JOINT ZERO-FORCING AND MATCHED FILTER PROCESSING IN AN ADAPTIVE EQUALIZER USING THE LINEARLY-CONSTRAINED LEAST-MEAN SQUARE (LC-LMS) ALGRITHM[†]

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ABSTRACT

Digital receivers often contain a cascade of two filters in the base-band signal-conditioning path. The first is a square-root raised-cosine Nyquist matched filter designed to maximize SNR and suppress out-of-band interference, while the other is a decision-directed (T/2)-spaced equalizer which removes the spectral distortion introduced by the channel. This paper presents an equalizer modified to permit a single filter to converge simultaneously to a (T/2)-spaced zero forcing equalizer and a matched filter.

I. INTRODUCTION

Adaptive equalizers operate in a digital receiver to minimize intersymbol interference (ISI) due to channel-induced distortion of the received signal. The equalizer operates in cascade with a matched filter (MF), synchronous sampler, and decision device (slicer) operating at symbol rate f_{sym}. A gradient descent process such as the LMS algorithm adjusts the equalizer weights to minimize the difference between the input and output of the decision device. In the signal-conditioning path analog and digital filters limit the noise bandwidth to a two-sided bandwidth of $2f_{sym}$. The noise-burdened signal is sampled at $2f_{sym}$ in order to satisfy the Nyquist criterion for the subsequent digital processing. The remaining two filtering tasks are traditionally performed by a digital filter operating at $2f_{sym}$ (with or without down-sampling to f_{sym}) to minimize the effects of receiver noise, and by an equalizer to minimize the effects of channel distortion. In modern receivers the sampling process precedes the MF, and in order to satisfy the Nyquist criterion for the MF the sample rate is greater than the symbol rate by a ratio of small integers p-to-q such as 3-to-2 or 4-to-3. This ratio is often selected to be 2-to-1 to simplify the subsequent task of down-sampling prior to the slicer. If the down-sampling occurs prior to the equalizer, the equalizer operates at 1-sample-per-symbol and it is termed a symbol-spaced equalizer (SSE), and if the down sampling occurs after the equalizer, the equalizer operates on (p/q)-samples-per-symbol and it is termed a fractional-spaced equalizer (FSE).

We may be tempted to either replace the cascade of the two digital filters, the MF and the equalizer, with a single filter that performs both tasks or to bypass the MF all together and plan for the equalizer to perform both tasks, noise suppression and channel inversion. Applying the (T/2)-spaced adaptive equalizer, controlled by (T)-spaced decisions, results in full band equalization but in only a partial band match to the MF. When operating in this manner, there is no suppression of out-of-band noise; consequently, the single filter exhibits a 3dB noise penalty relative to the cascade of the two filters. Thus, cascading a MF with an equalizer filter is the standard architecture of most receivers.

In this paper we develop and demonstrate a technique that uses constrained optimization to purchase back the noise penalty. In doing so the zero-forcing (ZF) equalizer is modified such that its out-of-band frequency response mimics that of the MF. This method enables the single adaptive filter to converge to the composite MF and inverse channel and thus exhibit the same performance as the traditional cascade two-filter solution. With the performance preserved, the single filter solution permits a single bank of FPGA multipliers to service the demands of both the ZF and MF processing via a background time-multiplexing scheme. As a result, the pre-equalizer MF can be eliminated from the standard demodulator architecture defining the primary advantage of the joint ZF and MF adaptive digital equalizer as conservation of FPGA real estate.

II. BACKGROUND

Shown in Figure 1 is the standard model of a simplified communications link consisting of a modulator with a spectral shaping filter, a distorting channel introducing noise, and a demodulator with a MF and fractional-spaced adaptive equalizer. Random binary 2-tuples are presented to the base-band modulator at the symbol rate, f_{sym}, to form a QPSK constellation. These constellation points are

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[†]**United States Patent #60/293,423 –** *"A Joint Zero-Forcing and Matched Filter Adaptive Digital Equalizer".*

shaped and further interpolated to form samples of the waveform presented to the up-converter. The pulse-shaping filter is a square-root raised-cosine (RRC) response operating at $2f_{sym}$ with 10% excess bandwidth. The filter length is 61 coefficients spanning 30 symbols.

