NEW CONTRASTS FOR BLIND SEPARATION OF NON IID SOURCES IN THE CONVOLUTIVE CASE

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ABSTRACT

This paper deals with the separation of convolutive mixtures of independent source signals. Starting from a frequency point of view, we consider a joint diagonalization criterion. Integration of the latter over the frequency domain leads to contrasts which are valid for both iid and non iid sources. A generalization of these contrasts is proposed: it aims at obtaining contrasts with improved statistical performances. A link with existing time-domain contrasts is established. A simple gradient based method for the optimization of the contrasts is proposed and evaluated through simulation tests.

1 INTRODUCTION

Source separation has been a research field of increasing interest in the last years. Possible applications can be found in many scientific and engineering fields such as radio-telescopy, data communication, seismic exploration, ...

The separation of instantaneous mixtures, also known as the ICA problem (Independent Component Analysis) was first studied in [2]; some generalizations of the contrasts were proposed in this case [9]. However, the separation of convolutive mixtures remains a challenging problem. Under the assumption of iid sources, time-domain approaches proved to be successfull in designing various contrast functions [3, 6]. A frequency-domain framework has also been proposed [1]. This approach has been extended in order to exploit the time-dependent structure of the signals and consequently help us overcome the difficulty of separating non iid sources [8]. Nevertheless, this approach leads to contrasts which may be difficult to estimate statistically and require a large data size. The goal of this work is to try to simplify these contrasts, and relate them to simpler time-domain contrasts.

We first formulate the problem and recall some existing results. We then introduce some new frequency-domain contrasts which allow us to derive time-domain ones in Section 4. A simple parametrization of the separating system is proposed in Section 5 and a gradient descent optimization of the resulting cost function is then realized and applied to simulation examples.

2 PROBLEM FORMULATION

We consider a convolutive mixture of $N \in \mathbb{N}^*$ unknown source signals. The output of the mixture is represented by the N-dimensional observation vector:

$$\begin{aligned}\mathbf{x}(n) &\triangleq (x_1(n), \ldots, x_N(n))^T \\ &= \sum_{k\in\mathbb{Z}} \mathbf{c}(k)\mathbf{s}(n-k) + \mathbf{b}(n)\end{aligned} \tag{1}$$

where $\mathbf{s}(n) = (s_1(n), \ldots, s_N(n))^T$ is a $N \times 1$ source vector, $\mathbf{b}(n)$ is a $N \times 1$ noise vector and $\mathbf{c}(n)_{n\in\mathbb{Z}}$ represents the unknown linear time invariant (LTI) mixing system.

The following assumptions are made in this paper:

A.1 The sources $s_i(n)$, $i \in \{1, \ldots, N\}$ are mutually independent random sequences which are uncorrelated, and have unit variance. The trispectrum of source $s_i(n)$ (i.e. the Fourier transform with respect to $(\tau_1, \tau_2, \tau_3) \in \mathbb{Z}^3$ of $\mathrm{Cum}[s_i(n), s_i^*(n+\tau_1), s_i(n+\tau_2), s_i^*(n+\tau_3)]$) is assumed to be defined and will be denoted by $\Gamma_i^4(\omega_1, \omega_2, \omega_3)$.

A.2 For all $(i, j) \in \{1, \ldots, N\}^2$, the filter with impulse response $(c_{ij}(n))_{n\in\mathbb{Z}}$ is stable. The frequency response matrix $\mathbf{C}(\omega) \triangleq \sum_{k=-\infty}^{\infty} \mathbf{c}(k)e^{-ik\omega}$ therefore exists and is supposed invertible for all $\omega \in [-\pi, \pi)$.

A.3 The noise $(\mathbf{b}(n))_{n\in\mathbb{Z}}$ is Gaussian, zero-mean, independent of the source vector $(\mathbf{s}(n))_{n\in\mathbb{Z}}$ and stationary with known spectrum density matrix.

The multichannel blind deconvolution problem consists in estimating a LTI filter $(\mathbf{h}(n))_{n\in\mathbb{Z}}$ such that the vector:

$$\mathbf{y}(n) \triangleq \sum_{k\in\mathbb{Z}} \mathbf{h}(k)\mathbf{x}(n-k) \tag{2}$$

restores the N input signals $(s_i(n))_{n\in\mathbb{Z}}, i \in \{1, \ldots, N\}$.

