A FAST DESIGN METHOD FOR BIORTHOGONAL MODULATED FILTER BANKS

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ABSTRACT

A method is proposed to solve the unconstrained optimization problem related to the design of perfect reconstruction biorthogonal modulated filter banks. The basic idea is to take advantage of some attractive properties of the parametrization of the underlying lifting scheme-based structure. This approach allows the design of prototypes with thousands of coefficients. Nearly optimal results are presented for two different optimization criteria.

1 Introduction

Among the many different filter banks now available in the literature, the critically decimated modulated filter banks offer the possibility of a low cost implementation and the availability of various design techniques. In the case of biorthogonal modulated filter banks, the design problem is restricted to the computation of two filters: the analysis and synthesis prototype filters. Furthermore, the resulting filter banks can be used in many different contexts, e.g. for subband coding using cosine modulated or modified discrete Fourier transform (MDFT) [1], or for multicarrier transmission [2]. We only concentrate here on the case of the perfect reconstruction (PR) biorthogonal cosine modulated filter bank (CMFB), as it can be used in subband coding.

Denoting the number of subbands by M, the overall system delay $D = 2sM + d$, s and d being two integers ($0 \leq d \leq 2M - 1$), is variable within given limits imposed by the prototype filter's length [1], which is a significant advantage of biorthogonal CMFB compared to orthogonal ones. In our case, as is also the case for the examples presented in [1], the length is such that $L = 2mM$, m being a positive integer. The design problem is to find these L coefficients by optimization of a given criterion. All commonly used design criteria lead to highly non linear problems that cannot be easily solved. Consequently, most design techniques fail if one wants to design very long (i.e. a few thousand coefficients) PR biorthogonal prototypes that are nearly optimal.

In this paper, the design method is validated for two different optimization criteria: the out-of-band energy and the time-frequency localization. In the first step we take advantage of the cascade structure resulting from the application of the lifting scheme to the polyphase components of the prototype filter [3]. This leads to an unconstrained optimization problem with respect to a set of $(2m+1)\frac{M}{2}$ parameters. In the second step, the optimization of short or medium length prototypes reveals that, for both optimization criteria considered, the optimized set of parameters have some attractive properties. We then take advantage of these different features to derive new sets of parameters, named *compact representations* or *codes*, which can accurately represent the prototype filter. In this way we are able to simultaneously reduce the number of parameters to optimize, from $(2m+1)\frac{M}{2}$ to $(2m+1)K$, with $K < \frac{M}{2}$, while nearly maintaining the quality of the results. Several examples illustrate the efficiency of the compact representation approach.

2 Biorthogonal CMFBs

A general M-band maximally decimated filter bank is depicted in Fig. 1. In this scheme, the PR condition is satisfied if the z-transforms of the output and input signals are such that $\hat{X}(z) = z^{-r}X(z)$, $\forall z$, with r an integer delay.

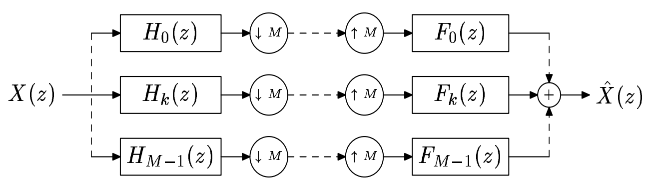

Figure 1: An M-band parallel filter bank.

2.1 The PR conditions

Biorthogonal CMFBs are a special case of such a parallel filter bank, in which all the analysis and synthesis filters are derived by modulation from one or two prototype filters. We denote by $p(n)$ in discrete-time, or by $P(z)$

in the z-domain, the common prototype to the analysis and synthesis sections. Without loss of generality, we assume that the modulated filters are obtained by a type IV discrete cosine transform (DCT-IV). Thus, for $0 \leq n \leq L-1$, the analysis and synthesis filters are given by [1]

$$h_k(n) = 2p(n)\cos((2k+1)\frac{\pi}{2M}(n-\frac{D}{2})+\theta_k), \quad (1)$$

$$f_k(n) = 2p(n)\cos((2k+1)\frac{\pi}{2M}(n-\frac{D}{2})-\theta_k), \quad (2)$$

respectively, where $\theta_k = (-1)^k\frac{\pi}{4}$.

