

# PERFORMANCE OF SINGLE-USER AND MULTI-USER CONSTANT-ENERGY BIT LOADING IN HIPERLAN CHANNELS

Ana García Armada <sup>(1)</sup> and John M. Cioffi <sup>(2)</sup>

<sup>(1)</sup> Department of Signal Theory and Communications, Carlos III University of Madrid, Spain

<sup>(2)</sup> Electrical Engineering Department, Stanford University, Stanford, CA 94305, U.S.A.

## ABSTRACT

Several WLAN standards, such as Hiperlan-2, have chosen OFDM as a transmission technique due to its multipath performance. However, in these standards spectrum is not optimized. This paper analyzes the performance of single-user and multi-user constant-energy bit loading in Hiperlan channels. We show that bit rates may be increased up to 12 times in the single-user case and a gain of more than 100% may be achieved relative to Hiperlan-2 in multi-user scenarios.

## 1. INTRODUCTION

Multicarrier modulation techniques have been standardized for Wireless Local Area Networks (WLAN) due to their good performance in multipath environments. In recent standards, such as Hiperlan type 2 and IEEE 802.11a, the number of bits modulating each sub-carrier can be chosen among several possibilities to adapt the transmission to the channel quality. However, the same number of bits is used for every sub-carrier, that is, the signal spectrum is not optimized.

The fact that the energy distribution to achieve capacity in a single-user spectrally-shaped Gaussian channel is found by waterfilling [1] has motivated several FDMA approaches that can be found in the literature for multi-user environments [2-4]. These approaches assume some kind of coordination among users, what is applicable to ADSL/VDSL systems, multi-service single-user scenarios or cellular downlink. However, WLAN transmitters are in general uncoordinated.

In [5] a practical and simple solution using Carrier Sense Multiple Access (CSMA) combined with Orthogonal Frequency Division Multiple Access (OFDMA) was introduced for uncoordinated environments and its performance was compared to optimum multi-user bit loading and conventional CSMA in the two-user case and frequency selective fading channels. In this paper the performance of the algorithm of [5] is analyzed in Hiperlan channels. The channel models

developed within the ETSI Project BRAN [6] are used as a basis for comparison of different modulation schemes proposed for Hiperlan-2.

## 2. HIPERLAN CHANNEL MODELS

Five channel models have been developed within the ETSI Project BRAN [6]. Their impulse responses have uniformly distributed taps with exponential power-delay profile (PDP). For this study, those channels developed for office non-line-of-sight (NLOS) and open space NLOS environments have been selected, what we will call A and B respectively throughout the paper. Channel A has an rms delay spread of 50 nsec and a maximum delay of 390 nsec; these parameters for channel B are 250 nsec and 1760 nsec respectively.

In both cases, the channel impulse response is:

$$h(t) = \sum_{k=1}^K a_k \delta(t - \tau_k) \quad (1)$$

where the  $K$  channel taps  $a_k$  are statistically independent complex Gaussian random variables with zero mean and power given by PDP:

$$p(\tau_k) = \frac{1}{2\pi\sigma_\tau} e^{-\tau_k / \sigma_\tau} \quad (2)$$

We are interested in the characteristics of the channel transfer function  $H(f)$  in order to derive the power allocation for each sub-channel. Since  $H(f)$  is the Fourier Transform of  $h(t)$ , which is a sum of Gaussian variables, it is Gaussian and  $|H(f)|$  has a Rayleigh pdf.

## 3. SINGLE-USER WATERFILLING IN HIPERLAN CHANNELS

In this section single-user waterfilling and constant-energy bit loading are reviewed and the achievable rates in Hiperlan channels are analyzed. Since transmission rates depend on the signal-to-noise ratio, the required transmitted power to achieve a given bit rate with a given outage probability when using fixed modulation will be

considered as a way of comparing these systems under the same circumstances.

