Space-Time Correlation of MIMO Flat Rayleigh Fading Channels

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ABSTRACT

In order to analyze and design mobile communication systems having multiple transmit and receive antennas, a statistical multiple-input-multiple-output channel model that characterizes the received signal in both spatial and temporal domains is needed. In this work, we derive a correlation function that takes into account both the time lag, the base station antenna separation, and the mobile terminal antenna separation. The mobile terminal is assumed to be surrounded by a ring of scatterers. The properties of the space-time correlation function in some practical cases are also analyzed.

1 Introduction

In recent years, the idea of employing multiple transmit and/or multiple receive antennas (spatial diversity techniques) has become very popular. Making use of the larger number of propagation paths between the transmitter and the receiver can substantially improve the quality of the wireless link. However, in order to design efficient space-time (ST) signal processing techniques and analyze their performance, it is important to develop multiple-input-multiple-output (MIMO) channel models that can capture both the spatial and the temporal variations of the fading radio signal.

The first work on statistical modeling [1] derived closed form expressions for the instantaneous spatial correlation of the received signal at a mobile receiver and for the temporal correlation at a fixed receiver location. An expression for the instantaneous spatial correlation of the received signal at the base station was provided in [2]. The propagation environment was modeled by a ring of scatterers around the mobile unit.

A channel model that considered the effect of both the base station and the mobile terminal antenna separation on the received signal correlation for Rayleigh fading channels was described in [3]. This work was also based on the ring of scatterers model, and quasi-static fading was assumed (no time domain variations). Closed form expressions were only provided in the case of special transmitter-receiver arrangements.

In [4], both the spatial domain at the base station, and the temporal domain were taken into account. The mobile receiver was assumed to be surrounded by a ring of scatterers and was assumed to have one receive antenna. The authors provided a closed form expression for the received signal correlation as a function of antenna spacing at the base station and the time difference between the signal receptions. However, due to the asymmetric model geometry, which arises from the scatterers around the mobile unit, the results of [4] cannot be used to examine the dependence of the correlation on the mobile antenna separation.

This work proposes a propagation model that incorporates the effects of both the base station antenna separation, the mobile unit antenna separation and the temporal variations on the received signal correlation using a ring of scatterers model. A closed form expression for the space-time correlation of the path gains between the transmit and receive antennas is derived assuming flat (frequency nonselective) Rayleigh fading propagation environment.

The paper is organized as follows. The MIMO channel model will be introduced in Section 2. The received signal model will be described in Section 3. Section 4 will contain the detailed derivation of the space-time correlation function. Some numerical results will be shown in Section 5. The discussion will be provided in Section 6, and the conclusion will be drawn in the last section.

2 The MIMO Channel Model

The statistical model of the MIMO channel is based on the geometry shown in Figure 1. Both the mobile terminal (MT) and the base station (BS) are assumed to have a linear array of isotropic antennas mounted horizontally. In this model, all angles are measured with respect to the line connecting the middle points of the BS and the MS antenna arrays. The MT is surrounded by a ring of N scatterers located randomly at distance R. The ith ($i = 0, 1, \dots, N-1$) scatterer lies at an angle θ_i to the middle point of the MT antenna array. Assuming that the radius of the scatterer ring (R) is much larger than the distances between the MT antennas, the ith scatterer will approximately be at an angle θ_i to all the MT antennas. The scatterers are independently and uniformly distributed around the MT (i.e. $f(\theta_i) = 1/(2\pi)$, $\theta_i \in [-\pi, \pi]$). The effect of scatterer i is modeled as multiplication of the incident signal with a scattering coefficient S_i. The scattering coefficients are assumed to be independent, complex, zero mean, circularly symmetric Gaussian random variables with variance $1/N$. We assume that the differences between the arrival times of the radio waves from different scatterers are small compared to one channel symbol period (flat fading). The values of the scattering coefficients are also independent of their location. The mobile is moving with a constant speed v in the direction σ. The model parameters can be summarized as follows:

d_B	BS antenna separation
d_M	MT antenna separation
D	distance between the BS and the MT
R	radius of the scatterer ring
N	number of scatterers
β	direction of the BS antenna array
γ	direction of the MT antenna array
σ	direction of the MT movement
v	magnitude of the MT speed

The effect of the MT movement is taken into account through the Doppler shift. The rest of the model geometry is assumed to be constant over one frame period.

