

# SPACE-TIME DECORRELATING RAKE RECEIVER IN MULTIPATH CDMA CHANNEL

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## ABSTRACT

This paper studies a space-time generalization of decorrelating RAKE receiver recently proposed by Liu and Li for blind multiuser detection of direct sequence code division multiple access (DS-CDMA) signals under frequency selective fading channel. Multiple receiving antennas are employed to further enhance the performance of the DRAKE receiver through diversity combining. Simulations show that the proposed space-time DRAKE (ST-DRAKE) receiver performs significantly better than the temporal-only DRAKE receiver and its performance is close to that of the MMSE receiver. The use of multistage Wiener filter to reduce the complexity of the ST-DRAKE is also studied.

## 1. INTRODUCTION

Multiple access interference (MAI) and multipath fading are two major issues that affect the capacity of direct sequence code division multiple access (DS-CDMA) systems. Conventional matched filtering technique suffers from the near-far problem and user's signature waveform mismatch of fading channels, resulting in poor performance. Advanced signal processing algorithms, including the RAKE receiver, multiuser detection and space-time processing using multiple antennas, have been proposed to overcome these impairments. A RAKE matched filter coherently combined replicas of the desired signal in resolvable multipaths [5]. On the other hand, multiuser detection (MUD) technique suppresses the MAI using known users' signature waveforms [6]. With antenna array, these techniques can be enhanced through spatial diversity to provide better performance [7].

Recently, adaptive minimum output energy (MOE) receiver has been proposed for blind CDMA reception which requires only the knowledge of the desired user's signature waveform [1]. In practice, the effective users' signature waveforms vary in a multipath fading channel. Periodic training is therefore needed to estimate the signature waveforms at the expense of lower bandwidth efficiency.

A decorrelating RAKE (DRAKE) receiver that allows multiuser detection in frequency-selective fading channels was presented in [3]. It has the advantage of low implementation complexity and only the desired user's spreading sequence is required. The DRAKE receiver is based on a constrained adaptive filtering approach similar to the MOE detector, in the sense that the constrained adaptive filter minimizes the energy of the output signal while constraining the output gain of the desired user to be a constant. To reduce the implementation complexity of the adaptive filter, rank reduction algorithm was proposed to reduce the dimension of adaptation to the signal subspace. A reduced rank decorrelating RAKE receiver based on multistage Wiener filter (MSWF) [8] was proposed in [4], where the MSWF and the generalized sidelobe canceller are used to implement the constrained adaptive filter for interference suppression. It is very effective when the

processing gain is larger than the dimension of the signal subspace, because the MSWF can be terminated at a very small number of stages. Hence, the MSWF aims to provide near full-rank performance with reduced receiver's complexity. Previous study showed that the performance of the DRAKE receiver is comparable to that of the MMSE receiver [9] in frequency-selective fading channels.

In this paper, we further extend the idea of the DRAKE receiver to multiple receiving antennas. In particular, a space-time DRAKE (ST-DRAKE) receiver is proposed for the reception of CDMA signals using multiple receiving antennas under frequency selective fading channel. The use of multiple receiving antennas further enhances the performance of the DRAKE receiver through diversity combining. Simulations show that the proposed space-time DRAKE receiver performs significantly better than the temporal-only DRAKE receiver and its performance is close to that of the MMSE receiver.

## 2. SYSTEM MODEL

Consider the reverse link of a DS-CDMA system with  $K$  users and  $M$  receiving antenna elements. The received complex signal at the  $m$ -th antenna is given by

$$y^{(m)}(t) = \sum_{k=1}^K \sum_{n=-\infty}^{\infty} b_k(i) g_k^{(m)}(t - iT_s - \nu_k) + u^{(m)}(t), \quad (1)$$

where  $b_k(i)$  is the  $i$ -th data symbol,  $\nu_k$  is the delay,  $g_k^{(m)}$  is the signature waveform of the  $k$ -th user. The symbol period is denoted by  $T_s$  and  $u(t)$  is the additive white Gaussian noise (AWGN) with covariance  $\sigma_u^2$ . The users' signature waveform is given by