### 3.1. Single-user waterfilling

With the insertion of a sufficiently long cyclic prefix in the OFDM signal, the number of bits that can be carried by each of the  $N$  AWGN sub-channels with noise variance  $\sigma^2$  and frequency response  $H_n$  is given by:

$$R_n = \log_2 \left( 1 + \frac{SNR_n}{\Gamma_n} \right) = \log_2 \left( 1 + \frac{E_n \cdot G_n}{\Gamma_n} \right) \quad (3)$$

$E_n$  is the energy transmitted over  $n$ -th sub-channel and  $G_n$  is the channel-to-noise ratio given by:

$$G_n = \frac{|H_n|^2}{\sigma^2} \quad (4)$$

The SNR gap  $\Gamma_n$  is used to measure the reduction of SNR with respect to capacity. It depends on the objective bit error rate. If the objective BER is the same for all sub-channels, then  $\Gamma_n = \Gamma$ .

The optimum distribution of  $E_n$  that achieves maximum rate transmission under a maximum energy constraint ( $E_{TOT}$ ) is obtained by waterfilling [1]:

$$E_n = \left[ K - G_n^{-1} \right]^+ \quad (5)$$

where a generalized  $G_n^* = G_n / \Gamma_n$  has been introduced and

$$[x]^+ = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (6)$$

The value of  $K$  (water level) is chosen so that the total energy constraint  $E_{TOT}$  is satisfied.

Given that frequency bands dedicated to WLAN have strict power spectrum density constraints, the algorithm of [5] uses a constant-energy approach. In this case, the energy distribution is:

$$E_n = \frac{E_{TOT}}{N} \quad (7)$$

and different numbers of bits  $R_n$  are allocated to each sub-channel following (3) so as to maximize the total bit rate:

$$R = \sum_{n=1}^N R_n \quad (8)$$

### 3.2. Achievable rates in Hiperlan channels

Let us examine to what extent may transmission rate be increased in Hiperlan channels with these two approaches. Since transmission rate depends on signal-to-noise ratio (SNR), we will assume that all three systems have the same noise power spectrum density and that they use the

same total energy in order to perform a fair comparison. With this total energy, the bit rates achievable with waterfilling and constant-energy distributions will be obtained and compared to Hiperlan-2.

In Hiperlan-2 every sub-channel carries the same number of QAM-modulated bits.  $R_0$  will be used to denote bit rate when fixed modulation is used, implying that  $R_n$  has the same value  $R_0$  for all  $n$ . Similarly, sub-index '0' will be used for any parameter that is set equal for all sub-channels throughout the paper.

The probability that achievable bit rate  $R_n$  does not equal nor exceed  $R_0$  in a given sub-channel  $n$  is obtained from (3) and (4) as:

$$p_n = \text{Prob}\{R_n < R_0\} = \text{Prob}\{|H_n| < C_n\} \quad (9)$$

$$C_n = \sqrt{\left(2^{R_0} - 1\right) \frac{\sigma^2 \cdot \Gamma}{E_n}} \quad (10)$$

This is often called the outage probability. Since  $|H_n|$  is Rayleigh distributed, it follows that:

$$p_n = 1 - \exp\left(-\frac{C_n^2}{2\sigma_H^2}\right) \quad (11)$$

With  $\sigma_H^2$  equal to the variance of real and imaginary parts of  $H_n$ .

If we impose that the probability of not achieving  $R_0$  be equal to  $p_0$  in all sub-channels, the same necessary energy is obtained in every sub-channel  $E_0$  for a given noise:

$$E_0 = \frac{1}{C_0^2} (2^{R_0} - 1) \cdot \sigma^2 \cdot \Gamma \quad (12)$$

with  $C_0$  satisfying (11). This means that, knowing  $R_0$ , the amount of energy  $E_0$  that is required in each sub-channel can be found following (12) so that they all have a given outage probability  $p_0$ .

