Low Complexity Multiple Description Coding Scheme using Wavelet Transform

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ABSTRACT

A challenge of image communication over unreliable channels is to achieve good compression rates and be effective in presence of channel failures.
In this work we use the Multiple Description Coding (MDC) techniques, based on Wavelet Transforms, that have been shown to be powerful against channel failures. We propose a Bit Allocation procedure that dispatch redundancy between the different channels when compressing to a target bit rate with a bounded side distortion. In this way we develop a MDC scheme well adapted to channel noise.

1 Introduction

1.1 Motivation and Related work

Image compression and robustness are essentials for transmission.

The volume of data required to describe images greatly slow down transmission. The information contained in the images must therefore, be compressed by extracting only the visible elements. Compression can be achieved by transforming the data, projecting it on a basis function, and then encoding this transform. The quantity of data involved is thus reduced substantially while maintaining good fidelity or image quality.

The problem of transmitting data over heterogeneous networks has recently received considerable attention. A typical scenario might require data to move from a finer link to a wireless link, which necessitates dropping packets to accommodate the lower capacity of the later. The situation is similar if packets are lost due to transmission errors or congestion. This problem is a generalization of the Multiple Description (MD) problem. In the MD problem (reduced to the simplest case of two descriptions), a source is described by two descriptions at side rates R_1 and R_2. These two descriptions individually lead to reconstructions with side distortions D_1 and D_2, respectively; the two descriptions together yield a reconstruction with central distortion D_0.

Same MDC framework have been proposed by Vaishampayan in [7, 8, 9, 6], where MD scalar, vector, or trellis quantizers are designed to produce two descriptions, using a generalized Lloyd-like clustering algorithm that minimizes the Lagrangian of the rates and expected distortions R_1, R_2, D_1, D_2, D_0. Jiang and Ortega propose an MD extension to SPIHT coder of Said and Perlman [5], by separating Zerotrees into polyphase components [2]. Miguel, Mohr and Riskin propose a scheme using SPIHT in a generalized multiple description framework[3].

1.2 Main Contributions

We use a Multiple Description scheme based on Wavelet Transforms and a new Bit Allocation technique to find an optimal compromise between efficient compression and robustness from losses due to communications using unreliable channels.

The new Bit Allocation technique makes possible a central distortion minimization for specified side distortions and bit rates. Our Bit Allocation scheme takes into account the channel noise to compute the amount of redundancy to include when compressing. In this way we found a scheme well adapted to channel noise.

To evaluate the performance of our system we propose an image coding application with two descriptors. We compare our application to some Multiple Description Coding techniques presented in [6, 2, 3].

Section (2) presents the problem statement and is followed by our Bit Allocation for Multiple Description scheme (section 3) and the proposed algorithm (section 4). The results we obtain are presented in section 5. We conclude and propose some future work in section 6.

2 Problem Statement

Our MD scheme focus on the special case in which there are two channels of equal capacity between a transmitter and a receiver. In such a scheme, a sequence of source symbols is given to an encoder to produce two independent bitstreams of equal importance. These bitstreams are transmitted to three decoders over two noisy channels. One decoder (the central decoder) receives infor-

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mation sent over both channels while the remaining two decoders (the side decoders) receive information only from their respective channels.

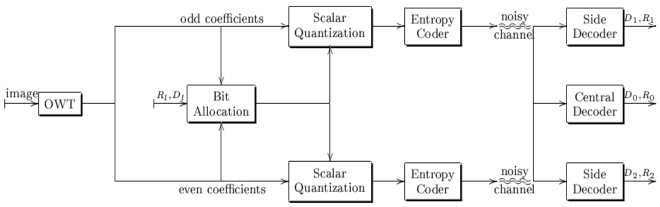

Figure 1: General MDC Scheme. R_l and D_l are respectively the side bit rate and the side distortion $\{l = 1, 2\}$.

