RATIONAL CHARACTERIZATION FOR MEMORYLESS ADAPTIVE PRE-DISTORTION

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ABSTRACT

In this paper we investigate an alternative characterization of the inverse AM/AM and AM/PM transfer curves of a high power amplifier (HPA), defining an adaptive algorithm for the optimum adjustment of the coefficients in a memoryless rational-based model used to pre-distort a base band signal with complex modulation in a quadrature non-linear model. Input signal statistics are unknown so its influence in the estimation is analyzed and compensation results are shown and compared with other previously proposed methods.

1. INTRODUCTION

The increasing utilization of non-constant amplitude modulations in digital communications motivates the main idea of this work[1] which is the identification of the transfer characteristic of a non-linear system (i.e. an HPA) and its subsequent application in a pre-distortion algorithm whose formulation, by means of a cost function, can achieve convergence according to some specific restrictions. The modeling and estimation of the inverse transfer characteristics of a non-linear device is an active research area looking for the maximum reduction of the harmful effects introduced by the amplitude (AM/AM) and phase (AM/PM) distortion in digital communications [1]. A comparison of some research results for the practical application of pre-distortion techniques suggest that different proposed ways for the estimation feature also different benefits in contrast to the overall system cost, where some related key points to evaluate such benefits are the power efficiency improvement, accuracy of the estimated curves and computational complexity of the implemented process. In previous work we have studied only the compensation of AM/AM distortion using coarse estimation of the statistics taken out from sampled signals at the input and output of the HPA. This current work also consider such discrete versions of the input and output of the HPA as the only available information to design and apply pre-distortion over the complex base-band input signal in a non-constant amplitude modulation scheme, such as M-QAM in OFDM. Here we spread the scope of the experiment for the compensation of the AM/PM distortion too. We note that the treatment and modeling of the AM/PM effect would strictly require to consider the signal delay introduced by analog stages in the transmission chain. However, since our main purpose is to evaluate how an specific polynomial approach fits the inverse transfer curves, this "memory" effect is constrained to be ideal defining no time delays between input and output samples at the HPA. According to that, a well known memoryless AM/AM and AM/PM non-linear model is applied to distort data in the simulations [2]. However, the algorithm herein described must be of general applicability since in a real scheme there is not a priori knowledge about the time-varying non linear response of the HPA due, for instance, to temperature changes and age of the equipment. The latter means that the adaptive capability, desirable to make the system able to follow variations in the performance of the HPA, will be determined by the convergence rate of the algorithm, which in turns depends on the input signal statistic distribution and how the non-linearity is modeled. Use of adaptive algorithms, such as LMS or RLS, have already been proposed to compensate non-linear distortions by estimating coefficients in polynomial non-linear models [3] [4]. Nevertheless, many problems have arisen when such optimization algorithms have been directly applied. Therefore its original updating structure requires modifications, and some restrictions must be stated especially when dealing with rational-based cost functions, as in this case, since they are plagued by the appearance of local minima that result in a threatened convergence capacity.

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2. NON-LINEAR MODEL DEFINITIONS

The transfer of a communications signal through an HPA could be seen either as a product between the input signal b_x and the amplifier's complex gain *G*, or in terms of the application of this signal in a modulus-dependent transfer function $A(b_x)$. For digital pre-distortion the multiplicative model results more suitable. Thus, as we intend to compensate both amplitude and phase non-linear distortion over complex modulations we define first the non-linear model for the later formulation of the algorithm. Let $b_x(t)$ and $b_y(t)$ be the equivalent analog base-band HPA input and output signals respectively. The input base-band signal and its phase and quadrature components are given by,

$$b_x(t) = u_x(t)e^{q_x(t)} = x_p(t) + jx_q(t) \qquad (1)$$
$$x_p(t) = u_x(t)\cos\left(q_x(t)\right)$$
$$x_q(t) = u_x(t)\sin\left(q_x(t)\right)$$

where $u_x(t) = |b_x(t)|$, the modulus of the equivalent input base-band signal. Then, the non-linear distortion over (1) can be applied using the quadrature model shown in figure 1.