Because of some remaining indeterminacies, the best one can expect is to obtain a separating LTI filter such that:

$$\mathbf{H}(\omega)\mathbf{C}(\omega) = \mathbf{P}\, e^{i(\mathbf{\Theta}+\mathbf{D}\omega)} \tag{3}$$

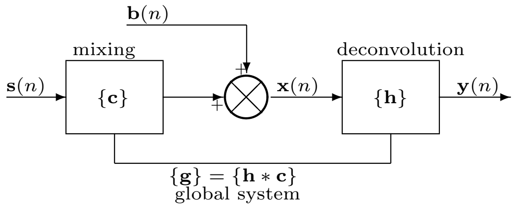


Figure 1: The global system

where $\mathbf{P}$ is a permutation matrix, $\mathbf{\Theta}$ is a real diagonal matrix and $\mathbf{D}$ is an integer diagonal matrix. When such a property holds, we will say that a type-I separation is achieved. Sometimes however, we are only able to guarantee that

$$\mathbf{H}(\omega)\mathbf{C}(\omega) = \mathbf{P}\, e^{i\mathbf{\Phi}(\omega)} \tag{4}$$

where $\mathbf{\Phi}(\omega)$ is a real diagonal matrix, and we will say that a type-II solution is obtained.

3 FREQUENCY DOMAIN CONTRASTS

3.1 Contrasts

Contrasts were first introduced in [2]. We use here a definition similar to that given in [8].

Let $\mathcal{A}$ denote the set of $N \times 1$ random vectors satifying assumption A.1 and $\mathcal{G}$ be the set of systems composed of a mixing system satisfying A.2 followed by a separating system $\{\mathbf{h}\}$. Let $\mathcal{Y}_{\mathcal{A}}$ be the set of random vectors obtained with relations (1) and (2) where $\mathbf{s}$ lies in $\mathcal{A}$ and the global system lies in $\mathcal{G}$. The subset of $\mathcal{G}$ such that equation (3) (resp. (4)) holds is denoted $\mathcal{P}_{\mathrm{I}}$ (resp. $\mathcal{P}_{\mathrm{II}}$).

Definition 1 *A type-I (resp. type-II) contrast is a real valued function on $\mathcal{Y}_{\mathcal{A}}$ satisfiying the following two requirements:*

$$\forall \mathbf{s} \in \mathcal{A}\ \forall \mathbf{g} \in \mathcal{G} \quad \mathcal{I}([\mathbf{g}]\mathbf{s}) \leq \mathcal{I}(\mathbf{s})$$
$$\forall \mathbf{s} \in \mathcal{A}\ \forall \mathbf{g} \in \mathcal{G} \quad \mathcal{I}([\mathbf{g}]\mathbf{s}) = \mathcal{I}(\mathbf{s}) \Rightarrow \mathbf{g} \in \mathcal{P}_I \quad (resp. \mathcal{P}_{II})$$

Consequently, the maximisation of a type-I contrast with respect to $\{\mathbf{g}\}$ leads to the separation of the sources. The maximisation of a type-II contrast leaves an all-pass filtering ambiguity; we should mention however that if we restrict ourselves to finite impulse response (FIR) filters, the all-pass filtering ambiguity reduces to a delay ambiguity.

3.2 A joint diagonalization criterion

Let us define:

$$\mathcal{I}(\omega_1,\omega_2,\omega_3) = \sum_{i,j} |\Gamma_j^4(\omega_1,\omega_2,\omega_3)|^2 |G_{ij}(-\omega_1-\omega_2-\omega_3)|^2 |G_{ij}(-\omega_1)|^2$$
$$= \sum_{il_1l_2} |K^4_{il_1l_2}(\omega_1,\omega_2,\omega_3)|^2 \tag{5}$$

where $(K^4_{i_1i_2l_1l_2}(\omega_1,\omega_2,\omega_3))_{i_1,i_2,l_1,l_2}$ correspond to the cross-trispectra of the outputs i_1, i_2, l_1, l_2 of the global system. $\mathcal{I}$ was introduced in [8] and was shown to be equal to a joint diagonalization criterion of a given set of matrices.

It is important to note that, $\mathcal{I}(\omega_1,\omega_2,\omega_3)$ is a function of $\mathbf{G}$. In practice, the mixed signals $\mathbf{x}(n)$ are prewhitened with a pre-whitening filter $\{\mathbf{v}\}$. The separating system can thus be split in $\{\mathbf{h}\} = \{\mathbf{f} * \mathbf{v}\}$; the criterion $\mathcal{I}$ then depends on the remaining para-unitary separating system $\{\mathbf{f}\}$ only. For notation concision, we will not make this dependence explicit for $\mathcal{I}(\omega_1,\omega_2,\omega_3)$ or any other criteria which will be derived from it.