The generation of the two descriptions is constrained to some conditions that we detail in the following.

condition 1 The central decoder has to reconstruct the original image from the two descriptors with **minimal central distortion** D_0.

condition 2 A MDC coder must generates two descriptors each with a **maximal side rate** R_l;

condition 3 The side decoders must reconstruct the original image from a single descriptor with a **maximal side distortion** D_l;

The problem is then to **minimize the central distortion D_0 (condition 1) when condition 2 and 3 are verified**. This is, for $\#SB$ the number of subbands, we have to find the sets of bit rates $\{R_{i,1}\}, \{R_{i,2}\}$ that minimize the central distortion D_0, where $R_{i,j}$ is used for coding subband $i \in \{1, .., \#SB\}$, of descriptor $j \in \{1, 2\}$. More precisely, we have to find the quantization steps $\{q_{i,1}\}, \{q_{i,2}\}$ that produce the bit rates $\{R_{i,1}\}, \{R_{i,2}\}$. This problem is known as the *Bit Allocation Problem.*

3 Bit Allocation

3.1 General Case

Bit Allocation schemes intend to determine the set of quantization steps $\{q_i\}$ that minimize the distortion at a given bit rate. Choosing a given quantizer q_i will produce a distortion D_i and a rate R_i for a subband i. The problem is to find which combination of quantizers in the various subbands will produce the minimum distortion while satisfying the side bit rates constraint and side distortion.

3.2 Our Bit Allocation for Multiple Description scheme

The purpose of our bit allocation for MD scheme is to determine the optimal sets of quantization steps $\{q_{i,1,i=1,\ldots,\#SB}\}$, $\{q_{i,2,i=1,\ldots,\#SB}\}$ for descriptors 1 and 2. More precisely, the bit allocation finds which combination of quantizers in the various subbands minimizes the central distortion -D_0- (condition 1) for a determined bit rate -R_l- (condition 2) and for a side distortion not greater than a given distortion -D_l (condition 3). The parameters R_l and D_l are given for the Bit Allocation (Fig. B.1).

Recall that the central distortion, is the distortion of the decoded image when using both descriptors at decoding. When the decoder receives both descriptors, each subband appears twice, with different bit rates (different associated quantization steps). In this case, if the subbands are noiseless, the decoder choose the subband with the smaller quantization step (better coded). Hence, we can calculate, the distortion of the decoded image as $\sum_{i=1}^{\#SB} min(D_{i,1}, D_{i,2})$, where $D_{i,1}, D_{i,2}$ are the distortions of subband i for descriptors 1 and 2, respectively.

In the general case we have to take into account channels noise. More the channels are noisy, more we need redundancy between descriptors. This is easily understandable if we think in the two extreme cases:

1. the channels are almost noiseless: in this case we want to *minimize the redundancy*, i.e., each subband is very well coded in one description, but not in the other;

2. the channels are really noisy: in this case we want to *maximize the redundancy* to warrant that the decoder can at least recover each subband once.

In that way the central distortion depends on channels noise. We use a parameter $r_N \in [0, 1]$ to reflect this dependency. $r_N = f(\frac{p_\epsilon}{1-p_\epsilon})$ where p_ϵ is the probability of channels noise. r_N is 0 when $p_\epsilon = 0$, hence we minimize the redundancy. When $r_N \approx 1$ the redundancy becomes maximal.

We compute the central distortion as

$$D_0 = \sum_{i=1}^{\#SB} \Delta_i \sigma_{i,0}^2 D_{i,0} \left(\frac{q_{i,1}}{\sigma_{i,1}}, \frac{q_{i,2}}{\sigma_{i,2}} \right) \quad (1)$$

and we assume

$$\begin{aligned} D_{i,0} \left(\frac{q_{i,1}}{\sigma_{i,1}}, \frac{q_{i,2}}{\sigma_{i,2}} \right) &= \frac{1}{\sigma_{i,0}^2} min(\sigma_{i,1}^2 D_{i,1}, \sigma_{i,2}^2 D_{i,2}) \quad (2) \\ &+ r_N \times max(\sigma_{i,1}^2 D_{i,1}, \sigma_{i,2}^2 D_{i,2}). \end{aligned}$$

where, $\sigma_{i,j}^2 D_{i,j}(\frac{q_{i,j}}{\sigma_{i,j}})$ is the Mean Square Error for the ith subband and Δ_i is an optional weight for frequency selection [4]. The side distortions D_1, D_2 are defined by

$$D_j = \sum_{i=1}^{\#SB} \Delta_i \sigma_{i,j}^2 D_{i,j} \left(\frac{q_{i,j}}{\sigma_{i,j}} \right) \quad (3)$$

Condition 1 implies to find the quantization steps $\{q_{i,1}\}$, $\{q_{i,2}\}$ that minimize this central distortion D_0.