3 The Received Signal Model

Assume that the BS is the transmitter and the MT is the receiver. To determine the space-time correlation, we proceed as follows. First, we assume that only one BS antenna, BS_1, transmits and only one MT antenna, MT_1, receives signals. The transmitting antenna transmits an unmodulated unit amplitude sine wave with carrier frequency f_c (This passband signal corresponds to the complex baseband equivalent signal $x(t) = 1$.). If the synchronization is perfect, the complex baseband equivalent signal received by antenna MT_1, $r_1(t)$, will be the path gain between the transmit antenna and the receive antenna due to the flat (frequency non-selective) nature of the channel. As a consequence of the MT movement, the transmitted signal frequency is altered by the Doppler effect. This frequency shift depends on the maximum Doppler shift, f_D ($f_D = v/\lambda_c$, where λ_c is the carrier wavelength), and the angle between the direction of the incident wave coming from the ith scatterer (θ_i) and the direction of the MT movement (σ). The scatterers introduce random amplitude and phase variations, according to the statistics of the scattering coefficients. The length of the distance traveled by the radio wave from the transmitter to the receiver also introduces a phase shift in the received signal. Therefore, the received baseband equivalent signal can be expressed as

$$r_1(t) = \sum_{i=0}^{N-1} S_i e^{j2\pi f_D \cos(\theta_i - \sigma)t - j\frac{2\pi}{\lambda_c}(d_{11}(\theta_i)+d_{12}(\theta_i))}, \quad (1)$$

where $d_{11}(\theta_i)$ denotes the distance between antenna BS_1 and the ith scatterer (See also Figure 1). The distance between the ith scatterer and antenna MT_1 is $d_{12}(\theta_i)$.

Then, we assume that only antenna BS_2, which is located at d_B distance away from antenna BS_1, transmits the same $x(t)$ signal. The only receiver antenna will be MT_2, separated by a distance of d_M from antenna MT_1. Thus, the complex baseband equivalent received signal, $r_2(t)$, will be the path gain between the second pair of transmit and receive antennas. Similarly to the previous case, the received complex baseband equivalent signal is given by

$$r_2(t) = \sum_{k=0}^{N-1} S_k e^{j2\pi f_D \cos(\theta_k - \sigma)t - j\frac{2\pi}{\lambda_c}(d_{21}(\theta_k)+d_{22}(\theta_k))}. \quad (2)$$

Both $r_1(t)$ and $r_2(t)$ are zero mean, complex Gaussian random processes with unit variance (Rayleigh fading channel). Our objective is to determine the cross-correlation between $r_1(t)$ and $r_2(t)$.

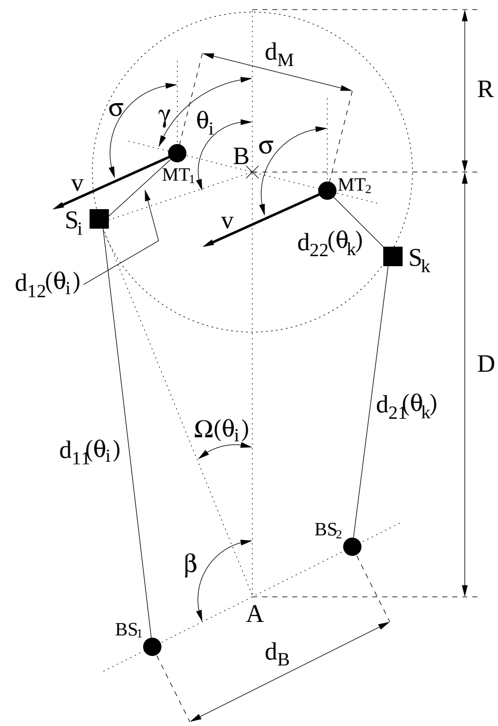


Figure 1: The geometrical model

4 The Correlation Function

In this section, we will derive a closed form expression for the space-time correlation function. First, we assume that the scatterer angles $\theta_0, \theta_1, \ldots, \theta_{N-1}$ are known and determine the conditional correlation given $\theta_0, \theta_1, \ldots, \theta_{N-1}$ by taking expectation over the values of the scattering coefficients:

$$R(\tau, d_B, d_M|\theta_0, ..., \theta_{N-1}) = E\left[r_1(t) r_2^*(t+\tau)\right]. \quad (3)$$