It was stated in [8] that,

$$\mathcal{I}_{\mathcal{E}} \triangleq \int_0^{2\pi}\int_0^{2\pi}\Big(\int_{\mathcal{E}(\omega,\nu)} \mathcal{I}(\omega, \frac{\nu+\alpha}{2}, \frac{\nu-\alpha}{2})\, d\alpha\Big)\, d\omega\, d\nu \tag{6}$$

is a type-II contrast under the following assumption:

A.4 For almost all $(\omega,\nu) \in [0,2\pi)^2$, there exists a set $\mathcal{E}(\omega,\nu) \subset [-2\pi, 2\pi)$ such that, for at least $N-1$ sources:

$$\int_{\mathcal{E}(\omega,\nu)} \Big|\Gamma_j^4(\omega, \frac{\nu+\alpha}{2}, \frac{\nu-\alpha}{2})\Big|^2 d\alpha \neq 0. \tag{7}$$

4 GENERALIZED CONTRASTS

4.1 New contrasts

Let us recall the following property [7, 11]:

Property 1 *Let $\mathcal{I}_1$ be a real-valued function on $\mathcal{Y}_{\mathcal{A}}$ and $\mathcal{I}_2$ be a contrast. If*

$$\forall \mathbf{s} \in \mathcal{A} \qquad \mathcal{I}_1(s) = \mathcal{I}_2(s) \tag{8}$$
$$\forall \mathbf{y} \in \mathcal{Y}_{\mathcal{A}} \qquad \mathcal{I}_1(\mathbf{y}) \leq \mathcal{I}_2(\mathbf{y}) \tag{9}$$

then, $\mathcal{I}_1$ is a contrast.

Let us define:

$$\mathcal{J}(\omega_1,\omega_2,\omega_3) \triangleq \sum_{il_1l_2} \eta_{il_1l_2} |K^4_{iil_1l_2}(\omega_1,\omega_2,\omega_3)|^2 \tag{10}$$

$$\text{where:} \quad \eta_{il_1l_2} \begin{cases} = 1 & \text{if } i = l_1 = l_2 \\ \leq 1 & \text{otherwise.} \end{cases} \tag{11}$$

We can generalize the contrast (6) and consider now:

$$\mathcal{J}_{\mathcal{E}} \triangleq \int_0^{2\pi}\int_0^{2\pi}\Big(\int_{\mathcal{E}(\omega,\nu)} \mathcal{J}(\omega, \frac{\nu+\alpha}{2}, \frac{\nu-\alpha}{2})\, d\alpha\Big)\, d\omega\, d\nu \tag{12}$$

According to (5), (6) and the preceding definition, we have $\mathcal{J}_{\mathcal{E}} \leq \mathcal{I}_{\mathcal{E}}$. Furthermore, when the outputs of the global system are equal to the sources up to some permutation and all-pass filtering operations, the terms in (10) and (5) for which one of the indices i, l_1, l_2 differs from the two others vanish. Consequently we have in this case $\mathcal{J}_{\mathcal{E}} = \mathcal{I}_{\mathcal{E}} = \max_{\mathbf{g}\in\mathcal{G}} \mathcal{I}_{\mathcal{E}}$. Using Property 1, we conclude that:

Proposition 1 *Under assumption A.4, $\mathcal{J}_{\mathcal{E}}$ is a type-II contrast.*

Using the same arguments, and results from [8] it can be proved that:

Proposition 2 *Under assumption*

A.5 For almost all $(\omega, \nu) \in [0, 2\pi)^2$, there exists $\alpha_{\omega,\nu} \in [-2\pi, 2\pi)$ such that for at least $N-1$ sources:

$$\Gamma_j^4\Big(\omega, \frac{\nu + \alpha_{\omega,\nu}}{2}, \frac{\nu - \alpha_{\omega,\nu}}{2}\Big) \neq 0, \qquad (13)$$

the following expression is a type-II contrast:

$$\bar{\mathcal{J}} \triangleq \int_0^{2\pi}\int_0^{2\pi} \mathcal{J}\Big(\omega, \frac{\nu + \alpha_{\omega,\nu}}{2}, \frac{\nu - \alpha_{\omega,\nu}}{2}\Big)\, d\omega\, d\nu \qquad (14)$$

4.2 Connection with known time-domain contrasts

Using the 2π-periodicities of $\mathcal{J}(\omega_1, \omega_2, \omega_3)$, we can prove after some changes of variables that:

$$\frac{1}{2}\mathcal{J}_{[-2\pi,2\pi)} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \mathcal{J}(\omega_1, \omega_2, \omega_3)\, d\omega_1\, d\omega_2\, d\omega_3 \qquad (15)$$

