Semiblind Direct Equalisation using Mutually Referenced Equalisers

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ABSTRACT

Wireless ad-hoc networks must accept the presence of unsynchronized interfering signals. To identify the user of interest, some form of training must be used, *e.g.*, colour code or a short training burst. Instead of a single block of training, it is better to have the training symbols dispersed over the packet. We propose a semiblind algorithm for the separation and equalization of the packet of the user of interest, received over multipath fading channels. The algorithm is based on a direct equaliser approach, combined with ideas from blind "mutually referenced equalisers" (MRE).

1 Introduction

Wireless ad-hoc networks in the unlicenced band must accept any interference from other devices, e.g. users of similar equipment, users with other types of equipment, and even microwave ovens (Figure 1). The close operation of Bluetooth and WLAN devices is already causing problems, and it is expected that this problem will grow to significant proportions in the near future.

To be able to cancel interfering signals, we propose to use receivers equipped with multiple antennas, so that appropriate null steering will deliver only the desired sequence. To identify the user of interest among other signals, it must code its signal in a unique way, e.g., as is done in CDMA [1]. To avoid unnecessary loss of spectral efficiency, we propose in this paper to provide the user with a unique but short training sequence. The training should not be localized in a mid-amble, but has to be spread over the complete packet, because bursts from other users can appear and disappear at any point (Figure 2). It has been shown that dispersed training can lead to more accurate channel estimates in similar scenarios [2], as based on an analysis of the Cramer-Rao Bound (CRB). However, it appears that there are no algorithms available to compute the channel estimates, or the separating equalisers.

For small amounts of training, it is necessary to combine this information with structural properties of the other received symbols, leading to semiblind algorithms. There are many such algorithms (e.g. [3, 4]), but none

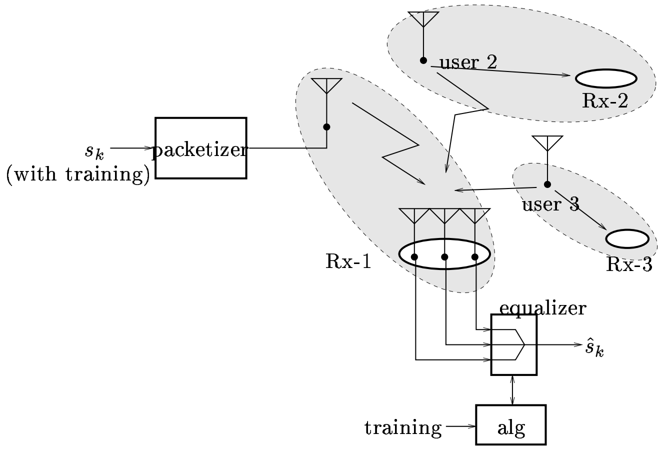


Figure 1: Wireless ad-hoc communication scenario

for source separation with equalization based on dispersed training symbols. Our objective here is to derive a semiblind algorithm for this case.

The algorithm is based on a "direct equaliser approach", in which not the channel is estimated, but the signal at the output of the beamformer/equaliser. In this framework, it is straightforward to pose conditions on the resulting sequence (the training symbols), and to exploit the structure in the signal matrix due to the convolutive FIR channel model. The latter is based on ideas from "mutually referenced equalisers" (MRE), in which a bank of equalisers is defined, such that the output of one is a shift of the next equaliser. This approach was proposed in [5], and later shown to be equivalent to another direct blind sequence estimator based on row span intersections [6].

Notation: ^{T} denotes a matrix transpose, ^{H} the matrix complex conjugate transpose, † the pseude inverse, $\mathbf{0}$ an appropriatly sized vector or matrix of all 0s, and $\mathbf{1}$ an appropriatly sized vector or matrix of all 1s.

2 Data Model

For the user of interest, we consider a single-transmit / multiple-receive antenna (SIMO) model with a con-

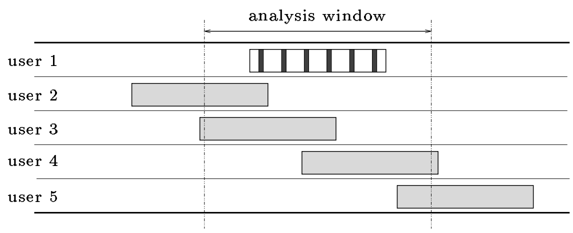


Figure 2: Slot structure

volutive channel. The user transmits a digital symbol sequence $[s_i]$ through a medium, which is received by an array of $M \geq 1$ sensors. The received signals are sampled $P \geq 1$ times faster than the symbol rate, normalized to $T = 1$. During each symbol period, a total of MP measurements are available, which can be stacked into MP-dimensional vectors $\mathbf{x}_i = [x_i^1, \cdots, x_i^{MP}]^T$. Assuming an FIR channel, we can model $\mathbf{x}_i$ as the output of an MP-dimensional vector channel with impulse response $[\mathbf{h}_0, \mathbf{h}_1, \cdots, \mathbf{h}_{L-1}]$, where L denotes the channel length. In the interference- and noise-free case, $\mathbf{x}_i$ is then given by