A convenient way to express the PR condition is based on the polyphase decomposition of the prototype filter. Rewriting $P(z)$ using the type 1 polyphase decomposition of order $2M$ [4], we get $P(z) = \sum_{l=0}^{2M-1} z^{-l}G_l(z^{2M})$, where $G_l(z)$ are the $2M$ polyphase components of $P(z)$.

A common feature of the modulated filter banks presented in [1] for subband coding, or in [2, 5], for multicarrier transmission, is that they lead to an identical set of biorthogonality conditions. Here, as we focus on the more interesting case in a design context [1], we set $d = 2M-1$. Then, for $0 \leq l \leq M-1$ and $L = 2mM$, the PR condition is given by

$$G_l(z)G_{2M-1-l}(z) + G_{M+l}(z)G_{M-1-l}(z) = \frac{z^{-s}}{2M}, \quad (3)$$

with s an integer parameter between 0 and $m-1$.

2.2 Factorization with the lifting scheme

Using the lifting scheme, which was originally proposed for the construction of biorthogonal wavelets [6], one can obtain, as in [3], a factorization form of the PR condition (3). We denote by $\boldsymbol{A}$ and $\boldsymbol{B}$ the matrices used, with their inverses, for lifting and dual lifting, respectively, which do not affect the delay and, similarly by $\boldsymbol{C}$ and $\boldsymbol{D}$, the ones which, with their inverses, increase the delay. In the simplest case each matrix only depends upon a real parameter, denoted by α

$$\boldsymbol{A}(\alpha) = \begin{bmatrix} 1 & 0 \\ \alpha z^{-1} & 1 \end{bmatrix},\ \boldsymbol{B}(\alpha) = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}, \quad (4)$$

$$\boldsymbol{C}(\alpha) = \begin{bmatrix} z^{-1} & 0 \\ \alpha & 1 \end{bmatrix},\ \boldsymbol{D}(\alpha) = \begin{bmatrix} 1 & \alpha \\ 0 & z^{-1} \end{bmatrix}. \quad (5)$$

$$\text{with } \boldsymbol{F}_0 = \begin{bmatrix} 1 & 0 \\ \alpha_1^l & 1 \end{bmatrix}\begin{bmatrix} 1 & \alpha_2^l \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \alpha_3^l & 1 \end{bmatrix}. \quad (8)$$

When l goes from 0 to $\frac{M}{2}-1$, $2m+1$ parameters, denoted α_i^l, $i = 1,\ldots,2m+1$, correspond to each quadruplet $(G_l, G_{M-1-l}, G_{M+l}, G_{2M-1-l})$. Let us present the corresponding factorization when s is even, knowing that the one when s is odd can be similarly derived. If s is even we can set $s = 2j_1$, $m = i_1 + j_1 + 1$, i_1 and j_1 being integers. Then the PR condition can be rewritten using equations (6)-(7).

2.3 Design criteria

The factorization given by (6)-(7) for s even structurally ensures the PR property as does its counterpart for s odd. The design problem is to optimize the prototype P with respect to the set of parameters such that $\boldsymbol{\alpha} = (\alpha_i^l)_{i=1,\cdots,2m+1, l=0,\cdots,\frac{M}{2}-1}$. As in [1], the first criterion we have selected is related to the out-of-band energy. Using a normalized frequency, i.e. a sampling frequency equal to 1, the objective function to be minimized is written as

$$J(\boldsymbol{\alpha}) = \frac{E(f_c)}{E(0)} \text{ with } E(x) = \int_x^{\frac{1}{2}} |P(e^{j2\pi\nu})|^2 d\nu, \quad (9)$$

where f_c is the cutoff frequency and $0 \leq J(\boldsymbol{\alpha}) \leq 1$.