According to the quadrature model, the non-linear distortion of the HPA could be expressed as a complex modulus-dependent multiplicative gain *G(u)*, considering simultaneously both AM/AM and AM/PM distortions,

$$G\left(u_x(t)\right) = P\left(u_x(t)\right) + jQ\left(u_x(t)\right) \quad (2)$$

Thence, the corresponding output (distorted) signal is written as

$$\begin{aligned} b_y(t) &= u_y(t)e^{[q_x(t)+\Phi(u_x(t))]} = y_p(t) + jy_q(t) \quad (3)\\ y_p(t) &= u_y(t)\cos\left[q_x(t)+\Phi\left(u_x(t)\right)\right]\\ y_q(t) &= u_y(t)\sin\left[q_x(t)+\Phi\left(u_x(t)\right)\right] \end{aligned}$$

whence we extract the AM/AM distortion,

$$u_y(t) = A[u_x(t)] = u_x(t)\cdot\sqrt{P^2\left(u_x(t)\right)+Q^2\left(u_x(t)\right)} \quad (4)$$

and the AM/PM characteristic of the model defined as,

$$\Phi\left(u_x(t)\right) = \arctan\left[\frac{Q\left(u_x(t)\right)}{P\left(u_x(t)\right)}\right] \quad (5)$$

The same non-linear distortion can also be modeled with the following relationships corresponding to the so-called amplitude-phase Saleh model [2].

$$A[u_x(t)] = K\cdot A_{sat}^2\frac{u_x(t)}{u_x^2(t)+A_{sat}^2} \quad (6)$$

$$\Phi[u_x(t)] = \frac{p}{3}\frac{u_x^2(t)}{u_x^2(t)+A_{sat}^2} \quad [rad] \quad (7)$$

This latter HPA model is normalized to distort data in the simulations constraining the input and output range of the HPA to be the same. Note that it is applied only as a particular case since the algorithm is derived independently from the HPA model. For digital treatment all the involved continuous time signals are sampled at $f_s = 1/T_s$, then we have that,

$$\begin{aligned} b_y(kT_s) &= b_x(kT_s)G\left(u_x(kT_s)\right)+\Delta(kT_s) \quad (8)\\ &\cong b_x(kT_s)G\left(u_x(kT_s)\right) \end{aligned}$$

where the aliasing term $\Delta(kT_s)$, related to the expanded bandwidth of the signal after non-linear distortion, is considered negligible compared to the magnitude of distortion itself, if f_s is high enough. Finally, we take from (2) the final discrete quadrature model to express non-linear distortion in the formulation of the algorithm, from now on we skip the dependence on the discrete time variable, $n=kT_s$, to simplify the expressions,

$$G(u_x) = P(u_x) + jQ(u_x) \quad (9)$$

It is important to note that these non-linear characteristics are solely dependent on the modulus of the input pass-band signal.

The normalized versions of (6), with K=2 and A_{sat}=1, and the corresponding AM/PM characteristic (7) of the distortion model used in our simulations appear depicted in Figure 2.

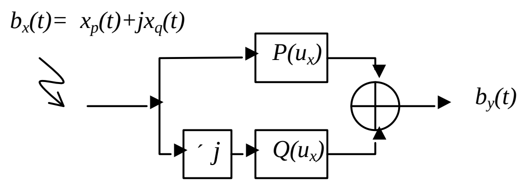

Figure 1: Quadrature nonlinear model of HPA.

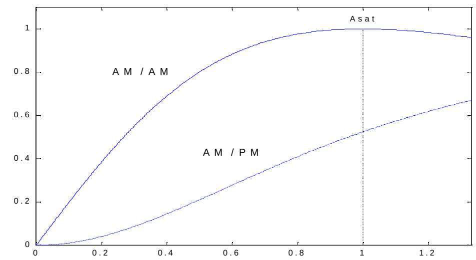

Figure 2: Normalized non-linear model characteristics.

Our purpose, in order to execute digital pre-distortion, is to estimate an discrete inverse complex multiplicative function $G^{-1}(u)$ such that, $b_x = b_y\, G^{-1}(u_y)$, thus providing compensation of amplitude and phase distortion.