$$\mathbf{x}_i = \sum_{k=0}^{L-1} \mathbf{h}_k s_{i-k} \,. \tag{1}$$

Define corresponding matrices

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \dots & \mathbf{x}_N \end{bmatrix} \\ \mathbf{H} &= \begin{bmatrix} \mathbf{h}_0 & \mathbf{h}_1 & \cdots & \mathbf{h}_{L-1} \end{bmatrix} \\ \mathbf{S} &= \begin{bmatrix} s_0 & s_1 & \cdots & s_N \\ s_{-1} & s_0 & \cdots & s_{N-1} \\ \ddots & \ddots & \ddots & \ddots \\ s_{-L+1} & s_0 & \cdots & s_{N-L+1} \end{bmatrix}, \end{aligned} \tag{2}$$

such that we can write $\mathbf{X}$ as a factorisation $\mathbf{X} = \mathbf{HS}$. With noise $\mathbf{N}$ and additional users interference $\mathbf{Y}$, the model becomes

$$\mathbf{X} = \mathbf{HS} + \mathbf{Y} + \mathbf{N} \,. \tag{3}$$

Furthermore, we will assume that $\mathbf{H}$ is tall and full column rank, and $\mathbf{S}$ is wide and full row rank L, so that $\mathbf{HS}$ is rank L. In addition, we assume that rank$(\mathbf{Y}) \leq MP - L$, so that row$(\mathbf{X})$ contains row$(\mathbf{S})$. (If $\mathbf{H}$ is not tall, then it can be made tall by shifting and stacking rows of $\mathbf{X}$ [7].) These conditions are essential to the existence of (zero-forcing) equalisers $\mathbf{w}$ that can reconstruct rows of $\mathbf{S}$ via $\mathbf{w}^H\mathbf{X}$, at least in the noise free case. Let d be the rank of $\mathbf{X}$ in the noise free case. To avoid equalisers in the null space of $\mathbf{X}$, in all algorithms to follow a preprocessing is necessary, consisting of a prewhitening and dimension reduction to the rank of $\mathbf{X}$. The processing consists of computing a singular value decomposition of $\mathbf{X} = \mathbf{U\Sigma V}$, and replacing $\mathbf{X}$ by the first d rows of $\mathbf{V}$. Refer to [6, 7] for further details.

3 Algorithm

The proposed semiblind algorithm for direct equalisation is presented in detail in Section 3.3, after an introduction to mutually referenced equalisers (MRE) in Section 3.2. Although not hard to generalise, we assume from now on a simplified case where the channel has length $L = 2$ symbols.

3.1 Training and sequence shifts

Consider a finite block of data and define the $MP \times N$ submatrices of $\mathbf{X}$,

$$\mathbf{X}^{(i)} = \begin{bmatrix} \mathbf{x}_i & \mathbf{x}_{i+1} & \dots & \mathbf{x}_{i+N-1} \end{bmatrix} .$$

where the superscript $^{(i)}$ denotes an offset (we will consider $i = 0$ and $i = 1$). From (3), $\mathbf{X}^{(i)}$ has a factorisation as $\mathbf{X}^{(i)} = \mathbf{HS}^{(i)}$, where $\mathbf{H}$ is the $MP \times 2$ channel matrix defined before, and $\mathbf{S}^{(i)}$ is a $2 \times N$ submatrix of $\mathbf{S}$,

$$\begin{aligned} \mathbf{H} &= \begin{bmatrix} \mathbf{h}_0 & \mathbf{h}_1 \end{bmatrix} \\ \mathbf{S}^{(i)} &= \begin{bmatrix} s_i & s_{i+1} & \cdots & s_{i+N-1} \\ s_{i-1} & s_i & \cdots & s_{i+N-2} \end{bmatrix} . \end{aligned} \tag{4}$$