$$g_k^{(m)}(t) = \sum_{j=1}^{L_c} c_k(j) h_k^{(m)}(t - jT_c), \quad (2)$$

where  $\{c_k(j)\}_{j=1}^{L_c}$  and  $h_k^{(m)}(t)$  are the spreading sequence and chip waveform of the  $k$ -th user, respectively.  $T_c$  is the chip interval and  $L_c = T_s / T_c$  is the processing gain. For a multipath channel with delay spread  $L_m \geq 1$ , the users' chip waveforms are given by

$$h_{k,l}^{(m)}(t) = \sum_{l=1}^{L_m} \alpha_{k,l}^{(m)}(t) \mathbf{a}(\theta_{k,l}^{(m)}) \rho_k(t - \tau_{k,l}^{(m)}), \quad (3)$$

where  $\alpha_{k,l}^{(m)}(t)$ ,  $\tau_{k,l}^{(m)}$  and  $\theta_{k,l}^{(m)}$  are the gain, delay and AOA, respectively, of the  $l$ -th path of the  $k$ -th user's signal received at the  $m$ -th antenna. The antenna response due to the signal arriving at an angle of  $\theta$  is denoted as  $\mathbf{a}(\theta)$ , and  $\rho_k(t)$  is the original chip waveform. We assume that the fading rate of the channel is slow so that the channel is approximately constant over several symbol intervals.

The received baseband signal  $y^{(m)}(t)$  is sampled at chip rate over symbol duration of  $N = L_c + L_m - 1$ . The  $N$ -vector of chip-sampled signal samples is written as

$$\begin{aligned} \mathbf{y}^{(m)}(n) &= [y^{(m)}(nL_c) \dots y^{(m)}(nL_c + N - 1)]^T \\ &= b_1(n) \mathbf{C}_1 \mathbf{h}_1^{(m)}(n) + \sum_{k=2}^K b_k(n) \mathbf{C}_k \mathbf{h}_k^{(m)}(n) + \mathbf{u}^{(m)}(n), \end{aligned} \quad (4)$$

where

$$\begin{aligned} \mathbf{C}_k &= [\mathbf{c}_{k,1} \dots \mathbf{c}_{k,L_m}] \\ &= \begin{bmatrix} c_k(1) & & \mathbf{0} \\ \vdots & \ddots & c_k(1) \\ c_k(L_c) & & \vdots \\ \mathbf{0} & \ddots & c_k(L_c) \end{bmatrix} \end{aligned} \quad (5)$$

$$\text{and } \mathbf{h}_k^{(m)}(n) = [h_k^{(m)}(t)|_{t=1} \dots h_k^{(m)}(t)|_{t=L_m T_c}]^T. \quad (6)$$

Without loss of generality, we assume that the desired user in the system is user 1 (i.e.  $k = 1$ ). The sampled signal vector  $\mathbf{y}(n) = [\mathbf{y}^{(1)}(n) \dots \mathbf{y}^{(M)}(n)]^T$  from the  $M$  antennae at time  $n$  can be expressed as

$$\mathbf{y}(n) = \mathbf{g}_1 b_1(n) + \mathbf{i}(n) + \mathbf{u}(n), \quad (7)$$

where  $\mathbf{i}(n)$  denotes the MAI,  $\mathbf{u}(n)$  is the additive white noise vector and

$$\mathbf{g}_1 = \tilde{\mathbf{C}} \mathbf{h}_1, \quad (8)$$

where

$$\mathbf{h}_1(n) = [\mathbf{h}_1^{(1)} \dots \mathbf{h}_1^{(M)}]^T, \quad (9)$$

$$\tilde{\mathbf{C}} = \mathbf{I}_{MN \times ML_m} \cdot \mathbf{C}_1, \quad (10)$$

and  $\mathbf{I}_{m,n}$  is an  $m \times n$  identity matrix.