3. ALGORITHM DERIVATION.

While the amplitude-phase model in (6) and (7) are applied for input base-band symbols distortion, the equivalent quadrature model is considered to estimate the pre-distortion characteristic, assuming the existence of two inverse functions P' and Q' such that,

$$G^{-1}(u) = P'(u) + jQ'(u) \quad (10)$$

hence, the inverse AM/AM and AM/PM will be respectively given by,

$$u\cdot\left|G^{-1}(u)\right| = A^{-1}[u] = u\cdot\sqrt{\left(P'(u)\right)^2+\left(Q'(u)\right)^2} \quad (11)$$

$$\Phi^{-1}\left(u\right) = \arctan\left[\frac{Q'(u)}{P'(u)}\right] \quad (12)$$

Applying (10), when the optimal pre-distortion is obtained we can verify the following relationship between input and output,

$$b_x = b_y\cdot G_{opt}^{-1}(u_y) = (y_p + jy_q)\cdot\left\{P'_{opt}(u_y) + jQ'_{opt}(u_y)\right\} \quad (13)$$

Other authors use either odd or even high order power memoryless polynomials to model separately the inverse AM/AM and AM/PM characteristics for typical HPAs. We propose now to approximate $G^{-1}(u_y)$ and consequently the P' and Q' curves with a quotient of two complete polynomials of order L-1 (a rational complex function) instead of a single polynomial, thus expecting to achieve similar compensation performance with a lesser power order estimation and letting the coefficients adaptively choice the odd or even nature of such orders. The general expression for this two polynomials modeling for $G^{-1}(u)$ is defined as follows (eq.14),

$$G^{-1}(u_y)=\frac{P_2(u_y)}{P_1(u_y)}=\frac{b_0^*+b_1^*u_y+b_2^*u_y^2+\cdots+b_{L-1}^*u_y^{L-1}}{a_0+a_1u_y+a_2u_y^2+\cdots+a_{L-1}u_y^{L-1}} \tag{14}$$

where we can define vectorial simplified expressions of the polynomials in (14) given by

$$P_2(u_y)=\underline{b}_L^H\,\underline{u}_{y,L} \tag{15}$$

$$P_1(u_y)=\underline{a}_L^T\,\underline{u}_{y,L} \tag{16}$$

with,

$$\underline{u}_{y,L}=[1\ u_y\ u_y^2\cdots u_y^{L-1}]^T=[1\ \underline{u}_{y,L-1}^T]^T \tag{17}$$

$$\underline{a}_L=[a_0\ a_1\cdots a_{L-1}]^T=[a_0\ \underline{a}_{L-1}^T]^T \tag{18}$$

$$\underline{b}_L=[b_0\ b_1\cdots b_{L-1}]^T \tag{19}$$

Then, using (15) and (16), and according to the optimum pre-distortion shown in (13) we can define an instantaneous square error and its corresponding cost function J featuring dependence on the estimation of the coefficients in $\underline{a}_L$ and $\underline{b}_L$,

$$J=E\left\{|e|^2\right\}=E\left\{\left|b_x\cdot P_1(u_y)-b_y\cdot P_2(u_y)\right|^2\right\}$$
$$=E\left\{\left|a_0b_x+b_x(\underline{a}_{L-1}^T\cdot\underline{u}_{y,L-1})-b_y(\underline{b}_L^H\cdot\underline{u}_{y,L})\right|^2\right\}$$

Letting the constant term of $P_1(u_y)$ be constrained (without loss of generality) to a_0 =1, and rearranging terms we have,

$$J=E\left\{\left|b_x-\left\{\underline{b}_L^H[b_y\cdot\underline{u}_{y,L}]-\underline{a}_{L-1}^T[b_x\cdot\underline{u}_{y,L-1}]\right\}\right|^2\right\} \tag{20}$$

which is an expression that matches well with the conventional quadratic cost function for a typical optimization with an steepest descent algorithm,

$$J=E\left\{\left|d-\underline{w}^H\underline{x}\right|^2\right\}$$

This structural correspondence is shown in figure 3 where the elements are assigned into a block diagram configuration making the non-linear compensation become a system identification problem. So that, we can consider b_x as the reference term d, while the input data and weights ("filter coefficients") vectors, $\underline{x}$ and $\underline{w}$ respectively, are given by,

$$\underline{x}_{2L-1}=\begin{bmatrix}b_y\cdot\underline{u}_{y,L}\\-b_x\cdot\underline{u}_{y,L-1}\end{bmatrix}\qquad \underline{w}_{2L-1}=\begin{bmatrix}\underline{b}_L\\\underline{a}_{L-1}\end{bmatrix} \tag{21}$$

Thus, with these definitions we develop a LMS procedure to find the "filter " coefficients that minimize the mean square output error , that is E{ $|e|^2$}, by estimating the gradient of the MSE error with its instantaneous expression in (20). From the well-known update equation given by

$$\underline{w}_{k+1}=\underline{w}_k-m\,\nabla_{\underline{w}^H}|e|^2 \tag{22}$$

we calculate the gradient after setting a second restriction (first one is that a_0 =1) in order to insure convergence of the algorithm.