This allocation problem is a constrained problem which can be solved by introducing the Lagrange operators. The Lagrangian functional for the constrained optimization problem, is given, for $j = 1, 2$ by

$$J_{\lambda_j,\mu_j}(\{q_{i,j},\}) = D_0 + \lambda_j(R_j < R_l) + \mu_j(D_j < D_l). \quad (4)$$

Conditions 2 ($R_{j=1,2} < R_l$) and 3 ($D_{j=1,2} < D_l$) have to be defined for each descriptor. For the different descriptors $j = 1, 2$, we write **condition 2** as a constraint,

$$\left(\sum_{i=1}^{\#SB} a_i R_{i,j}\left(\frac{q_{i,j}}{\sigma_{i,j}}\right) - R_l\right), \quad (5)$$

where $a_i = \frac{1}{2^{2\lceil i/3 \rceil}}$ is the quotient of the size of the subband divided by the size of the whole image for a dyadic wavelet decomposition. $R_{i,j}(q_{i,j})$, is the bit rate for the ith subband.

Condition 3 is forced using a penalty. The penalty methods are simple and efficient. Consider a constraint $x > 0$ the penality is written as $P(x) = (\frac{|x|-x}{2})^2$. If the constraint is verified then $x > 0$ and $P(x) = 0$. Otherwise, $x < 0$ and $P(x) = x^2$. Considering (3), the constraint is $(D_j - D_l) < 0$. The penalty is written as

$$\left[\frac{|D_j - D_l| + (D_j - D_l)}{2}\right]^2, \text{ for all } j \in \{1, 2\} \quad (6)$$

This penalty function allows us to find a solution with $\sum_{i=1}^{\#SB} \Delta_i \sigma_{i,j}^2 D_{i,j}\left(\frac{q_{i,j}}{\sigma_{i,j}}\right) \leq D_l$. In this case we say that the penalty is verified.

Considering (1) and the constraints (5) and (6) the Lagrangian functional (4) can be rewritten as

$$\begin{aligned} J_{\lambda_1,\lambda_2,\mu_1,\mu_2}(\{q_{i,1}, q_{i,2}\}) &= \sum_{i=1}^{\#SB} \Delta_i \sigma_{i,0}^2 D_{i,0}\left(\frac{q_{i,1}}{\sigma_{i,1}}, \frac{q_{i,2}}{\sigma_{i,2}}\right) \\ &+ \sum_{j=1}^{2} \lambda_j \left(\sum_{i=1}^{\#SB} a_i R_{i,j}\left(\frac{q_{i,j}}{\sigma_{i,j}}\right) - R_l\right) \\ &+ \sum_{j=1}^{2} \mu_j \left[\frac{|D_j - D_l|}{2} + \frac{(D_j - D_l)}{2}\right]^2 \end{aligned} \quad (7)$$

Solution of (7) is obtained when $\frac{\partial J_{\lambda_1,\lambda_2,\mu_1,\mu_2}(\{q_{i,1},q_{i,2}\})}{\partial q_{i,1}} = 0$ and $\frac{\partial J_{\lambda_1,\lambda_2,\mu_1,\mu_2}(\{q_{i,1},q_{i,2}\})}{\partial q_{i,2}} = 0$. The differentiations are similar. For the first one we have

