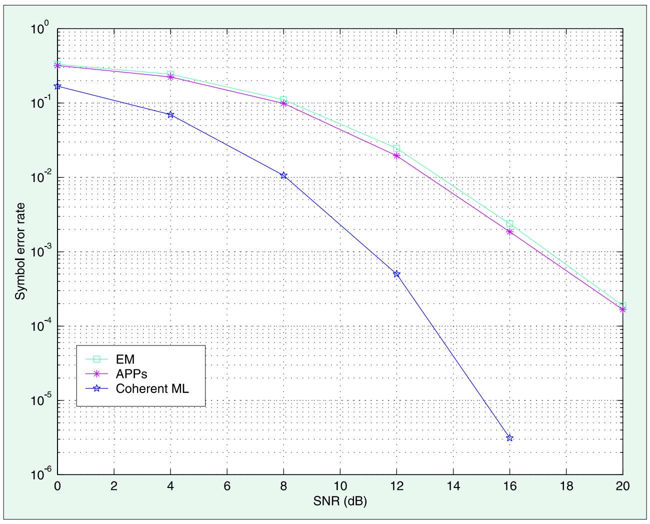

Figure 5: Receivers' performance for $B_d T = 0.25$ and $T_m F = 0.5$

AR-based channel model defined in 3.2 with classical channel estimation methods gives good performance.

5 Conclusion

In this paper, we presented a simple AR-based channel model for OFDM systems taking into account physical characteristics of the channel, such as time and frequency correlation between adjacent OFDM symbols and subcarriers.
We have shown that even if this model is obtained from a rough approximation of the time-correlation behavior, it is possible to get simple symbol by symbol receivers with good performance.
Next, further studies will lead to the identification of appropriate receiving techniques for this new channel model.

References

[1] O. Edfors and *et al.* OFDM channel estimation by singular value decomposition. *IEEE Transactions on Communications*, 46(7), Jul. 1998.

[2] B. Le Floch, M. Alard, and C. Berrou. Coded orthogonal frequency division multiplex. *Proceedings of the IEEE*, 83(6):982–996, June 1995.

[3] E. Jaffrot and M. Siala. Turbo-channel estimation for OFDM systems on highly time and frequency selective channels. In *International Conference on Accoustics, Speech and Signal Processing, ICASSP'2000*, volume 5, pages 2977–2980, Istanbul, Turkey, Jun. 2000.

[4] Y. Li. Pilot-symbol-aided channel estimation for OFDM in wireless systems. In *IEEE Vehicular Technology Conference, VTC'99*, volume 2, pages 1131–1135, 1999.

[5] B. Muquet and *et al.* A subspaced based blind and semi-blind channel indentification method for OFDM systems. In *IEEE Workshop on Signal Processing Advances in Wireless Communications*, pages 170–173, 1999.

[6] S. B. Weinstein and P. M. Ebert. Data transmission by frequency-division multiplexing using the discrete fourier transform. *IEEE Transactions on Communications Technology*, COM-19(5):628–634, Oct. 1971.