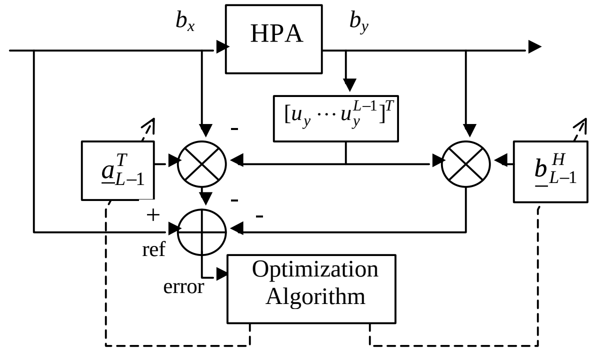

Figure 3: Block diagram of the system identification problem for the HPA's base-band model.

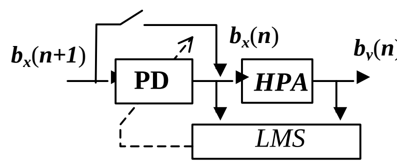

Figure 4: Block diagram of the adaptive pre-distortion.

So that, coefficients in $\underline{a}_L$ are restricted to be real valued while the coefficients in $\underline{b}_L$ will be the complex valued part of the filter weights vector able to compensate the AM/PM distortion.

With these restrictions we find separate update equations for these real and complex coefficients in $\underline{w}_{2L-1}$. The difference from typical LMS and consequently the two piece update equation shown below, arises when we evaluate the gradient separately for the real coefficients sub-vector and for the complex coefficient sub-vector as $\nabla_{\underline{w}^T}$ or $\nabla_{\underline{w}^H}$ respectively.

Thus the general expression in (23) is modified and becomes,

$$\underline{w}_{k+1}=\underline{w}_k-\begin{cases}m\nabla_{\underline{w}^H}|e|^2=me^*\cdot\underline{x}\\ m\nabla_{\underline{w}^T}|e|^2=2m\,\mathrm{Re}\{e^*\cdot\underline{x}\}\end{cases} \tag{23}$$

Thence, the final update equations for the coefficients in $P_1(u_y)$ and $P_2(u_y)$ are given by,

$$\underline{a}_{k+1}=\underline{a}_k+2m\,\mathrm{Re}\left[e^*(-b_x\cdot\underline{u}_{y,L-1})\right] \tag{24}$$

$$\underline{b}_{k+1}=\underline{b}_k+me^*b_y\cdot\underline{u}_{y,L-1} \tag{25}$$

The goal is to use this restricted steepest descent to estimate as few coefficients as possible to achieve best fit to the real P' and Q' inverse curves thus minimizing the mean square error (MSE) and making the HPA appear linear over the full dynamic range.

The fact that the AM/AM distortion characteristic shown in figure 2 increases monotonously within the range defined for the estimation of the pre-distortion curve [0 , A_{sat} = 1], ensures that the corresponding inverse characteristics exists and therefore the real-valued denomitator in (14) has no singularities in such range wich is a necessary condition for the stability of the model. The inverse characteristics of AM/AM and AM/PM distortion are jointly obtained in a frame-oriented adaptive process according to the block diagram shown in figure 4 where the pre-distorter is bypassed only at the initial iteration *n=1* (training frame) and then two base-band signal vectors of the same length *N*, $\boldsymbol{b_x}(n)$ and $\boldsymbol{b_y}(n)$, are sampled at the input and output of the HPA respectively, thus obtaining the L-1 filtering coefficients (a_0 =1) that remain fixed to pre-distort the next *k* input sample vectors $\boldsymbol{b_x}(n+1)$ ... $\boldsymbol{b_x}(n+k)$. Then, after every *k* samples the pre-distorted $\boldsymbol{b_x}$ is applied to update the PD estimation. The expected changes in the non-linear reponse of the HPA and therefore the updating frequency are very slow. Thence, this scheme is self-adaptive and provides better estimations when previously estimated coefficients are applied as the initial condition for the algorithm.