$$\begin{aligned} \frac{\partial J_{\lambda_1,\lambda_2,\mu_1,\mu_2}(\{q_{i,1}, q_{i,2}\})}{\partial q_{i,1}} &= \Delta_i \sigma_{i,0}^2 \frac{\partial}{\partial q_{i,1}} D_{i,0}\left(\frac{q_{i,1}}{\sigma_{i,1}}, \frac{q_{i,2}}{\sigma_{i,2}}\right) \\ &+ \lambda_1 a_i \frac{\partial}{\partial q_{i,1}} R_{i,1}\left(\frac{q_{i,1}}{\sigma_{i,1}}\right) + \mu_1 \frac{\partial}{\partial q_{i,1}} A\left(\frac{q_{i,1}}{\sigma_{i,1}}\right) = 0 \end{aligned} \quad (8)$$

where A is 0 when $D_j \leq D_l$, and $(D_j - D_l)^2$ otherwise.

The minimization of (8) yields to

$$\begin{aligned} (C_1 + \mu_1 E_1)\, \Delta_i \sigma_{i,1}^2 \frac{\partial}{\partial q_{i,1}} D_{i,1}\left(\frac{q_{i,1}}{\sigma_{i,1}}\right) & \\ + \lambda_1 a_i \frac{\partial}{\partial q_{i,1}} R_{i,1}\left(\frac{q_{i,1}}{\sigma_{i,1}}\right) &= 0 \end{aligned} \quad (9)$$

where C_1 is 1 if $min(D_{i,1}, D_{i,2}) = D_{i,1}$ or r_N otherwise; and E_1 is 0 if $D_1 \leq D_l$ or $2 \times (D_1 - D_l)$ otherwise.

Simplifying (9) and performing similar calculus for $\partial q_{i,2}$ permit to obtain the system (10) (for $j \in \{1, 2\}$) for a two channels scheme.

$$\begin{cases} \frac{\partial D_{i,j}}{\partial R_{i,j}}\left(\frac{q_{i,j}}{\sigma_{i,j}}\right) = \frac{-\lambda_1 a_i}{\Delta_i \sigma_{i,j}^2 (C_j + \mu_1 E_j)} & \textbf{(a)} \\ \sum_{i=1}^{\#SB} a_i R_{i,j}\left(\frac{q_{i,j}}{\sigma_{i,j}}\right) - R_l = 0 & \textbf{(b)} \end{cases} \quad (10)$$

The resolution of system (10) give us the optimal sets of quantization steps $\{q_{i,1}\}$, $\{q_{i,2}\}$.

3.3 C_j parameter

The C_j parameter in (10) depends on $min(D_{i,1}, D_{i,2})$, that are unknown before system (10) is solved due to their dependence on the quantization steps $\{q_{i,1}\}$, $\{q_{i,2}\}$. To solve this problem we calculate the system for all the different possibilities of $min(D_{i,1}, D_{i,2})$.

For each descriptor we consider a combination of $\#SB$ 0's and 1's, indicating which one's of the subbands have minimal distortion. We name such combinations, a path. For $\#SB$ subbands we have $2^{\#SB}$ different paths.

Our Bit Allocation procedure compute the system (10) for each different path. The optimal path is the one which provides the smaller distortion D_0.

In the next section is presented the global bit allocation algorithm.

4 Proposed Algorithm

Require: μ_{ind}, λ_{ind} and $r_N = f(p_\epsilon)$
  repeat
    for each path(section 3.3) **do**
      for $j = 1, 2$ (descriptor 1 and 2) **do**
        repeat
          for each subband **do**
            From **(a)**$_{j=1,2}$, in (10) compute the bit rates $R_{i,j}$, using $\frac{\partial D}{\partial R}$ function.
          end for
          Compute a new λ_{ind} by dichotomy.
        until (b)$_{j=1,2}$ in (10) verified
        for each subband **do**
          Compute $q_{i,j}$ using the bit rate R functions.
          Compute D_j (3) using the D function.
        end for
      end for
      Compute the minimal central distortion D_0 (1).
    end for
    Compute the new μ_j as $\mu_j^{new} = \mu_j^{old} + \eta_k d_j$, with η_k the displacement value and $d_j = D_j - D_l$.
  until $D_j <= D_l, j = \{1, 2\}$
  Result: $\{q_{i,1}\}$, $\{q_{i,2}\}$

5 Results

5.1 Coder

Our coder use Wavelet Transforms, with a 9-7 biorthogonal filter [1] and a three level wavelets decomposition, to generate the descriptions. The Bit Allocation procedure, presented in section 3, is followed by a simple scalar quantization and the encoding of each subband uses EBCOT as entropy coding. The encoding of each subband of each descriptor is done independently and can be performed simultaneously.