### 3. SPACE-TIME DECORRELATING RAKE RECEIVER

The structure of the ST-DRAKE receiver is shown in Figure 1. In the rest of this paper, the desired user's subscript is dropped for simplicity. As shown in Figure 1, the received signal at the  $m$ -th antenna is first filtered by weight vectors  $\{\mathbf{w}_{m,l}\}_{l=1}^{L_m}$  such that the desired signal along each code vector  $\mathbf{c}_l$  is extracted while suppressing the MAI and channel noise. It can be readily shown that this constrained optimization criterion can be written as

$$\mathbf{w}_{m,l} = \arg \min_{\mathbf{w}_{m,l}} \mathbf{w}_{m,l}^H \mathbf{R}_{\mathbf{y}^{(m)} \mathbf{y}^{(m)}} \mathbf{w}_{m,l} \quad \text{s.t.} \quad \mathbf{C} \mathbf{w}_{m,l} = \mathbf{I}_l, \quad (11)$$

where  $\mathbf{R}_{\mathbf{y}^{(m)} \mathbf{y}^{(m)}}$  is the covariance matrix of  $\mathbf{y}^{(m)}$  and the weight vector  $\mathbf{w}_{m,l}$  is given by

$$\mathbf{C} \mathbf{w}_{m,l} = [0 \dots 1 \dots 0]^T = \mathbf{I}_l, \quad l = 1, \dots, L_m. \quad (12)$$

The optimal solution to the problem in (11) is given by

$$\tilde{\mathbf{w}}_{m,l} = \mathbf{R}_{\mathbf{y}^{(m)} \mathbf{y}^{(m)}}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{R}_{\mathbf{y}^{(m)} \mathbf{y}^{(m)}}^{-1} \mathbf{C})^{-1} \mathbf{I}_l. \quad (13)$$

Each weight vector in the  $l$ -th arm accounts for signal received from different delayed paths. The output of the  $l$ -th arm of the adaptive filter is given by

$$x_{m,l}(n) = \mathbf{w}_{m,l}^H \mathbf{y}^{(m)}(n), \quad m = 1, \dots, M. \quad (14)$$

Using (4) and (11), equation (14) can be rewritten to emphasize the desired user's signal as

$$\begin{aligned} x_{m,l}(n) &= h^{(m)}(l) b(n) + \mathbf{w}_{m,l}^H \mathbf{u}^{(m)}(n) \\ &= h^{(m)}(l) b(n) + \varepsilon_{m,l}(n), \end{aligned} \quad (15)$$

where  $\varepsilon_{m,l}(n)$  is the effective MAI and channel noise after filtering. With  $L_m$  number of paths for each user and  $M$  receiving antennae, all signals scattered through different multipaths are extracted and stacked together such that

$$\begin{aligned} \mathbf{x} &= [\mathbf{x}^{(1)} \dots \mathbf{x}^{(M)}]^T \\ &= [\mathbf{h}^{(1)} \dots \mathbf{h}^{(M)}]^T \cdot b(n) + [\boldsymbol{\varepsilon}^{(1)} \dots \boldsymbol{\varepsilon}^{(M)}]^T, \\ &= \mathbf{h} \cdot b(n) + \boldsymbol{\varepsilon} \end{aligned} \quad (16)$$

where  $\mathbf{h}^{(m)}$  is given by (6),  $\mathbf{x}^{(m)}(n) = [x_{m,1}(n) \dots x_{m,L_m}(n)]^T$  and  $\boldsymbol{\varepsilon}^{(m)}(n) = [\varepsilon_{m,1}(n) \dots \varepsilon_{m,L_m}(n)]^T$ . The filter output vectors  $\{\mathbf{x}^{(m)}\}_{m=1}^M$  are then coherently combined to obtain the estimate of transmitted symbols. The optimum combining vector  $\mathbf{w}_{coh}$  is given by

$$\mathbf{w}_{coh} = \mathbf{R}_{\mathbf{xx}}^{-1} \mathbf{h}. \quad (17)$$

Since the MAI and channel noise are decorrelated and suppressed in the constrained adaptive filter, the signal-to-interference-and noise ratio (SINR) of the filter outputs is high such that the optimum combining vector can be approximated by the principle eigenvector of  $\mathbf{R}_{\mathbf{xx}}^{-1}$ . The combined output  $z\{n\}$  is given by