Another criterion, in use in the context of multicarrier modulation [2], is time-frequency localization. The objective function to be maximized is given by

$$\xi(\boldsymbol{\alpha}) = \frac{1}{\sqrt{4m_2M_2}}, \quad (10)$$

with m_2 and M_2, as defined in [7], the modified second order moments in time and frequency, respectively. In this case we know that $0 \leq \xi(\boldsymbol{\alpha}) \leq 1, \xi(\boldsymbol{\alpha}) = 1$ being the optimum.

3 Compact representation

3.1 Analysis of the optimal solutions

A similar analysis to the one proposed in [8] for orthogonal CMFBs shows that, with the lifting scheme, the optimized solutions lead, to a relatively smooth function $l \mapsto \alpha_i^l$ for all $i \in \{1,\ldots,2m+1\}$. Let us define the application ϕ_M from the set $\{0,\ldots,\frac{M}{2}-1\}$ to the interval $[0, 0.5)$ by

$$\phi_M(l) = \frac{2l+1}{2M}. \quad (11)$$

$$[G_l(z), G_{M+l}(z)] = \frac{1}{\sqrt{4M}}[1\ 1]\,\boldsymbol{F}_0 \prod_{i=1}^{i_1} \boldsymbol{A}(\alpha_{2i+2}^l)\boldsymbol{B}(\alpha_{2i+3}^l) \prod_{j=1}^{j_1} \boldsymbol{C}(\alpha_{2i_1+2j+2}^l)\boldsymbol{D}(\alpha_{2i_1+2j+3}^l), \quad (6)$$

$$\begin{bmatrix} G_{2M-1-l} \\ G_{M-1-l} \end{bmatrix} = \frac{1}{\sqrt{4M}} \prod_{j=j_1,-1}^{1} z^{-2}\boldsymbol{D}^{-1}(\alpha_{2i_1+2j+2}^l)\boldsymbol{C}^{-1}(\alpha_{2i_1+2j+1}^l) \prod_{i=i_1,-1}^{1} \boldsymbol{B}^{-1}(\alpha_{2i+3}^l)\boldsymbol{A}^{-1}(\alpha_{2i+2}^l)\ \boldsymbol{F}_0^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}. \quad (7)$$

Fig. 2 and 3 illustrate the smoothness of the function $l \mapsto \alpha_i^l$ with respect to $\phi_M(l)$.

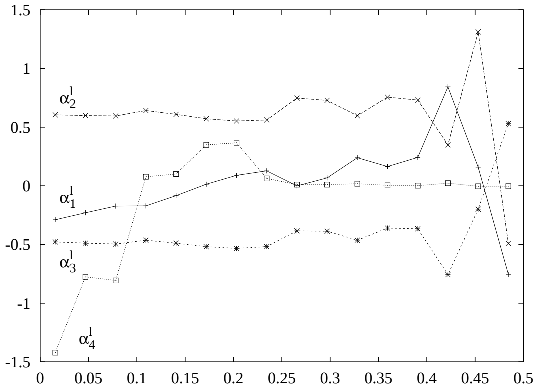

Figure 2: Optimization of the localization: The first four α_i^l coefficients (as a function of $\phi_M(l)$ and for $i = 1, \cdots, 4$) of the lifting scheme with $M = 32$, $L = 192$, $s = 0$.

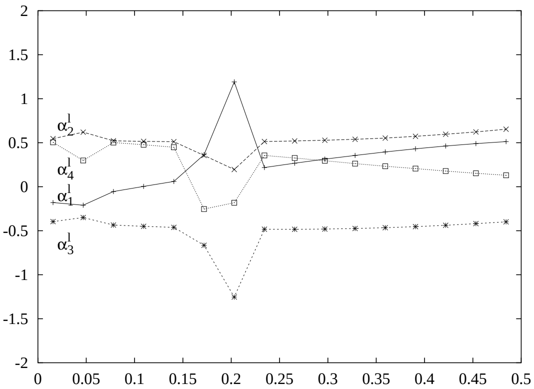

Figure 3: Optimization of the out-of-band energy: The first four α_i^l coefficients (as a function of $\phi_M(l)$ and for $i = 1, \cdots, 4$) of the lifting scheme with $M = 32$, $L = 192$, $s = 0$.