For the purposes of this paper the effects of channel multi-path are excluded and so distortion occurs solely from AWGN whose source is the receiver's low-noise input amplifier. However, ISI is induced during spectral shaping if the shaping filter takes the form of an RRC response. In Figure 1 the MF is to be combined with the equalizer so that it also performs as a two-sample-per-symbol filter RRC pulse-shaping filter. The equalized data is formed from the inner product of the input data and the weights of the equalizer, and the weights are obtained by an LMS based gradient descent as shown in equations (1), (2), and (3).

$$y(n) = \underline{\boldsymbol{w}}^{\boldsymbol{H}}(n)\underline{\boldsymbol{u}}(n) \quad (1)$$

$$e(n) = \tilde{d}(n) - y(n) \quad (2)$$

$$\underline{\boldsymbol{w}}(n+1) = \underline{\boldsymbol{w}}(n) + \mu e^{*}(n)\underline{\boldsymbol{u}}(n) \quad (3)$$

III. THE LC-LMS ALGORITHM

In order for the equalizer to replace the RRC MF its spectral side-lobes must suppress the out-of-band interference as would the MF we are trying to replace. We can achieve this desired effect by modifying the LMS algorithm to satisfy a constraint – that of rejecting out-of-band interference. The manipulation of the weights of an LMS update subject to a constraint is the basis for the linearly constrained LMS algorithm (LC-LMSA). We term a FSE that utilizes the LC-LMSA to perform joint ZF and MF processing a linearly constrained least-mean-square equalizer (LC-LMS-E). The LC-LMS-E performs minimization of the decision error in the same manner as the unconstrained approach. From (1) and (2),

$$e(n) = \tilde{d}(n) - \underline{\boldsymbol{w}}^{\boldsymbol{H}}(n)\underline{\boldsymbol{u}}(n) \quad (4)$$

Equation (4) satisfies the first and primary constraint of the LC-LMS-E, that of ISI cancellation. This is achieved by controlling the pass band and roll-off of the response of the equalizer. What remains is the proposed control of the equalizer's spectral side-lobe response to achieve levels comparable to those of the MF. This is achieved by requiring the equalizer's impulse response $\underline{\boldsymbol{w}}$ to be uncorrelated with a high frequency signal $\underline{\boldsymbol{c}}$ located in the desired stop band.

To decorrelate $\underline{\boldsymbol{w}}$ from $\underline{\boldsymbol{c}}$ we manipulate the inner product[1] of the two sequences defined in (5).

$$\underline{\boldsymbol{w}}^{\boldsymbol{H}}\underline{\boldsymbol{c}} = \beta \qquad \{0 \le \beta < 1\} \quad (5)$$

[1] The inner product of two sequences is formed by the convolution of the two sequences evaluated at the midpoint of the response.

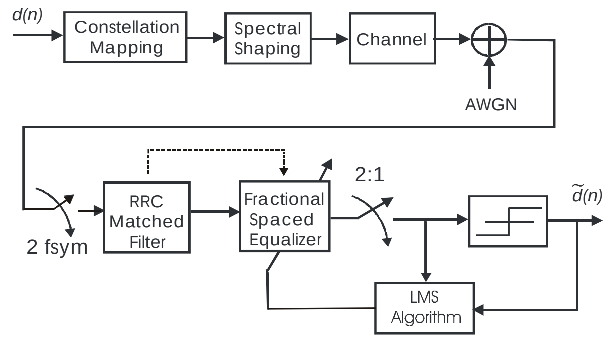

Figure 1. Standard Communication Model Showing Receiver Matched Filter and Fractional Spaced Equalizer to be Merged into a Single Combined Filter

Because scalar β is assigned a value less than unity the LC-LMSA forces $\underline{\boldsymbol{w}}$ from $\underline{\boldsymbol{c}}$ to become uncorrelated. If $\underline{\boldsymbol{c}}$ is defined to be samples of a complex sinusoid residing at an out-of-band frequency f_t/f_s a decorrelation of $\underline{\boldsymbol{w}}$ from $\underline{\boldsymbol{c}}$ would result in a spectral notch of the equalizer at f_t/f_s and would suppress a portion of the out-of-band noise contained within the bandwidth of that notch. Hence, vector $\underline{\boldsymbol{c}}$ is comprised of elements $c(k)$, where

$$c(k) = exp(j2\pi(f_t/f_s)\,k) \qquad \{0 \le k < L\} \quad (6)$$

Together, (5) and (6) constitute the constraint imposed on the equalizer weights. During the update of the equalizer weights the LC-LMSA modifies the weights such that the difference between $\underline{\boldsymbol{w}}^{\boldsymbol{H}}\underline{\boldsymbol{c}}$ and β is minimized. We therefore say that the inner product $\underline{\boldsymbol{w}}^{\boldsymbol{H}}\underline{\boldsymbol{c}}$ measures to what degree constraint sinusoid $\underline{\boldsymbol{c}}$ is uncorrelated with the equalizer's finite impulse response $\underline{\boldsymbol{w}}$ as compared to a desired target β. The minimization of (4) subject to the constraint (5)-(6) results in the joint minimization whose cost function is described using the Lagrange multiplier as indicated in (7) where λ is the Lagrange multiplier [3,4].