Substituting (1) and (2) into (3) and using the independence of the scattering coefficients, we obtain

$$R(\tau, d_B, d_M|\theta_0, ..., \theta_{N-1}) = \frac{1}{N}\sum_{i=0}^{N-1} e^{-j2\pi f_D \cos(\theta_i-\sigma)\tau} \cdot e^{j\frac{2\pi}{\lambda_c}(d_{21}(\theta_i)-d_{11}(\theta_i)+d_{22}(\theta_i)-d_{12}(\theta_i))}. \quad (4)$$

The enlarged model of the BS antennas can be observed in Figure 2a. Let $\Omega(\theta_i)$ denote the angle at which the ith scatterer is situated, as viewed from the center of the BS antenna array, relative to the line connecting the center points of the BS and MT antenna arrays. Assuming that D is much larger than R, and R is much larger than d_B, the transmitted waves incident to the ith scatterer are approximately parallel to the line connecting the center of the BS antenna and the ith scatterer. Therefore, the difference of the distances between the BS antennas and the ith scatterer can be determined as

$$\begin{aligned} d_{21}(\theta_i) - d_{11}(\theta_i) &\approx -d_1 = -d_B\cos(\Omega(\theta_i) + \pi - \beta) \\ &= d_B\cos(\beta)\cos(\Omega(\theta_i)) + d_B\sin(\beta)\sin(\Omega(\theta_i)). \end{aligned} \quad (5)$$

Considering the geometry of Figure 1 and assuming that $D \gg R$, the distance between the ith scatterer and the middle point of the BS antenna array (the $\overline{AS_i}$ line segment) is approximately D long. Thus, using the law of sines on the ABS_i triangle gives us the relationship

$$\sin(\Omega(\theta_i)) \approx \frac{R}{D}\sin(\theta_i). \tag{6}$$

In case $D \gg R$, the angle $\Omega(\theta_i)$ will be very small. Therefore, the cosine of $\Omega(\theta_i)$ can be approximated by

$$\cos(\Omega(\theta_i)) \approx 1. \tag{7}$$

Equations (5), (6) and (7) together will yield the expression

$$d_{21}(\theta_i) - d_{11}(\theta_i) \approx d_B\cos(\beta) + d_B\frac{R}{D}\sin(\beta)\sin(\theta_i). \tag{8}$$

The geometry of the MT antennas is depicted in Figure 2b. Similarly to the BS side, the path length difference between the MT antennas and the ith scatterer can be approximated as

$$\begin{aligned} d_{22}(\theta_i) - d_{12}(\theta_i) &\approx -d_2 = -d_M\cos(\gamma+\pi-\theta_i) \\ &= d_M\cos(\gamma)\cos(\theta_i) + d_M\sin(\gamma)\sin(\theta_i). \end{aligned} \tag{9}$$

The expression for the conditional correlation can be obtained by substituting (8) and (9) into (4):

$$\begin{aligned} &R(\tau, d_B, d_M|\theta_0, ..., \theta_{N-1}) \\ &\qquad \approx \tfrac{1}{N}\sum_{i=0}^{N-1} e^{j2\pi(z_0+z_1\cos(\theta_i)+z_2 sin(\theta_i))}, \end{aligned} \tag{10}$$

with

$$\begin{aligned} z_0 &= \frac{d_B}{\lambda_c}\cos(\beta) \\ z_1 &= \frac{d_M}{\lambda_c}\cos(\gamma) - f_D\tau\cos(\sigma) \\ z_2 &= \frac{d_B}{\lambda_c}\frac{R}{D}\sin(\beta) + \frac{d_M}{\lambda_c}\sin(\gamma) - f_D\tau\sin(\sigma). \end{aligned}$$

To determine the correlation function, the last step is to take the expectation of the conditional correlation with respect to the scatterer angles:

$$R(\tau, d_B, d_M) = E[R(\tau, d_B, d_M|\theta_0, ..., \theta_{N-1})]. \tag{11}$$

Combining (11) and (10) and taking advantage of the independence of the scatterer angles, the correlation function becomes

$$R(\tau, d_B, d_M) \approx \frac{1}{N}\sum_{i=0}^{N-1}\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j2\pi(z_0+z_1\cos(\theta_i)+z_2 sin(\theta_i))}d\theta_i. \tag{12}$$