Let us denote the transmitted sequence by $\mathbf{s} = \begin{bmatrix} s_0 & s_1 & \cdots & s_N \end{bmatrix}$, and the shifted and truncated version by $\mathbf{s}^{(i)} = \begin{bmatrix} s_i & s_{i+1} & \cdots & s_{i+N-1} \end{bmatrix}$. The sequence $\mathbf{s}$ contains L_T training symbols $\overline{s}_i$, $i = 0 \cdots L_T$, in arbitrary positions. We define a selection matrix $\mathbf{J}$ such that $\mathbf{sJ}$ selects exclusively the training symbols of the sequence $\mathbf{s}$. $\mathbf{J}$ is an $N+1 \times L_T$ augmented identity matrix, where all-zeros rows were inserted in the positions corresponding to unknown symbols and

$$\mathbf{s}_T = [\overline{s}_0 \cdots \overline{s}_{L_T}] = \mathbf{sJ} \,. \tag{5}$$

Similarly to (4,5) we define $\mathbf{J}^{(i)}$ such that $\mathbf{s}^{(i)}\mathbf{J}^{(i)} = \mathbf{s}_T$, assuming that all training symbols are present in $\mathbf{s}^{(i)}$.

3.2 Mutually referenced equalisers

An equaliser can be viewed as a vector $\mathbf{w}$ acting on $\mathbf{X}^{(i)}$ to produce an output sequence $\mathbf{z} = \mathbf{w}^H\mathbf{X}^{(i)}$. Since $\mathbf{S}^{(i)}$ has two rows, there are two different equalisers, $\mathbf{w}_0$ and $\mathbf{w}_1$, to recover the source symbols at the specified delays,

$$\begin{cases} \mathbf{w}_0^H\mathbf{X}^{(i)} &= [s_i \;\; s_{i+1} \;\; \cdots \;\; s_{i+N-1}] \\ \mathbf{w}_1^H\mathbf{X}^{(i)} &= [s_{i-1} \;\; s_i \;\; \cdots \;\; s_{i+N-2}] \end{cases}$$

or

$$\mathbf{w}_0^H\mathbf{x}_k = \mathbf{w}_1^H\mathbf{x}_{k+1} \,. \tag{6}$$

Taking two delays of the inputs, we can write

$$\mathbf{w}_1^H\mathbf{X}^{(1)} = [s_0 \;\; s_1 \;\; \cdots \;\; s_{N-1}] = \mathbf{w}_0^H\mathbf{X}^{(0)} \tag{7}$$

Thus, the equaliser outputs can be paired, which is the idea behind the MRE technique [5]. The equalisers can be found in various ways, adaptively or using subspace intersections, *cf.* [6], essentially by solving

$$\min_{\mathbf{w}_0,\mathbf{w}_1} \ \| [\mathbf{w}_0^H \quad \mathbf{w}_1^H] \begin{bmatrix} \mathbf{X}^{(0)} \\ -\mathbf{X}^{(1)} \end{bmatrix} \|^2$$

with a suitable norm constraint on $[\mathbf{w}_0^H \ \ \mathbf{w}_1^H]$. The solution is given by the left singular vector corresponding to the smallest singular value of $\left[\begin{smallmatrix} \mathbf{X}^{(0)} \\ -\mathbf{X}^{(1)} \end{smallmatrix} \right]$. The corresponding right singular vector is the source sequence $\alpha[s_0 \ \ s_1 \ \ \cdots \ \ s_{N-1}]$, where α is an indetermined scaling.

3.3 Semiblind MRE implementation

The proposed MRE semiblind equalisation algorithm is designed to work on the first L rows of the signal row space $\mathbf{V}$, obtained from an initial svd of the received signal matrix, $\mathbf{X} = \mathbf{U\Sigma V}$. In a multiuser scenario, d rows of $\mathbf{V}$ corresponding to the signal subspace would be selected, where d is the rank of $\mathbf{X}$.

Let us define $\mathbf{V}^{(i)}$ in a similar manner as $\mathbf{X}^{(i)}$. For the particluar case of $L = 2$ shifts, $\mathbf{V}^{(0)}$ and $\mathbf{V}^{(1)}$ are the $L \times N$ shifted and truncated copies of $\mathbf{V}$.

The semiblind MRE implementation must include both blind and training-based equaliser conditions, corresponding to the pairing of both equaliser outputs for the several shifts of the transmitted sequence.

As in Section 3.2, the blind conditions are expressed by

$$\mathbf{w}_0^H \mathbf{V}^{(0)} = \mathbf{w}_1^H \mathbf{V}^{(1)} \,.$$

The training-based equalisation conditions are expressed by

$$\mathbf{w}_0^H \mathbf{V}_T^{(0)} = \mathbf{w}_1^H \mathbf{V}_T^{(1)} = [\bar{s}_0 \cdots \bar{s}_{L_T}]$$

where $\mathbf{V}_T^{(0)} = \mathbf{VJ}^{(0)}$, and $\mathbf{V}_T^{(1)} = \mathbf{VJ}^{(1)}$.