$$\xi^{c} = \mathrm{E}[e(n)e^{*}(n)] + \lambda(\underline{\boldsymbol{w}}^{\boldsymbol{H}}\underline{\boldsymbol{c}} - \beta) \quad (7)$$

Solving for $\underline{\boldsymbol{w}}_{\mathbf{opt}}$ in (7) we obtain the following update equations.

$$e(n) = \tilde{d}(n) - \underline{\boldsymbol{w}}^{\boldsymbol{H}}(n)\underline{\boldsymbol{u}}(n) \quad (8)$$

$$\underline{\boldsymbol{w}}^{+}(n) = \underline{\boldsymbol{w}}(n) + \mu e^{*}(n)\underline{\boldsymbol{u}}(n) \quad (9)$$

$$\varepsilon(n) = \beta - (\underline{\boldsymbol{c}})^{H}\underline{\boldsymbol{w}}^{+}(n) \quad (10)$$

$$\underline{\boldsymbol{w}}(n+1) = \underline{\boldsymbol{w}}^{+}(n) + (1/\underline{\boldsymbol{c}}^{H}\underline{\boldsymbol{c}})\varepsilon^{*}(n)\underline{\boldsymbol{c}} \quad (11)$$

The terms $\underline{\boldsymbol{w}}^{+}$ and $\underline{\boldsymbol{w}}$ denote the equalizer's state after the ZF and MF updates, respectively. In this, a constrained adaptation, the equalizer's weights are adjusted to not only invert the channel in the signal's pass band but to also be approximately orthogonal to the complex sinusoid of (6). At steady-state, this orthogonality is to be observed as a

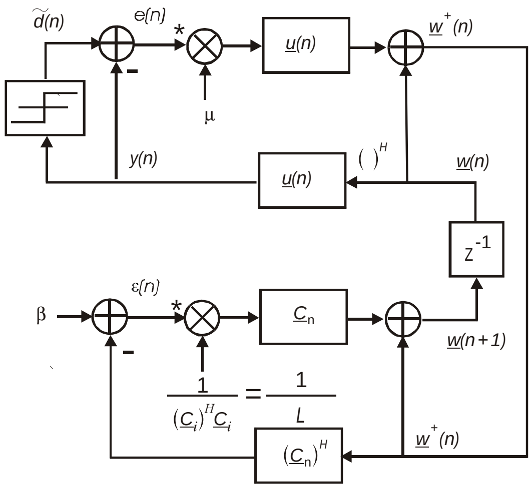

Figure 2. Block Diagram of LC-LMS-E in Terms of Vectored Updates

spectral null of the equalizer located at normalized frequency f_t/f_s in the out-of-band frequency response. The targeted depth for the spectral null is $20\log_{10}(\beta)$ and the null bandwidth is approximately the reciprocal of the equalizer length, or $1/L$. The difference between the desired strength of othogonality, β, and the actual measured othogonality, $(\underline{\boldsymbol{c}})^H\underline{\boldsymbol{w}}^+(n)$, is termed the constraint error $\varepsilon(n)$ as expressed in (10).

Intuitively, multiple constraints are required to uniformly suppress the out-of-band response comparable to that of the MF. Thus, the constraint vector $\underline{\boldsymbol{c}}$ must comprise a set of tones spanning the out-of-band frequency band. Simulations have shown that $\underline{\boldsymbol{c}}$ formed from the addition of numerous complex sinusoids of different frequencies is not sufficient to constrain the spectral mask of the equalizer to that comparable of a MF. A different approach involves cycling through N independent complex sinusoids on an iteration-by-iteration basis. If N represents the number of unique constraint frequencies to be used in the update then the i^{th} constraint sinusoid accessed by the LC-LMSA at iteration n is represented as in (12) for $k = 0,1,2, ..., L-1$,

$$c_i(k) = exp(j2\pi f_i\, k) \qquad (12)$$

where

$$i = (n-1)(mod)N + 1 \qquad (13)$$

Equations (10) and (11) are revised in (14) and (15) to account for multiple out-of-band CW signals.

$$\varepsilon(n) = \beta - (\underline{\boldsymbol{c}}_i)^H\underline{\boldsymbol{w}}^+(n) \qquad (14)$$

$$\underline{\boldsymbol{w}}(n+1) = \underline{\boldsymbol{w}}^+(n) + (1/\underline{\boldsymbol{c}}_i^H\underline{\boldsymbol{c}}_i)\varepsilon^*(n)\underline{\boldsymbol{c}}_i \qquad (15)$$

With $\underline{\boldsymbol{c}}_i$ representing samples of a complex sinusoid of unity amplitude the inner product $\underline{\boldsymbol{c}}_i^{\boldsymbol{H}}\underline{\boldsymbol{c}}_i$ is a scalar equal to the length of vector $\underline{\boldsymbol{c}}_i$, or L. Consequently, $(1/\underline{\boldsymbol{c}}_i^{\boldsymbol{H}}\underline{\boldsymbol{c}}_i) = 1/L$. The block diagram of Figure 2 implements update equa-tions (8), (9), (14), and (15) in a vectored format [5].