3.2 Introduction of the compact representation

We denote by $\mathcal{P}_K$ a K-dimensional vector subspace of the real functions defined on $[0, 0.5)$ and by $\mathcal{B} = (p_1, \ldots, p_K)$ a basis of this subspace. To any K-uple $X_i = (x_i^1, \ldots, x_i^K)$, with $i = 1, \cdots, 2m+1$, we associate the α_i^l coefficients by the application $\alpha_i^l = \alpha_i(\phi_M(l))$ with $\alpha_i = x_i^1 p_1 + \cdots + x_i^K p_K$. The sequence $X = (X_1, \cdots, X_{2m+1})$, belonging to $(\mathbf{R}^K)^{2m+1}$, is the whole set of all the compact representations. We will also say that the application $(x_i^k) \mapsto P_X(z)$ corresponds to a code or that, equivalently, $P_X(z)$ is coded by the set $(x_i^k), k = 1, \ldots, K, i = 1, \ldots, 2m+1$. For the simulations presented in this paper, we used the two following codes:

The trivial code– We choose $K = \frac{M}{2}$ and we set $x_i^{l+1} = \alpha_i^l$ for all $l = 0, \cdots, \frac{M}{2} - 1$, $i = 1, \cdots, 2m+1$.

The Chebyshev code– For $K \geq 1$, $\mathcal{P}_K$ is the subspace of polynomials of degree $\leq K - 1$ and the selected basis is derived from the set of Chebyshev polynomials $T_k, k = 0, \ldots, K-1$. We then set

$$\alpha_i^l = \sum_{k=1}^{K} x_i^k T_{k-1}(4\phi_M(l) - 1). \qquad (12)$$

3.3 Optimization

The choice of a compact representation with $K < \frac{M}{2}$, naturally leads to a reduction in size of the parameter space and, consequently, of the CPU design time. Let the set of variables associated to the compact representation be denoted by $\boldsymbol{x} = (x_i^k), 1 \leq i \leq 2m+1, 1 \leq k \leq K$, then the unconstrained optimization problems we have to solve are given by

$$\min_{\boldsymbol{x}} \tilde{J}(\boldsymbol{x}) \text{ or by } \max_{\boldsymbol{x}} \tilde{\xi}(\boldsymbol{x}), \qquad (13)$$

with $\tilde{J}(\boldsymbol{x}) = J(\boldsymbol{\alpha})$ and $\tilde{\xi}(\boldsymbol{x}) = \xi(\boldsymbol{\alpha})$ for the minimization of the out-of-band energy and the maximization of the time-frequency localization, respectively.

For both optimization problems, several experiments have shown that, if no good initial solution is available, the Chebyshev code can not only drastically reduce the CPU design time but also can provide optimized solutions of better quality than the trivial code. This problem, when using directly the trivial code, naturally tends to become more critical when the number of free parameters increases.

		$L = 4M$	$L = 6M$	$L = 8M$
$M = 16$	J	2.0332×10^{-3}	4.4090×10^{-4}	2.4713×10^{-4}
	$J^\star$	2.0332×10^{-3}	4.4089×10^{-4}	2.4260×10^{-4}
$M = 32$	J	2.0364×10^{-3}	4.4746×10^{-4}	2.5275×10^{-4}
	$J^\star$	2.0364×10^{-3}	4.4306×10^{-4}	2.4465×10^{-4}
$M = 64$	J	2.0372×10^{-3}	4.5111×10^{-4}	2.5238×10^{-4}
	$J^\star$	2.0371×10^{-3}	4.4890×10^{-4}	2.4980×10^{-4}

Table 1: Best out-of-band energy (J) for $s = 1$ found for the Chebyshev code with $K = 8$ followed by an optimization with the trivial code ($J^\star$).