The two conditions can be combined as a single condition, where α is a scaling for the training condition, and

$$\mathcal{V} \quad = \quad \left[\begin{array}{c|cc} \mathbf{V}^{(1)} & \alpha\mathbf{V}_T^{(1)} & \mathbf{0} \\ -\mathbf{V}^{(2)} & \mathbf{0} & \alpha\mathbf{V}_T^{(2)} \end{array} \right] ,$$

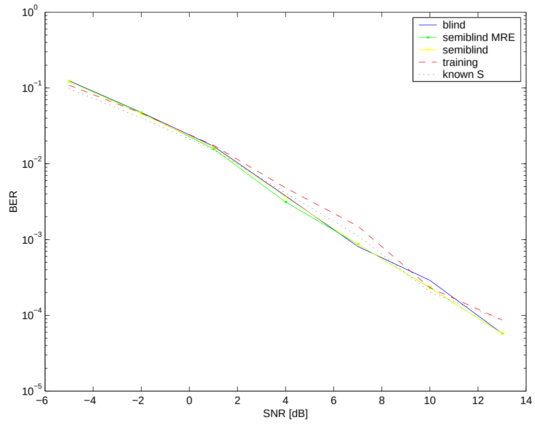
and, with $\mathbf{t} = [\mathbf{0} \quad \mathbf{s}_T \quad \mathbf{s}_T]$, the problem can be posed as

$$[\mathbf{w}_0^H \quad \mathbf{w}_1^H]\mathcal{V} = \mathbf{t} \tag{8}$$

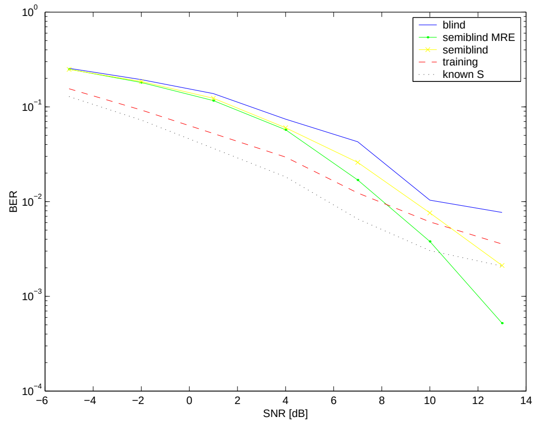
If $\mathcal{V}$ is full rank, the equaliser pair $[\mathbf{w}_0 \quad \mathbf{w}_1]$ is the unique solution to Eq. (8). The design and placement of the training symbols such that the full rank condition is always observed is an interesting and open issue. In the presence of noise, Eq. (8) is replaced by the minimisation of the cost function

$$\min_{\mathbf{w}_1,\mathbf{w}_2} \ \| [\mathbf{w}_1^H \quad \mathbf{w}_2^H] \left[\begin{array}{c|cc} \mathbf{V}^{(1)} & \alpha\mathbf{V}_T^{(1)} & \mathbf{0} \\ -\mathbf{V}^{(2)} & \mathbf{0} & \alpha\mathbf{V}_T^{(2)} \end{array} \right] - \mathbf{t} \ \|^2 \tag{9}$$

and the solution is $[\mathbf{w}_0^H \quad \mathbf{w}_1^H] = \mathbf{t}\mathcal{V}^{\dagger}$.



(a) Equal power 2-tap Rayleigh.



(b) Dominant path, 2-tap Rayleigh.

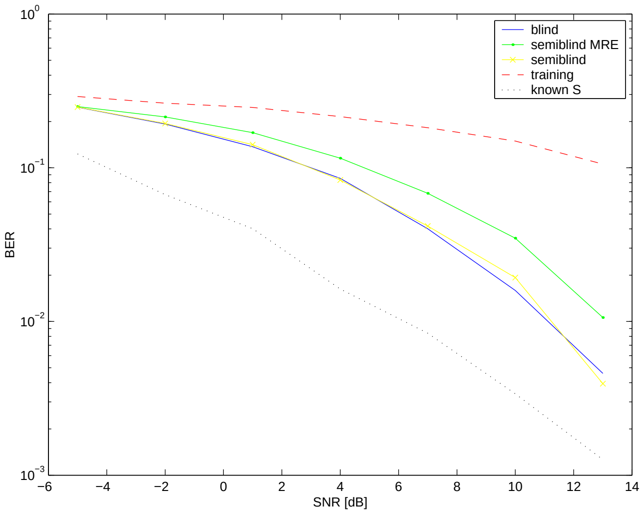
Figure 3: BER results for $N = 116, L_T = 26$.