$$z(n) = \mathbf{w}_{coh}^H \mathbf{x}(n). \quad (18)$$

An estimate of the transmitted information symbol is then obtained by making a decision based on  $z\{n\}$ . For BPSK modulation, the estimate of the transmitted symbol is given by

$$\hat{b}_1(n) = \text{sgn}[\text{Re}\{z(n)\}]. \quad (19)$$

The main difference between the proposed receiver and the DRAKE receiver is that the proposed receiver processes signal in both space and time domain while the DRAKE receiver operates in time domain only. By exploiting both spatial and temporal signatures using antenna array, better interference suppression can be achieved.

For MMSE receiver, the optimal weighting vector is given by

$$\mathbf{w}_{MMSE} = \arg \min_{\mathbf{w}} \{E \|\mathbf{w}^H \mathbf{y}(n) - b_1(n)\|^2\}. \quad (20)$$

In section 5, the performance of the proposed ST-DRAKE receiver is compared with the MMSE receiver.

#### 4. RANK-REDUCTION USING MULTISTAGE WIENER FILTER

The calculation of the weight vector  $\mathbf{w}_{m,l}$  in each arm is computationally intensive since the solution of  $\mathbf{w}_{m,l}$  involves a matrix inversion. An efficient solution is to decompose the weight vector into two orthogonal components as follows

$$\mathbf{w}_l = \mathbf{w}_{c,l} - \mathbf{B}^H \mathbf{w}_{a,l}, \quad (21)$$

where

$$\mathbf{w}_{c,l} = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{1}_l \quad (22)$$

is a vector which depends on the spreading sequence of the desired only. Note that the subscript of the  $m$ -th antenna element is omitted for clarity. Equation (21) is in fact analogous to the generalized sidelobe canceller (GSC) used in array processing. The weight vector  $\mathbf{w}_{c,l}$  is in the constraint subspace and thus allows the desired signal to pass through the adaptive filter with constant gain. The matrix  $\mathbf{B}$  is a blocking matrix that is orthogonal to  $\mathbf{C}$ , i.e.  $\mathbf{B}\mathbf{C} = \mathbf{0}$ . Since the spreading sequence of the desired user is assumed known, the blocking matrix  $\mathbf{B}$  can be determined by using eigen-decomposition [10]. The blocking matrix  $\mathbf{B}$  projects the received data sample  $\mathbf{y}(n)$  into a subspace with reduced dimension, thus reduces the number of taps required in the adaptive filter. In contrast, the weight vector  $\mathbf{w}_{a,l}$  depends on the received data and hence determines the complexity of the MOE detector. It is used to remove any interference and noise from appearing at the filter output. The adaptive weight vector  $\mathbf{w}_{a,l}$  can be determined by the recursive least squares (RLS) or the least mean squares (LMS) algorithms. Alternatively, eigen-based methods, such as principle components and cross spectral method can be used to reduce the dimension for adaptation.

Another effective method for dimension or rank reduction is the multistage Wiener filter (MSWF) [5]. The MSWF does not require matrix inversion or eigen-decomposition and therefore is more computationally efficient than the eigen-based methods. It decomposes a Wiener filter (i.e.  $\mathbf{w}_{a,l}$ ) into multiple stages and successively projects the desired signal onto lower dimension orthogonal subspaces. Near full rank performance can be achieved by using a small number of stages, greatly reducing the complexity of the receiver. Figure 2 shows the block diagram of the  $l$ -arm of the constrained adaptive filter utilizing a rank-4 MSWF. Using the MSWF structure, the adaptive weighting vector  $\mathbf{w}_{a,l}$  is decomposed into multiple stages, each of which consists of two orthogonal components;  $\mathbf{h}_{l,i}$  is a normalized cross-correlation vector given by