Using Parseval's equality and (10), equation (15) yields:

$$\frac{1}{16\pi^3}\mathcal{J}_{[-2\pi,2\pi)} = \sum_{il_1l_2}\sum_{\tau_1\tau_2\tau_3} \eta_{il_1l_2}|\kappa^4_{iil_1l_2}(\tau_1, \tau_2, \tau_3)|^2 \qquad (16)$$

where: $\kappa^4_{iil_1l_2}(\tau_1, \tau_2, \tau_3) \triangleq$
$\mathrm{Cum}[y_i(n), y_i^*(n+\tau_1), y_{l_1}(n+\tau_2), y_{l_2}^*(n+\tau_3)]$

By setting to zero some of the weighting factors $\eta_{il_1l_2}$, the expressions of the obtained criteria may appear simpler than that of $\mathcal{I}_{\mathcal{E}}$ since they contain fewer terms. As a first special case, we can choose $\eta_{il_1l_2} = \delta_{l_1-l_2}$, where δ stands for the Kronecker symbol. We have then:

$$\frac{1}{16\pi^3}\mathcal{J}_{[-2\pi,2\pi)} = \sum_{il}\sum_{\tau_1\tau_2\tau_3} |\kappa^4_{iill}(\tau_1, \tau_2, \tau_3)|^2 \qquad (17)$$

This reminds us contrasts introduced in [5] for instantaneous mixtures and in [11] for the convolutive iid case. In particular, extensions to the convolutive case of contrasts introduced by L. Delathauwer and E. Moreau for instantaneous mixtures can be obtained in this way. An alternative choice for the weighting factors is $\eta_{il_1l_2} = \delta_{l_1-i}$. Then, the contrast $\mathcal{J}_{[-2\pi,2\pi)}/(16\pi^3)$ can be written:

$$\frac{1}{16\pi^3}\mathcal{J}_{[-2\pi,2\pi)} = \sum_{il}\sum_{\tau_1\tau_2\tau_3} |\kappa^4_{iiil}(\tau_1, \tau_2, \tau_3)|^2 \qquad (18)$$

By choosing $\eta_{il_1l_2} = \delta_{i-l_1}\delta_{i-l_2}$, we obtain the simplest form of the contrast:

$$\frac{1}{16\pi^3}\mathcal{J}_{[-2\pi,2\pi)} = \sum_{i}\sum_{\tau_1\tau_2\tau_3} |\kappa^4_{iiii}(\tau_1, \tau_2, \tau_3)|^2 \triangleq \zeta \qquad (19)$$

4.3 Further simplifications for source cumulant fields with limited support

The contrasts proposed in the preceding section may be simplified for source signals which have only a restricted number of non-zero auto-cumulants. Invoking again Property 1, we can indeed see that:

Proposition 3 *If the sources have fourth order cumulants with support $\mathcal{S}$, one still obtains a contrast by replacing the infinite sum on τ_1, τ_2, τ_3 in (16), (17), (18) or (19) by sum over $\mathcal{S}$.*

It is clear that the support of the cumulant fields of the outputs $(y_i(n))_{n\in\mathbb{Z}}$, $i \in \{1, \ldots, N\}$, of the global system may be much larger than $\mathcal{S}$. This means that Proposition 3 often leads to a substantial reduction of the number of terms in the contrasts derived in the previous section.

Consider for instance that the sources are described by a so-called *stochastic volatility model*, i.e. $\forall i \in \{1, \ldots, N\}\ \ s_i(n) = a_i(n)w_i(n)$ where $a_i(n)$, $w_j(n)$ are stationnary, mutually independent random sequences with unit variance and $w_j(n)$ moreover is iid, zero-mean and Gaussian. One can check that this random process is uncorrated but non iid. The fourth-order auto-cumulant equals:

$$\mathrm{Cum}[s_i(n), s_i(n+\tau_1), s_i(n+\tau_2), s_i(n+\tau_3)] = (M_{a_i}(\tau_1, \tau_2, \tau_3) - 1)(\delta_{\tau_1}\delta_{\tau_2-\tau_3} + \delta_{\tau_2}\delta_{\tau_1-\tau_3} + \delta_{\tau_3}\delta_{\tau_1-\tau_2}) \qquad (20)$$

where, E{.} being the mathematical expectation:

$$M_{a_i}(\tau_1, \tau_2, \tau_3) \triangleq \mathrm{E}\{a_i(n)a_i(n+\tau_1)a_i(n+\tau_2)a_i(n+\tau_3)\}$$

