A Novel Channel Estimation Algorithm Applied to UTRA-FDD

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ABSTRACT

Channel estimation is a key step in the initial synchronisation of a RAKE receiver. In this paper we present an algorithm for multipath channel estimation based on an elegant new technique for resolving the multipath components. The performance of the algorithm is assessed through simulation in a full UTRA-FDD compliant uplink scenario. Test results are presented and compared with those of other channel estimation techniques.

1. INTRODUCTION

For applications such as 3G mobile communications, RAKE receivers are often used to utilise the diversity offered by a multipath channel. Such receivers use a matched filter to estimate the impulse response of the channel. The matched filter output consists of two elements: peaks associated with the multipath components and noise due to interference and the imperfect correlation properties of the spreading codes.

Peaks in this measurement can be distinguished from noise by applying a threshold. A common and mathematically sound approach is to base the threshold on the sample standard deviation of the matched filter output, multiplied by some scaling factor. Any peak of magnitude greater than the threshold is considered to be a multipath component. The identified paths constitute the channel delay profile which is used by the RAKE finger allocation mechanism.

2. SYSTEM MODEL

Let $x[k]$ be a known discrete complex sequence such as the chipped pilot symbols in a 3G FDD uplink scenario. Figure 1 illustrates a model of the distortion imposed on the sequence $x[k]$ taking into account a number of significant factors. The first of these is the pulse shaping filter which is usually implemented as two filters split between the transmitter and the receiver [1]. The respective impulse responses of these filters are $p_{tx}(t)$ and $p_{rx}(t)$. The second factor is the multipath channel. This is a complex baseband equivalent model with time-varying impulse response $h_c(t;\tau)$. The third factor is the additive, potentially non-Gaussian noise component $n_c(t)$ which is predominantly due to multiple access interference.

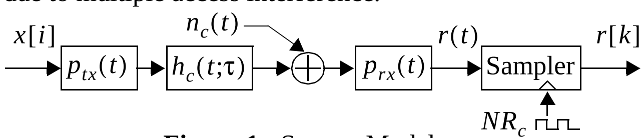

Figure 1: System Model

To simplify analysis it is assumed that the averaging time used to compute a single channel estimate does not exceed the channel coherence time [2]. Therefore, during a particular estimate the stationary impulse of the channel is denoted $h_c(t)$. The combined impulse response of the transmit filter, receive filter and channel is $h(t) = p_{tx}(t) * h_c(t) * p_{rx}(t)$ where $*$ is the convolution operator. Let $n(t)$ be the result of convolving the noise with the receive filter. The baseband waveform $r(t)$ can therefore be expressed as

$$r(t) = \sum_i x[i]h(t - iT_c) + n(t) \tag{1}$$

Note that in practice the receive filter would be implemented using both analogue and digital hardware. The low pass continuous waveform $r(t)$ is only defined here to assist in deriving an equivalent model of the N times oversampled output of the receiver filter. Denoting the chip period as T_c, $r[k]$ is given by

$$r[k] = r(kT_c/N) \tag{2}$$

Substituting Eq. 1 into Eq. 2 yields

$$r[k] = \sum_i x[i]h\left(\frac{kT_c}{N} - iT_c\right) + n\left(\frac{kT_c}{N}\right) \tag{3}$$

The discrete signal $r[k]$ is used along with prior knowledge of the pilot sequence $x[i]$ to perform channel estimation.

3. MATCHED FILTER

Figure 2 shows an efficient implementation of an oversampled filter matched to the pilot sequence. The purpose of this filter is to correlate $x[i]$ with $r[k]$ and obtain an estimate of the channel impulse response. Since the amount of averaging is limited by the coherence time of the

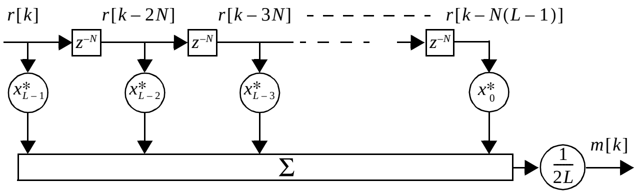

Figure 2: Oversampled Filter Matched to Pilot Sequence.