$$\mathbf{h}_{l,i} = \frac{E\{\mathbf{y}_{l,i-1}(n)d_{l,i-1}^*(n)\}}{\|E\{\mathbf{y}_{l,i-1}(n)d_{l,i-1}^*(n)\}\|}, \quad (23)$$

and  $\mathbf{B}_{l,i}$  is a blocking matrix such that

$$\mathbf{B}_{l,i}^H \mathbf{h}_{l,i} = \mathbf{0}, \quad (24)$$

where the subscript  $i$  denotes the  $i$ -th stage of the MSWF. The output  $d_{l,i}(n)$  of the filters  $\mathbf{h}_{l,i}$  at each stage are weighted by  $w_i$  and combined to obtain the final output  $x_l(n)$  of the  $l$ -arm. The weight at each stage can be obtained by

$$w_i = \frac{E\{d_{l,i-1}^*(n)e_{l,i}(n)\}}{E\{e_{l,i}(n)\|^2\}}. \quad (25)$$

A recursive algorithm for the MSWF is developed in [11]. The choice of  $D$  affects the output SINR of the receiver [12]. The recursive rank- $D$  MSWF is summarized as the follows.

$$d_{l,0}(n) = \mathbf{w}_{c,l}^H \mathbf{y}^{(m)}(n), \quad \mathbf{y}_{l,0}(n) = \mathbf{B}^H \mathbf{y}^{(m)}(n) \quad (26)$$

At each  $n$  for  $i = 1, \dots, D$ :

$$\mathbf{p}_{l,i}(n) = (1 - \mu)\mathbf{p}_{l,i}(n-1) + \mu d_{l,i-1}^*(n)\mathbf{y}_{l,i-1}(n) \quad (27)$$

$$\mathbf{h}_{l,i}(n) = \mathbf{p}_{l,i}(n) / \|\mathbf{p}_{l,i}(n)\| \quad (28)$$

$$\mathbf{B}_{l,i}(n) = \mathbf{I}_{N-i+1, N-i} - \mathbf{h}_{l,i}(n)\mathbf{h}_{l,i}^H(n) \quad (29)$$

$$d_{l,i}(n) = \mathbf{h}_{l,i}^H(n)\mathbf{y}_{l,i-1}(n) \quad (30)$$

$$\mathbf{y}_{l,i}(n) = \mathbf{B}_{l,i}^H(n)\mathbf{y}_{l,i-1}(n) \quad (31)$$

At each  $n$ , decrement from  $i = D, \dots, 1$ :

$$\xi_{l,i}(n) = (1 - \mu)\xi_{l,i}(n-1) + \mu |e_{l,i}(n)|^2 \quad (32)$$

$$w_i(n) = \|\mathbf{p}_{l,i}(n)\| / \xi_{l,i}(n) \quad (33)$$

$$e_{l,i-1}(n) = d_{l,i-1}(n) - w_i^H e_i(n), \quad (34)$$

where  $e_D(n) = d_D(n)$ . The subscript  $(l, N-i)$  of  $\mathbf{h}_i$  denotes its first  $(N-i)$  components.

#### 5. SIMULATION RESULTS

Consider an asynchronous CDMA system with BPSK modulation. The processing gain of the system is 31 and Gold code is used. The number of multipaths  $L_m$  for all users is 4 and are generated with delays chosen from  $\{0, T_c, 2T_c, 3T_c\}$ . It is also assumed that the AOA of each interfering users are randomly distributed over  $[0, \pi]$  with angular spread equals to zero. The rank of the MSWF is chosen as 10.

In Figure 3, the performance of the ST-DRAKE receiver is compared with the MMSE receiver. Both ST-DRAKE receivers with and without using MSWF are considered. The number of users, including the desired user, is 10. The result is shown as a function of bit-energy-to-noise-ratio  $E_b/N_o$  for number of antenna elements ranging from  $M = 1$  to 3. Note that the single antenna receiver (i.e.  $M = 1$ ) is equivalent to the DRAKE receiver in [7]. As shown in Figure 3, the ST-DRAKE has better BER performance than the DRAKE receiver, as a result of spatial diversity. The performance of the MSWF-based ST-DRAKE receiver has similar performance to the ST-DRAKE receiver without using MSWF. The ST-DRAKE receiver in general has comparable performance with the MMSE receiver.