Equation (12) can be further simplified by using the equality

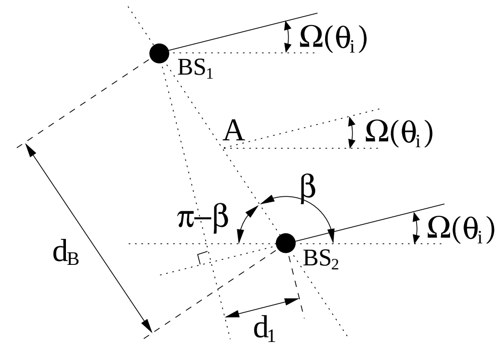
$$z_1\cos(\theta_i) + z_2\sin(\theta_i) = \sqrt{z_1^2+z_2^2}\cos(\theta_i+\Phi),$$

where $\Phi = \arctan(z_2/z_1)$. As a result, we obtain

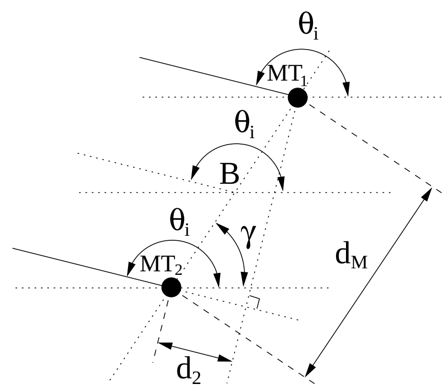
$$R(\tau, d_B, d_M) \approx e^{j2\pi z_0}\frac{1}{N}\sum_{i=0}^{N-1}\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j2\pi\sqrt{z_1^2+z_2^2}\cos(\theta_i)}d\theta_i. \tag{13}$$

The angle Φ could be neglected since the cosine function is integrated over a whole period. Finally, using the definition of the 0th order Bessel function of the first kind given by

$$J_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jx\cos(\theta)}d\theta,$$



a) The base station side



b) The mobile terminal side

Figure 2: The enlarged geometries

the correlation function can be expressed as

$$R(\tau, d_B, d_M) \approx e^{j2\pi z_0} J_0\left(2\pi\sqrt{z_1^2+z_2^2}\right). \tag{14}$$

Note that the above expression is independent of the number of scatterers, N.

5 Numerical Results

To illustrate the results obtained in the previous section, we present the space-time correlation function in the case of some practical parameter settings. The distance of the BS and the MT (D) was set to 1 km, and the scatterer radius (R) was 20 m. The carrier frequency (f_c) was 900 MHz ($\lambda_c = 0.33$ m), and the mobile was moving with a constant speed of $v = 70$ km/h (19.4 m/s). The MT moving direction was parallel with the line of the BS antennas, and the line connecting MT antennas was perpendicular to its moving direction ($\beta = 3\pi/4$ rad, $\sigma = 3\pi/4$ rad, $\gamma = \pi/4$ rad). The solid line in Figure 3 is the magnitude of the space-time correlation as a function of the time lag (τ), with BS antenna separation $d_B = 5\lambda_c = 1.67$ m, and MT antenna separation $d_M = 0.6\lambda_c = 0.2$ m. The dotted line shows the correlation without MT antenna separation ($d_M = 0$). Comparing the two curves, we observe that the MT antenna separation substantially decreases the maximum magnitude of the correlation function.,

The magnitude of the correlation as a function of the BS antenna separation, d_B, is plotted in Figure 4 for the same