channel this yields a noisy estimate. The output of the matched filter is

$$m[k] = \frac{1}{2L}\sum_{n=0}^{L-1} x^*_{L-n-1} r[k - Nn] \tag{4}$$

Substituting Eq. 3 into Eq. 4 and re-arranging results in the following decomposition of the matched filter output

$$m[k] = m_h[k] + m_{n_1}[k] + m_{n_2}[k] = m_h[k] + m_n[k] \tag{5}$$

$$m_h[k] = h\left(\frac{kTc}{N} - [L-1]Tc\right)\frac{1}{2L}\sum_{i=0}^{L-1} x_i^* x_i \tag{6}$$

$$m_{n_1}[k] = \frac{1}{2L}\sum_{n=0}^{L-1}\sum_{i \neq L-n-1} x^*_{L-n-1} x_i h\left(T_c\left[\frac{k}{N} - nT_c - iTc\right]\right) \tag{7}$$

$$m_{n_2}[k] = \frac{1}{2L}\sum_{n=0}^{L-1} x^*_{L-n-1} n([k-n]T_c/N) \tag{8}$$

The ideal output of the filter is described by Eq. 6. This is a sampled version of $h(t)$ delayed by the length of the filter and scaled by half the variance of the pilot sequence. Eq. 7 and 8 both represent additive noise at the filter output. $m_{n_1}[k]$ is due to the non-ideal autocorrelation properties of x_i over the averaging period (LT_c). The third component $m_{n_2}[k]$ is due to the non-ideal crosscorrelation properties of x_i with other users and interference. By applying central limit theorem it is clear that both $m_{n_1}[k]$ and $m_{n_2}[k]$ can be modelled as complex Gaussian processes as a result of the outer summation. This means that $|(m_{n_1}[k] + m_{n_2}[k])|$ will have a Rayleigh distribution. For the rest of this paper $m_n[k] = m_{n_1}[k] + m_{n_2}[k]$ will be referred to as "the noise".

4. PROBABILITY OF FALSE PATH DETECTION

In a RAKE receiver only a limited number of RAKE fingers, F, will be available to be assigned multipath components for decoding. Optimal performance can only be obtained if the RAKE fingers are tracking the F reflections with the highest signal to interference ratios. To perform an "F largest" search of the correlation magnitudes over the entire delay span of the matched filter output is impractical, and consequently a common approach is to obtain a reduced set of candidate paths by applying a threshold [3][4][5]. If the variance of the noise σ^2 could be measured accurately then applying a threshold $K\sigma$ to the magnitude of the matched filter output would yield the following probability of detecting a false path in the threshold stage, derived from the PDF of the Rayleigh distribution:

$$P(R > K\sigma) = e^{\frac{-r^2}{2\sigma^2}}\Bigg|_{r = K\sigma} = e^{\frac{-K^2}{2}} \tag{9}$$

This means that K and σ fully determine the probability that a delay at which there is no multipath component in the channel's delay profile will be mistakenly placed in the reduced set of candidate paths by the thresholding mechanism.

5. THE ALGORITHM

In this section an novel algorithm for the accurate determination of the noise variance σ^2 is presented. Consider the filter shown in Figure 3, used to average S successive delay profiles (one for each slot) obtained at the output of the matched filter [6].

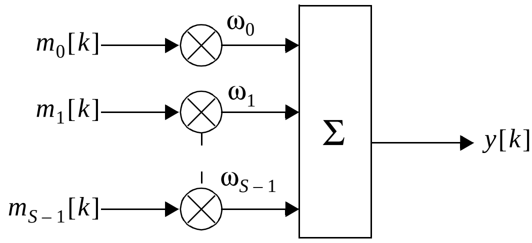

Figure 3: Weighted multi-slot average (WMSA) filter

Each delay profile is M samples in length i.e. $k = 0 \ldots M-1$. In order that this multi-slot averaging can be performed, the matched filter must produce estimates of the delay profile repeating every M samples. This is obtained by reloading the pilot sequence $\boldsymbol{x}$ in the matched filter every M samples with u_i, the pilot chip sequence for the i^{th} slot. It should be noted that this filter has resulted in more averaging at the output $y[k]$. Provided the total averaging time is within the coherence time, this output $y[k]$ is the preferred signal on which to perform thresholding, rather than $m[k]$.

Let vector $\boldsymbol{z}$ be defined as $\boldsymbol{z} = \boldsymbol{m_c} - \boldsymbol{y}$, where the vector $\boldsymbol{y}$ is the output of the WMSA filter across M samples, and $\boldsymbol{m_c}$ is one instantaneous delay profile. C is chosen to be the median element of the sequence $0 \ldots S-1$. To proceed with the analysis, the following assumptions must be made:

Considering the components of matched filter output in the c^{th} slot for the k^{th} sample in the delay profile as: $m_c[k] = m_{c,h}[k] + m_{c,n}[k]$

- The channel is stationary and therefore $\{m_{i,h}[k]\}$ has zero variance across successive slots.
- The noise $\{m_{i,n}[k]\}$ has zero mean across successive slots. Furthermore the individual variances σ_k^2 of the noise at each sample in the delay profile are all equal. i.e.$\sigma_k^2 = \sigma^2; \quad k = 0 \ldots M-1$
- Considering the delay profile as a series of random variables the following further assumptions result:
- $\boldsymbol{m_{c,n}}$ has zero mean, and has variance σ^2.
- The elements of vector $\boldsymbol{m_{c,h}}$ have different first-order statistics due to the channel, and therefore first-order statistics cannot be drawn across the elements of the

vector. However, the second-order statistics are the same: zero variance, due to the stationarity of the channel.
- The sum of the WMSA filter weights ω_i must be unity in order that the output of the WMSA has the same expected mean as the input.