Figure 4 shows the performance of the ST-DRAKE receiver versus the number of users in the system. The bit-energy-to-noise rate  $E_b/N_o$  is chosen as 5dB. Both the performance of ST-DRAKE and MMSE receiver degrade when the number of users in the system increases. It is due to the increase of MAI such that the SINR of the received signal is decreased, which leads to a lower BER.

## 6. CONCLUSIONS

A space-time generalization of the decorrelating RAKE (DRAKE) receiver is presented for blind multiuser reception of DS-CDMA signal using multiple receiving antennas under frequency selective fading channel. Through diversity combining, the performance of the DRAKE receiver is significantly improved. Simulations show that the proposed space-time DRAKE receiver performs significantly better than the temporal-only DRAKE receiver and its performance is close to that of the MMSE receiver. The use of multistage Wiener filter to reduce the complexity of the ST-DRAKE is also studied.

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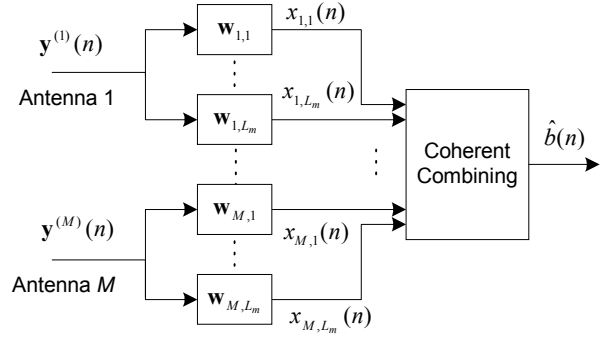


Figure 1. Space-time decorrelating RAKE receiver.

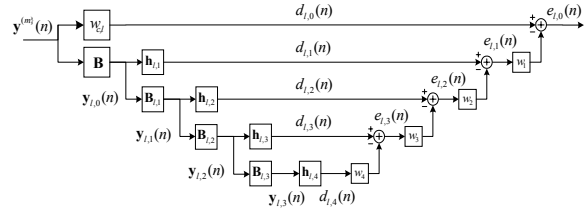


Figure 2. RAKE receiver for the  $l$ -th arm using MSWF.

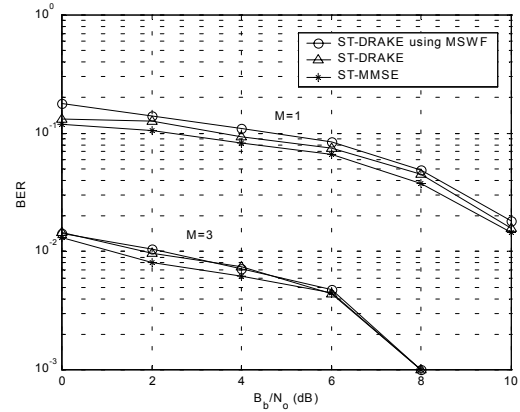


Figure 3. Bit-error-rate versus  $E_b/N_o$ .

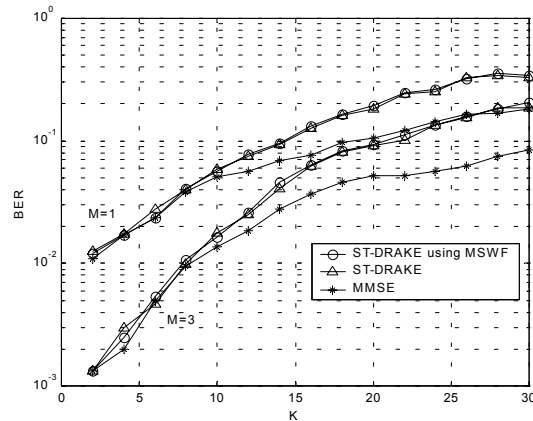


Figure 4. Bit-error-rate versus number of users  $K$ .