Using Proposition 3, the contrast given by (19) reduces to:

$$\begin{aligned}\zeta &= \sum_i\sum_\tau |\kappa^4_{iiii}(0,\tau,\tau)|^2 + \\ &\quad |\kappa^4_{iiii}(\tau,0,\tau)|^2 + |\kappa^4_{iiii}(\tau,\tau,0)|^2 \\ &= \sum_i\Big[|\kappa^4_{iiii}(0,0,0)|^2 + 3\sum_{\tau\neq 0}|\kappa^4_{iiii}(0,\tau,\tau)|^2\Big].\end{aligned} \qquad (21)$$

Furthermore, if the random sequence $a_i(n)$ is a q-th order moving average (MA) process, the fourth-order cumulants given by (20) vanish whenever one of the indices τ_1,τ_2 or τ_3 is greater than q. Consequently, the sum on τ in equation (21) can be further restricted to $|\tau| \leq q$.

5 SIMULATION

5.1 A simple gradient algorithm

As mentionned in paragraph 3.2, the separating filter can be decomposed in a prewhitening operation followed by a separating paraunitary filter $\{\mathbf{f}\}$. Since the contrasts presented in this paper depend on $\{\mathbf{f}\}$ only, a parametrization of $\{\mathbf{f}\}$ will lead to a parametrization of the outputs and of the contrasts. One can then apply classical optimization methods (gradient descent,...) in order to complete the source separation. In particular, a lossless lattice representation can be used, which in the case of two sources, reads:

$$\mathbf{F}(\omega) = \mathbf{R}(\theta_L)\mathbf{D}(\omega)\ \ldots\ \mathbf{R}(\theta_2)\mathbf{D}(\omega)\,\mathbf{R}(\theta_1) \quad \text{where}$$
$$\mathbf{R}(\theta) \triangleq \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad \mathbf{D}(\omega) \triangleq \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\omega} \end{pmatrix}.$$

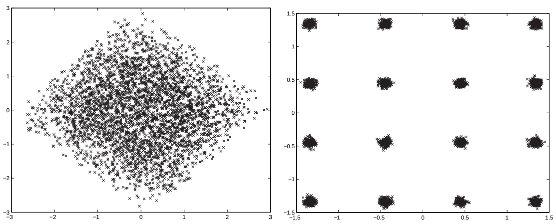
5.2 Simulation results

According to the preceding parametrization, FIR paraunitary filters $\{\mathbf{c}\}$ were generated, by choosing randomly the parameters $(\theta_1, \theta_2, \theta_3)$ in $[0, \frac{\pi}{2}]$. A gradient optimization of (21) was realized with stochastic volatility sources and PAM4 iid sources. Sample sizes of 4096 and 200 realizations were considered.

The optimization algorithm was initialized in the neighborhood of the true parameters (with a standard deviation of about 10 percent) so as to avoid local minima problems. The algorithm was then successful in separating the sources; mean square errors on the reconstructed sources, averaged on 200 realizations are given in Table 1. This shows that the proposed method is useful for separating possibly non iid sources, provided a rough initial guess of the solution is available. This prior information could for example result from a semi-blind approach.

sources	MSE
stochastic volatility	1.9×10^{-3}
PAM4 iid	5.1×10^{-5}

Table 1: Average mean square errors on reconstructed sources



(a) mixture (b) recovered sources

Figure 2: Separation of a mixture of PAM4 sources

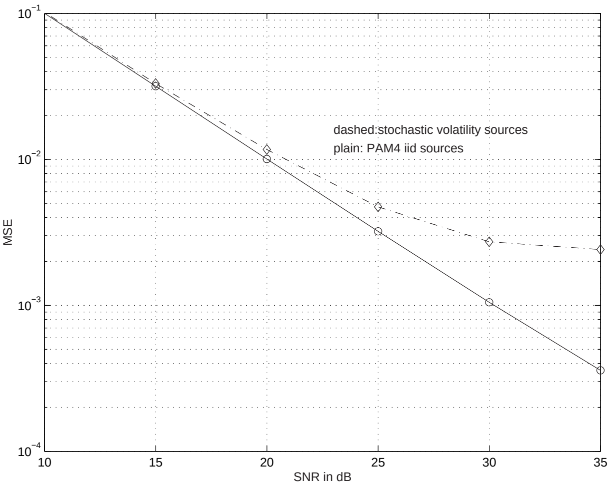


Figure 3: Average mean square error on reconstructed sources versus signal to noise ratio

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