4 Simulation Results

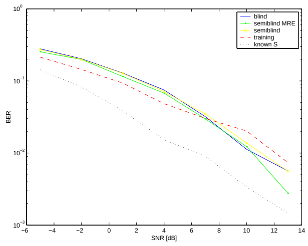
The performance of the proposed algorithm was simulated in a Rayleigh fading channel of lenght $L = 2$, for equal power paths and for a dominant path, in order to evaluate the resilience against an ill-conditioning of $\mathbf{H}$. Spatio-temporal sampling, or the product MP, was set to 2. Two equivalent implementations of the semiblind MRE algorithm are compared with the pure blind, the training-based, and the one assuming perfect knowledge of $\mathbf{S}$.

In Figure 3, for $N = 116$ samples, $L_T = 26$ training symbols (as in GSM), $\alpha = 1$, the average BER results for a Rayleigh fading channel are presented in a) for equal power paths, and in b) for a dominant path channel (where the difference in power is 10 dB).

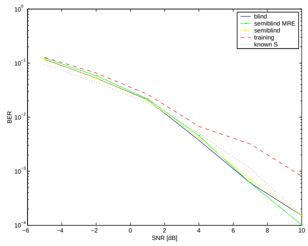
In Figure 4, for $N = 100$ samples, $\alpha = 1$, simulation results for a Rayleigh fading channel are presented in a) for a dominant path channel and $L_T = 2$ training symbols, in b) for a dominant path channel and $L_T = 10$ training symbols, and in c) for the same L_T but equal-power paths. Figure 5, presents for $N = 116$ samples, $L_T = 26$, the average BER results obtained for a training weight of $\alpha = \frac{1}{4}$.



(a) $L_T = 2$ and dominant path, 2-tap Rayleigh.



(b) $L_T = 10$, dominant path.

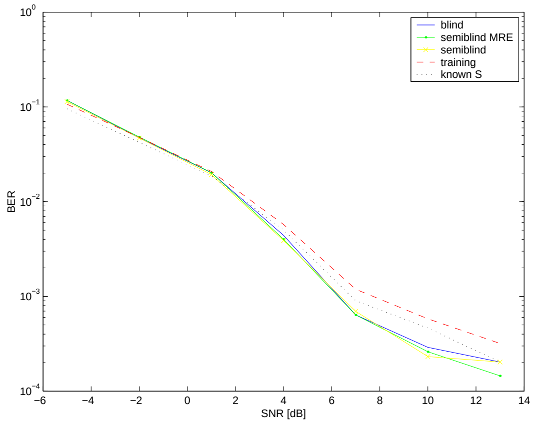


(c) $L_T = 10$, equal power paths.

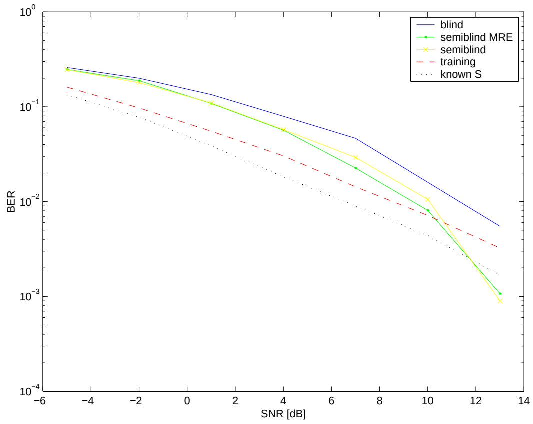
Figure 4: Bit Error Rate results for $N = 100$.

5 Conclusions

An accurate determination of training weight is needed, as semiblind algorithms can converge to the worst-case equaliser, either the blind or the training-based. Training sequence lenght and weight are directly connected with performance, and heavily dependent of the channel. BER improvements are noticed for strong dominant paths, and semiblind algorithms consitently performed better than training-only equalisers. The approach proposed in [8] has a second step for a search of the optimum weighting and length, only useful if the determinded coefficents can be re-used for the duration of several packets.



(a) Equal power 2-tap Rayleigh.



(b) Dominant path, 2-tap Rayleigh.

Figure 5: Bit Error Rate results, for $N = 116, L_T = 26$ and a weight $\alpha = \frac{1}{4}$.

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