4. SIMULATION RESULTS.

In the experiment we sampled a complex band-limited base-band signal distorted with the normalized model shown in Figure 2. Modulus of the sampled input vectors are Rayleigh distributed with a standard deviation of **s** = 1/5. As the corresponding theoretical expression holds that $F_{Rayl}(5s)$ = 0.999996273, almost every signal sample will be within the valid dynamic range {0,1} when $\mathbf{A_{sat}}$ = 1, whereupon we achieve an effective compensation of distortion. The estimated pre-distortion characteristic was applied according to eq. (14) over a second data frame with the same lenght, power and statistical distribution of the previous training set, but considering independent symbols generation Alternatively, we can extract P interpolation coordenates from the final estimated pre-distortion curve to execute digital pre-distortion but an interpolation error would be introduced in such case. Several realizations were done based on the general model in (14) defining the use of solely even, odd, or both power orders for the polynomials. The worst performance was obtained when using just odd orders while the best estimation to reach a –50dB average squared error was obtained using even and odd orders and also constraining the coefficient α_{L-1} (eq.18) to be zero, so that in such case the numerator is one order higher and fits better the vicinity of the saturation. The best passband compensation so obtained, measured as the input-to-output error spectrum using pre-distortion, was of 22dB below the input signal level, also featuring a different out-of-band behavior. Pre-distortion results obtained with a slow convergence LMS procedure are depicted in figures 5, 6 and 7. Polynomials of order 14 were used, so that 26 zero-initialized coefficients were estimated in the algorithm.

6. CONCLUSIONS.

In this paper we introduced and tested an alternative rational model to design memoryless complex pre-distortion. Adaptive algorithms have been suitably modified to satisfy some particular restrictions to insure convergence and stability in the estimation. Phase and amplitude pre-distortion are simultaneously estimated in a single process and then applied as a mutliplicative complex inverse gain thus obtaining an almost perfect linear HPA. The rational model fits better the vicinity of the saturation point than a single polynomial model and performs good compensation of the phase distortion assigning just the half of the coefficients for this purpose. Further analysis on the difference between passband and out-of-band compensation and the improvement in the estimation algorithm performance can be considered for a future work.

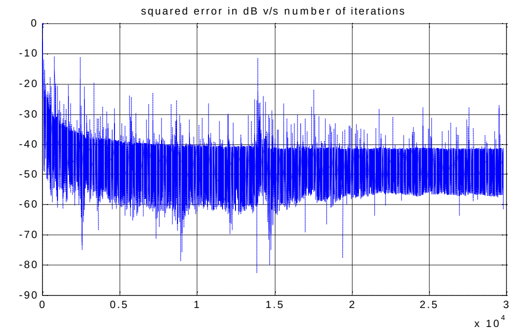

Figure 5: Evolution of the LMS squared error in dB.

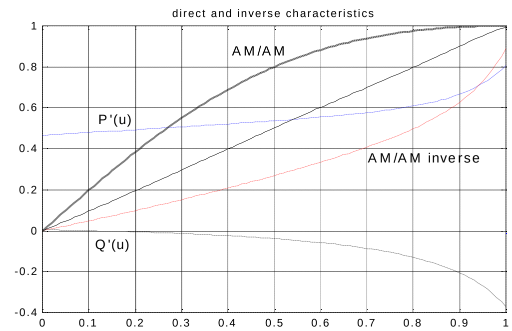

Figure 6: Direct (theoretical) and estimated inverse non-linear characteristics. P'(u) and Q'(u) curves (eq.)and linearization.

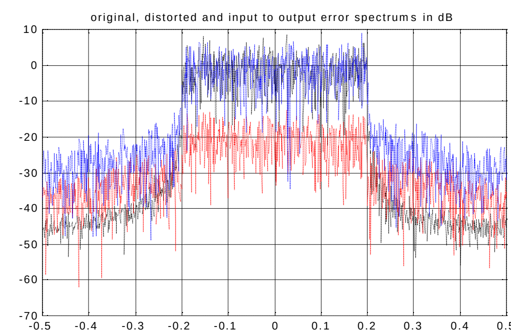

Figure 7: Input, distorted and compensation error spectrums.

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