In order for the fixed modulation to work properly  $p_0$  should be small. Since the  $N$  sub-channels are not independent (coherence bandwidth indicates how many sub-channels are correlated to some extent), the probability that  $R_0$  will not be achieved in one or more given sub-channels out of  $N$  (causing the OFDM symbol to be in error) will approximately satisfy:

$$p_0 \leq p \leq N \cdot p_0 \quad (13)$$

Which one of the lower of upper bound is closer to  $p$  depends on the channel. However, for  $N=64$  both bounds are close enough and  $p_0$  gives an idea of the outage probability. Moreover, either  $p_0$  or  $p$  may be used as a reference to compare different systems with the same channel conditions.

Once that  $E_0$  is known for all subchannels, the total energy that the fixed modulation scheme is using is obtained following (7):

$$E_{TOT} = N \cdot E_0 \quad (14)$$

With this total energy either waterfilling or constant-energy bit loading can be performed and if we call  $R_w, R_c$  the respectively achievable bit rates of (8), the ratio of achievable rates with respect to fixed  $R_0$  modulation is:

$$\eta_w = \frac{R_w}{R_0}, \quad \eta_c = \frac{R_c}{R_0} \quad (15)$$

Mean ratios averaged over 1000 channel realizations are represented in Fig. 1 for both channels A and B when fixed modulation uses  $R_0=2$  and  $R_0=3$  bits. The probability of not achieving the required bit rate in Hiperlan ( $p_0$ ) is represented in x-axis.

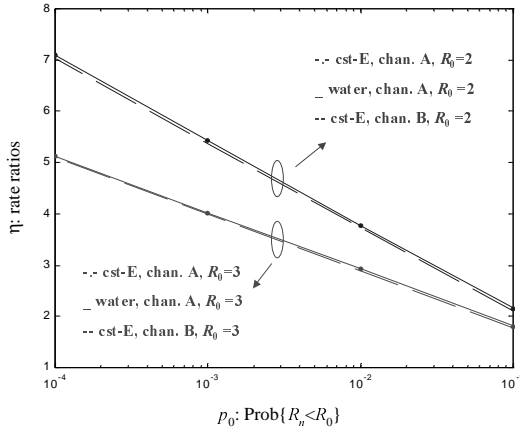


Fig. 1. Ratios  $\eta_w$  and  $\eta_c$  versus  $p_0$

It can be seen in Fig. 1 that constant-energy bit loading has negligible loss with respect to waterfilling. Higher ratios can be achieved when the outage probability is lower.

It is important to note that these ratios do not depend on required SNR, but just on  $R_0$ . However, the actual value of SNR is related to  $R_0$  and error probability requirements.

Mean ratios do not depend on the channel model either, as long as  $|H_n|$  are Rayleigh distributed. Fig. 2 shows mean ratios (averaged over 1000 channel realizations) for channels A and B. It can be seen that ratios of up to 12 may be achieved when fixed modulation uses 1 bit per symbol in every subchannel.

However, standard deviations are different for channels A and B. Since channel B has a lower coherence bandwidth, many different values of  $|H_n|$  can be expected to be found in each realization and the variance of ratios  $\eta_c$  should be lower. Simulations show that standard deviations in channel B are almost half the values of those in channel A. Anyway, standard deviations are small in both cases, so we can always expect an actual value of any realization to be very close to the mean.

It may be useful to illustrate these results with an example. The required  $E_b/N_0$  to transmit an 8-PSK modulation in channel A is shown in [7] to be 16.2 dB when convolutional encoding of rate 0.6 is used. It can be

found by simulation with channel A that  $R_0=3$  is achieved with an outage probability  $p_0=10^{-1}$  and in that case  $\text{SNR}_0=16.6$  dB giving a  $\text{BER}=10^{-5}$ . Since 512 bits per packet are used in [7], that gives a  $\text{PER}=0.5\%$ , that is less than 1% as required. In this case, a bit rate of 23.1 Mbps is transmitted, but looking at Fig. 2 we can see that the rate may be almost doubled (41.81 Mbps).