In order to illustrate the efficiency of the compact representation, when using the energy criterion, a first set of results is presented in Table 1. These results have been obtained with the CSFQP software [9], setting $s = 1$ and using the following procedure:

- Firstly, the optimization is run with a Chebyshev code such that $K = 8$;
- Secondly, the result obtained with the compact representation is used as an initial solution to solve the corresponding design problem with the trivial code.

It can be noted that the optimum found with the Chebyshev code (J) are very close from the ones provided by the trivial codes (J^*). Furthermore, even if for this highly non linear problem we do not know how close (or far) we are from a global optimum, we can clearly see that, for a given M, the performance regularly increases with the number of free parameters. But the main advantage of the compact representation, compared to the trivial code, is that we can get good results for practically any value of M.

4 Design examples

At first, we present an example corresponding to a design with $M = 2048$, $L = 16384$. The Chebyshev code is such that $K = 5$, i.e. only 45 free variables instead of 9216 are used. In this example, depicted in Fig. 4, the cutoff frequency is $f_c = \frac{1}{2M}$ and the out-of-band energy is equal to $2.52 \times 10^{-4}, 1.04 \times 10^{-4}, 8.63 \times 10^{-5}$ respectively for $s = 1, 2, 3$.

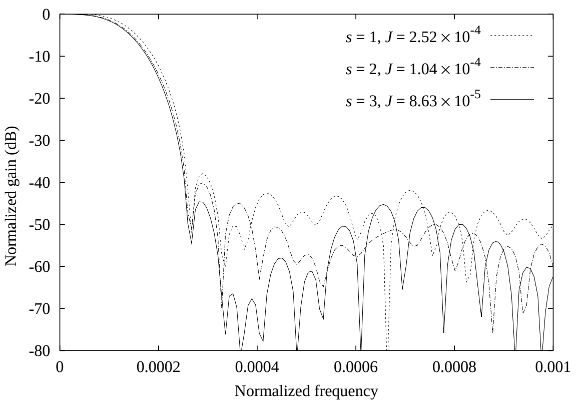

Figure 4: Frequency representations of biorthogonal prototype filters for $M = 2048, L = 16384, s = 1, 2, 3$ resulting from optimization of the out-of-band-energy criterion with the Chebyshev code ($K = 5$).

When using the time-frequency localization criterion, it can be shown [10] that degenerated solutions can be found that nearly attain the upper bound but, due to their poor frequency behaviour, are useless in practice. Such degeneracies can be controlled and, as shown in [5], useful prototypes that are nearly optimal can be obtained thanks to the compact representation. In table 2 we present the results obtained for a filter bank with 2048 subbands and prototype filters with different lengths. The biorthogonal CMFBs are such that $D = 4095$, which has to be compared to their orthogonal counterparts which are also designed using a compact representation approach [8] and have a delay given by $D = L - 1$.

5 Conclusion

A fast design method has been proposed for biorthogonal modulated filter banks which can provide biorthogonal prototypes being nearly optimal, with different optimization criteria, and for practically any number of subbands or subcarriers.

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	Orthogonal		Biorthogonal	
L	J^*	ξ^*	J^*	ξ^*
4096	1.898×10^{-2}	0.9057	1.898×10^{-2}	0.9057
8192	2.037×10^{-3}	0.9330	2.923×10^{-3}	0.9513
12288	3.182×10^{-4}	0.9763	1.370×10^{-3}	0.9563
16384	8.634×10^{-5}	0.9794	1.033×10^{-3}	0.9564

Table 2: Best measures found with $M = 2048$ for the out-of-band energy (J^*) and localization (ξ^*) criteria for long biorthogonal ($s = 0, D = 4095$) and orthogonal ($D = L - 1$) prototypes.