For the Bit Allocation procedure, described in section 4 we use analytical models for the distortion and rate functions as described in [4]. The distribution $p(x)$ of the wavelet coefficients are then modeled by generalized Gaussian [1].

5.2 Central PSNR vs. Side PSNR

For r_N values between 0 and 1 we found the Central PSNR vs. Side PSNR relation. We represent it for Lena image with 1bpp central bit rate in (2) and we compare our results with the referenced methods [3] - Ref1, [2] - Ref 2, [6] - Ref 3. We represent two different results. In the first one (New 1) we choose the path in the most simple way - 1 0 1 0 1 0 1 0 1 0 -, resulting in a very simple MDC scheme with a simple and efficient algorithm associated. When performing the search of the optimal path (result New 2) the algorithm provides the best results.

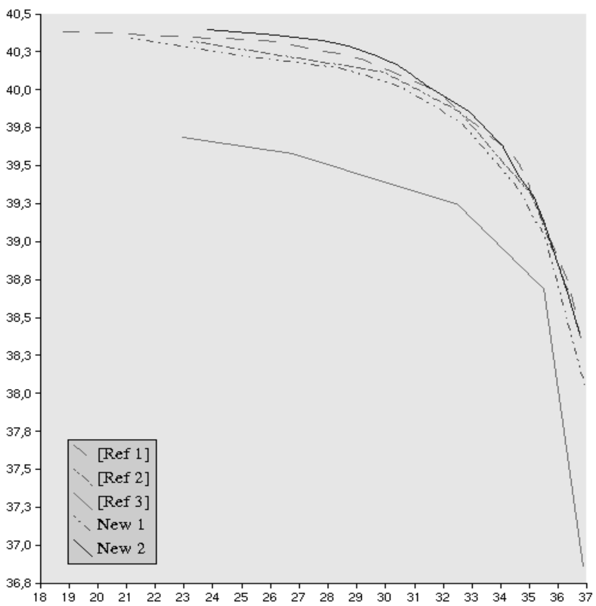

Figure 2: Side PSNR vs Central PSNR

5.3 Central PSNR vs Channel SNR

We simulate a Binary Symmetric Channel (BSC) with a probability the error p_ϵ. This is $P(Y=0|X=1) = P(Y=1|X=0) = p_\epsilon$ and $P(Y=0|X=0) = P(Y=1|X=1) = 1-p_\epsilon$. That means, the channel cause statistically independent errors in the transmitted binary sequence. We compare the performance of our model with the typical EBWIC [4] compression coder not robust to noise. This coder achieve similar results than JPEG for Lena image. Fig. 2 represents the PSNR performances for different probabilities p_ϵ.

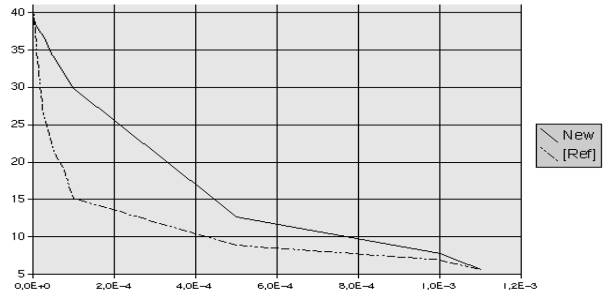

Figure 3: PSNR Image vs. SNR of a Rayleigh channel.

6 Conclusion and Future work

We propose a new Bit Allocation scheme that uses channels noise information to dispatch redundancy. In this way, we provide a low complexity MDC scheme well adapted to channel noise.

The method proposed can be generalized to more descriptions. This is our next work.

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