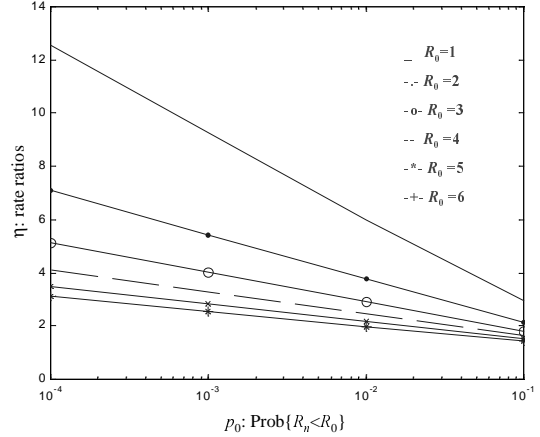


Fig. 2. Mean ratios  $\eta_c$  for channels A and B

However, a 10% probability that 8-level transmission in any sub-channel may fail is too lax. If we require a value  $p_0=10^{-2}$ , i.e. 99% sub-channels will achieve  $R_0$ , the rate may be increased three times and if we require a value  $p_0=10^{-3}$ , then it can be increased four times.

#### 4. PERFORMANCE OF CSMA-OFDMA IN HIPERLAN CHANNELS

Examination of Fig. 2 shows that Hiperlan-2 bit rates may be increased up to 12 times in some situations for the single-use case, so we are motivated for the study of the performance of a multi-user algorithm in such environments.

In this section the performance of the multi-user constant-energy CSMA-OFDMA bit loading algorithm will be examined. Its description may be found in [5]. Basically, any user that wishes to transmit senses the channel and performs a constant-energy algorithm in free sub-channels so as to achieve the required bit rate with the minimum number of sub-channels, so that they are free to be used by other terminals.

Since results vary depending on how much bit rate each user demands, random user rates uniformly distributed in the interval  $(0,R)$  have been generated to evaluate the probability that each user achieves the rate that is desired ( $P_H$  and  $P_C$  with Hiperlan-2 and constant-energy algorithm respectively). The gain of CSMA-OFDMA method relative to Hiperlan-2 for each user is defined as:

$$\text{gain}(\%) = \frac{P_C - P_H}{P_H} \cdot 100 \quad (16)$$

It is interesting to define  $R$  in relation to  $R_0$ , that is, the rate of fixed modulation in Hiperlan-2 (section 3). It is assumed that the total bit rate is divided between the users following a TDMA scheme in Hiperlan-2, so that, if  $L$  users are sharing the communication channel, the bit rate for each of them is  $NR_0/L$ . The ratio between the mean bit rate required by users and the achievable bit rate per user in Hiperlan-2 is defined as:

$$\rho_L = \frac{R \cdot L}{2 \cdot N \cdot R_0} \quad (17)$$

Since constant-energy bit loading achieves greater rates than Hiperlan-2 in the single-user case, we can expect some gain in a multi-user environment too. Depending on what probability  $p_0$  is required, the obtained gain will vary, as in the single-use case..

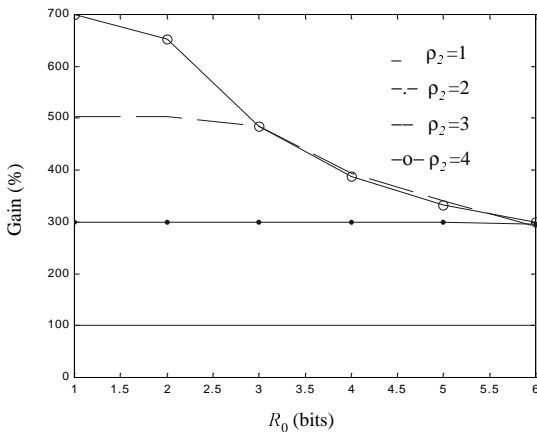


Fig. 3. Gain (%) for user 1 with  $p_0=10^{-2}$

Gains (%) obtained for user 1 when  $p_0=10^{-2}$  in a two-user scenario are represented in Fig. 3. User 1 is the first user to arrive to use the channel, so it will have every sub-channel available. User 2 arrives later and is allowed to transmit only on those channels not occupied by user 1. Gains for user 2 are shown in Fig. 4.