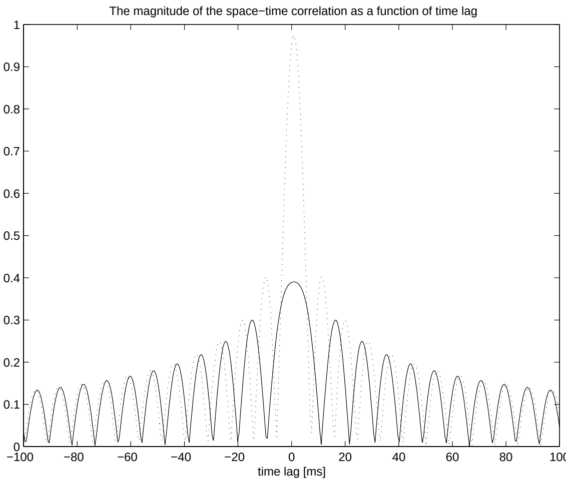


Figure 3: ST correlation as a function of the time lag

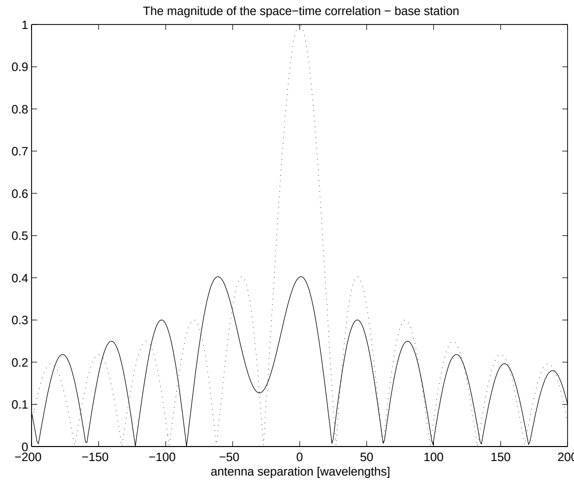


Figure 4: ST correlation as a function of BS antenna separation

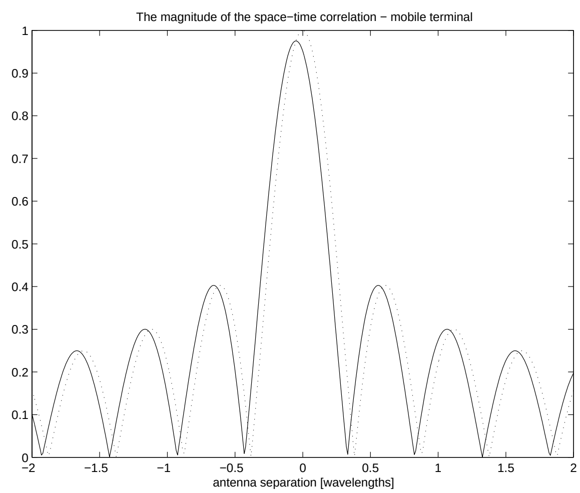


Figure 5: ST correlation as a function of MT antenna separation

geometrical arrangement. The value of the time lag (τ) was set to zero. The solid line corresponds to the $d_M = 0.6\lambda_c$ case, and the dotted line shows the correlation with $d_M = 0$. The effect of the MT antenna separation (d_M) on the ST correlation can be observed in Figure 5, assuming zero time lag and a BS antenna distance of $d_B = 5\lambda_c$ (solid curve) and $d_B = 0$ (dotted curve). Both curves demonstrate that to achieve a certain reduction in the maximum magnitude of the correlation function, much larger antenna separation is needed at the BS side than at the MT side.

6 Discussion

In the derivation of (14), it was assumed that the correlation does not depend on the absolute time (i.e. the processes $r_1(t)$ and $r_2(t)$ are jointly wide sense stationary). This resulted from the assumption that the parameters D, R, γ, β and σ are constant over the time period of interest. Using the parameters described in Section 5, the MT moves 9.7 cm during a frame period of 5 ms. This displacement is negligible compared to the BS-MT distance (1 km) and the radius of the scatterer ring (20 m). Therefore, it is a reasonable approximation to treat the above geometrical parameters constant in some practical situations.

Using analytical methods, it can be verified that our results approximately simplify to the special cases described in [3], assuming that $D \gg R$. The model geometry of [4] is very different from our approach, so only numerical comparisons could be made. It was found that in the range $0 \leq d_B \leq 100\lambda_c$, the difference between the correlation values produced by the two models is negligible. However, the derived correlation function is based on a simple, abstract geometrical and statistical propagation model, so the validity of the results will have to be verified via simulations and measurements.

7 Conclusion

We proposed a MIMO channel model for flat Rayleigh fading channels based on the ring of scatterers propagation model. A closed form expression for the space-time correlation of the path gains between the transmit and receive antennas was derived. The numerical results showed that in accordance with the expectations, the MT antenna separation has a much more pronounced effect on the correlation than the BS antenna separation.

References

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