Under these assumptions, the vector $\boldsymbol{z} = \boldsymbol{m_c} - \boldsymbol{y}$ can be shown (by simply taking expectations) to have the following statistics:

$$\mathrm{Var}\{\boldsymbol{z}\} = \frac{\mathrm{Var}\{\boldsymbol{y}\}(1 + B - 2\omega_c)}{B}; \quad \mathrm{E}\{\boldsymbol{z}\} = \boldsymbol{0} \qquad (10)$$

$$\mathrm{Var}\{y[k]\} = B \times \mathrm{Var}\{m_c[k]\} \qquad (11)$$
$$= B\sigma_k^2 = B\sigma^2; \quad k = 0 \ldots M-1$$

Where the constant coefficient B is defined as:

$$B = \sum_{i=0}^{S-1} \omega_i^2 \qquad (12)$$

The output $\boldsymbol{z}$ has an expected mean of zero for each element. By averaging the successive delay profiles, an increasingly accurate estimate of the means $\mathrm{E}\{\boldsymbol{m_h}\}$ is made, and when subtracting this from an instantaneous estimate $\boldsymbol{m_c}$, this effectively removes these means.

The crux of the algorithm is creating the vector $\boldsymbol{z}$ which has identical first and second-order statistics for each element, and further that these statistics are related to $B\sigma^2$ in deterministic manner. Note that $B\sigma^2$ is the variance of interest rather than σ^2 due to using $\boldsymbol{y}$ rather than $\boldsymbol{m}$ as the channel estimate.

6. ESTIMATION OF NOISE VARIANCE

In this section, methods for estimating the noise variance $B\sigma^2$ are described. Four methods are considered:

1. Making no adjustment for the presence of $\mathrm{E}\{\boldsymbol{m_h}\}$, calculate the sample variance across vector $\boldsymbol{y}$ (this is included for reference to assess the extent of the success of methods 2, 3 and 4 at removing $\mathrm{E}\{\boldsymbol{m_h}\}$).

2. Removing the four elements of $\boldsymbol{y}$ which have the largest magnitude, and calculate the sample variance across $\boldsymbol{y}$ [3].

3. Removing the six elements with largest magnitude in $\boldsymbol{y}$ and also those elements within $3T_C/4$ either side of those elements. In the case of $N = 2$ i.e. two times oversampling, this entails removing the samples adjacent to the six elements with the largest magnitudes. The sample variance is then calculated across vector $\boldsymbol{y}$ [4][5].

4. Performing the algorithm described in Section 5, and then calculating the sample variance across vector $\boldsymbol{z}$, and applying the relationship between the variances of $\boldsymbol{y}$ and $\boldsymbol{z}$ given in Eq. 10.

Let U denote the set of indices for which an element has been removed in method 2, and V denote the set of indices for elements removed in method 3. Let $\varepsilon(\boldsymbol{y}, \boldsymbol{W})$ be the estimate of the variance of the complex vector $\boldsymbol{y}$ (nominally of length M) when excluding the set of indices $\boldsymbol{W}$. $\varepsilon(\boldsymbol{y}, \boldsymbol{W})$ is defined as:

$$\varepsilon(\boldsymbol{y}, \boldsymbol{W}) = \frac{1}{R-1} \sum_{k=0; k \notin \boldsymbol{W}}^{M-1} (y_r^2[k] + y_i^2[k]) - \frac{1}{R(R-1)} \left(\sum_{k=0; k \notin \boldsymbol{W}}^{M-1} (y_r[k] + y_i[k]) \right) \qquad (13)$$

with $R = 2(M - |\boldsymbol{W}|)$. The four methods can then be stated mathematically as:

$1{:}\varepsilon(\boldsymbol{y}, \varnothing)$ $2{:}\varepsilon(\boldsymbol{y}, U)$ $3{:}\varepsilon(\boldsymbol{y}, V)$ $4{:}\dfrac{B \times \varepsilon(\boldsymbol{z}, \varnothing)}{(1 + B - 2\omega_c)}$

7. SIMULATION CONFIGURATION

Simulations were configured as follows: $m[k]$ was generated as described in Section 3. N was set to 2, and L was 512, hence spanning two pilot symbols. M was set equal to L. Noise $n_c(t)$ was added at a level of -5dB E_c/N_0. A bearer service of 12.2kbps was assumed. The WMSA filter was configured with $S = 5$ and each $\omega_i = 0.2$. Three models were used for $h_c(t;\tau)$:

- “Case 2” channel from the 3GPP specifications [7].
- Vehicular B from the ETSI UMTS specifications [8].
- “Case 3” channel from the 3GPP specifications [7].