We can see in Figs. 3 and 4 that the gain increases when the bit rate required by users increases until a point is reached in what there is not enough bandwidth to provide the required bit rates. Then, the gain decreases, but it is always greater than zero. However, in cases with small gain, the probability that any user achieves the required rate is very low, so it would be preferable to avoid such situations.

It can be seen too that gains of 100% are always obtained for both users when they require a mean bit rate two times the achievable bit rate in Hiperlan-2, independently of the number of bits per Hiperlan-2 sub-channel. Besides, very high gains can be obtained in some circumstances.

The gains follow a similar behavior for different values of the probability  $p_0$ . When  $p_0$  is smaller, higher gains may be achieved, as in the single-use case.

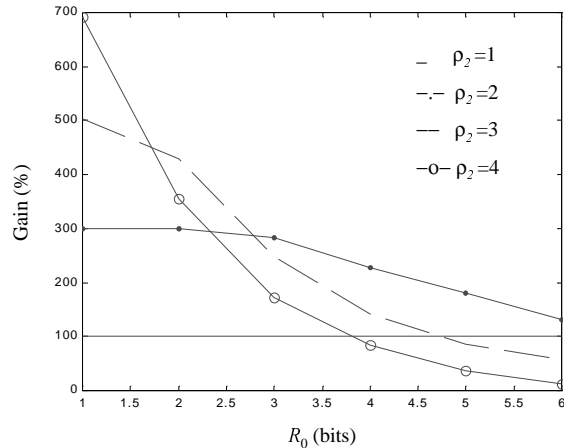


Fig. 4. Gain (%) for user 2 with  $p_0=10^{-2}$

## 5. CONCLUSIONS

The performance of single-user and multi-user constant-energy bit loading has been analyzed in Hiperlan channels. It has been shown that constant-energy has a negligible degradation with respect to optimum waterfilling and that bit rates can be increased up to 12 times when using these approaches instead of the fixed modulation used in Hiperlan-2. In multi-user scenarios, a simple algorithm such as CSMA-OFDMA can allow us to obtain a 100% gain in performance (and even higher in many cases).

## REFERENCES

- [1] R. G. Gallager, *Information Theory and Reliable Communication*, Wiley, New York, NY, 1968.
- [2] L. M. C. Hoo, J. Tellado, J. M. Cioffi, "Dual QoS Loading Algorithms for Multicarrier Systems Offering Different CBR Services", *Proceedings PIMRC*, vol. 1, pp. 278-282, September 1998.
- [3] W. Rhee, J. M. Cioffi, "Increase in Capacity of Multiuser OFDM System Using Dynamic Subchannel Allocation", *Proceedings VTC*, vol. 2, pp. 1085-1089, May 2000.
- [4] C. Y. Wong, C. Y. Tsui, R. S. Cheng, K. B. Letaief, "A Real-time Sub-carrier Allocation Scheme for Multiple Access Downlink OFDM Transmission", *Proceedings VTC*, vol. 2, pp. 1124-1128, September 1999.
- [5] A. G. Armada, "A Simple Multiuser Bit Loading Algorithm for Multicarrier WLAN", *Proceedings ICC*, June 2001.
- [6] J. Medbo, P. Schramm, "Channel Models for HIPERLAN 2", *ETSI/BRAN* document no. 3ERI085B, 1998.
- [7] U. Dettmar et al., "Modulation for Hiperlan Type 2", *Proceedings VTC*, vol. 2, pp. 1094-1100, September 1999.