Rather than modelling these environments at their maximum velocities (3km/h, 500km/h and 120km/h respectively) the propagation models were made stationary. Three different initialisations were used to yield nine instances in total, three for each propagation model. This enabled average statistics on the performance of each method under each of the nine $h_c(t)$ to be drawn over a extended period of time (0.1s). The three instances of each model were chosen to give a variety of different relative multipath component strengths.

8. RESULTS

The mean squared error in the nine standard deviations resulting from the different variance estimates are shown in Table 1. In order to compute the error, a perfect estimate of the variance was obtained by calculating $B \times \varepsilon(\boldsymbol{m_n}, \varnothing)$ for each of the nine propagation environments.

TABLE 1- Mean squared errors in standard deviations for the various estimation methods.

1	**2**	**3**	**4**
3.42×10^{-6}	1.95×10^{-7}	1.39×10^{-8}	4.39×10^{-9}

It can be seen that the method presented in Section 5, method 4, performs significantly better than other methods.

The selection of threshold factor K (denoted a in [5] and M in [3]) is, as described in Section 4, an expression of the risk of introducing noise into the candidate set of paths for RAKE finger allocation. [3] and [5] propose $K = 3$ or $K = 4$ to be suitable, and using Eq. 9, this results in noise introduction probabilities of 11.1×10^{-3}. and 0.335×10^{-3} respectively. Figure 4 illustrates the effect of inaccurate estimation of $B\sigma^2$, in terms of the degree (in orders of magnitude) of the resulting inaccuracy in the probability of introducing noise. Both worst case and average case

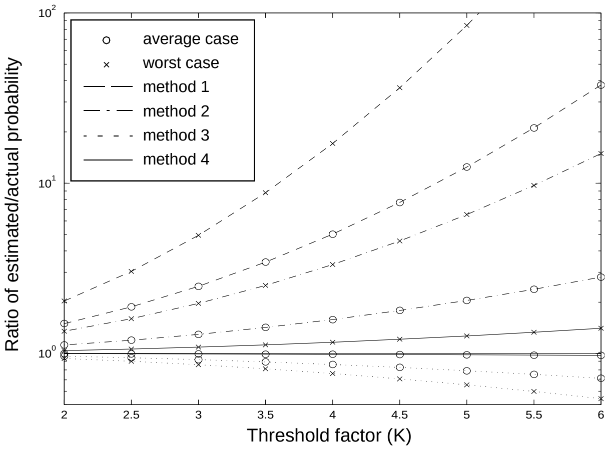

Figure 4: Effect of accuracy of variance estimate on the probability of noise following from threshold factor K

performance across the nine propagation environments are shown. Again method 4 can be seen to give the best performance. As there is a subtle relationship between diminishing the risk of noise and maintaining paths of weaker SNRs[5], it is important to make as accurate an estimate of $B\sigma^2$ as possible.

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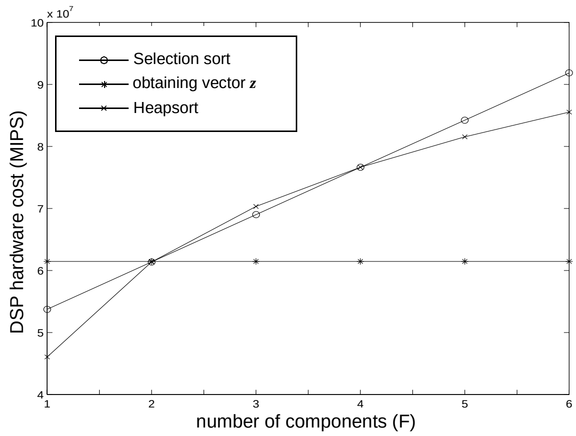

Figure 5: Cost comparison of different methods

Figure 5 shows a cost comparison of different methods of making the "F largest" selection [9] against the cost of obtaining the vector $\mathbf{z}$. The cost of the sample variance calculation has also been incorporated, as it varies depending on the excluded set $\mathbf{W}$.The figure in Million Instructions per Second (MIPS) has been derived assuming single cycle addition, multiplication and comparison operators. It can be readily seen that where 3 or more RAKE fingers are to be selected, obtaining $\mathbf{z}$ is cheaper than typical comparison algorithms for making the "F largest" exclusion present in methods 2 and 3.

9. CONCLUSIONS

This paper has presented an elegant solution to the problem of finding the statistics of the noise in the output of a channel estimation matched filter for UTRA-FDD. The commonly used technique of thresholding the matched filter output relative to the noise statistics has been discussed. Through simulation the novel solution presented in this paper has been shown to have cost and performance benefits over other techniques.

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