In order to demonstrate the LC-LMSA as applied to a joint ZF and MF update within the digital equalizer the LMS-based update in Figure 1 is enhanced using equations (8), (9), and (12)-(15). Simulation is performed using an adaptation constant $\mu = 0.0001$, constraint level $\beta = 0.001$, and 60 equidistantly spaced constraint sinusoids placed at out-of-band frequencies[2]. Figure 3a depicts the decision error for the linearly constrained adaptation and shows a slight increase in acquisition time vs. that of the unconstrained adaptation shown in Figure 3b. Remember that the unconstrained equalizer works in conjunction with the pre-equalizer MF. In illustrating the adaptation trend of the constraint error, Figure 4 makes inference to the equalizer's out-of-band spectral response levels achieved during the adaptation. The final attenuation level of –60dB matches the design goal.

Two conclusions can be drawn from the output SNR values achieved for the three configurations of the receiver tabulated in Table 1. First, the existence of a MF,

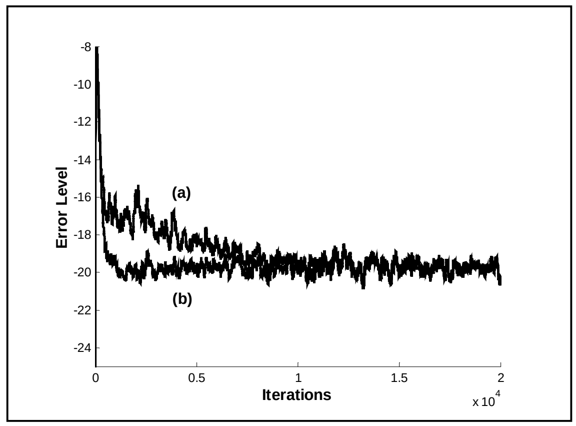

Figure 3. Decision MMSE for LC-LMS-E (a), LMS-E (b).

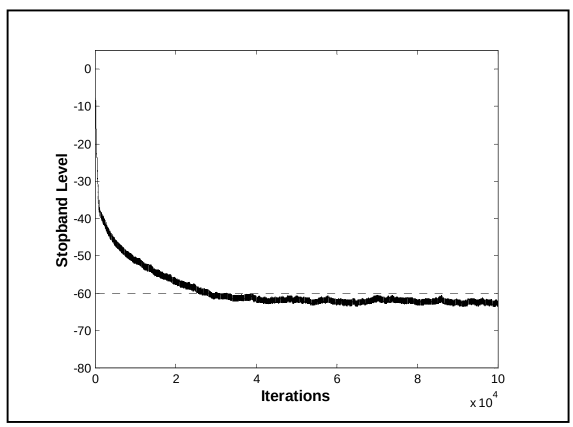

Figure 4. Constraint Error (Stop band Attenuation) for $\beta = 0.001$ (–60 dB).

[2] Although the selection of $\beta = 0$ surely optimizes performance, infinite side-lobe attenuation predicted by theory is not possible. A discussion of this case is left for future work [5].

SNR_{in} = 20 dB	SNR_{out} (dB)
LMS-E w/ MF	19.8836
LMS-E w/o MF	17.7746
LC-LMS-E	19.8742

Table 1. Comparison of SNR Values for Three Modes of Operation of the Receiver

either before or within the equalizer, emphasizes that an approximate 3dB improvement in SNR can be realized over the non-MF case[3]. Only a 2dB improvement is realized here since the side-lobes of the unconstrained FSE, Figure 5a, reject ~1dB of the out-of-band noise. Secondly, the LC-LMS-E restores full SNR if the case of the LMS-E plus pre-equalizer MF cascade design is considered to be the performance target. This is seen in Figure 5b where the spectral characteristics of the converged LC-LMS-E match those of the RRC MF.

As previously mentioned the primary advantage in forming a joint ZF and MF process within an adaptive equalizer using the LC-LMSA is the conservation of hardware associated with the pre-equalizer RRC MF. This can be seen if we observe that the FPGA multipliers responsible for performing the inner product computation in (8) can be shared to also perform the computations of the additional inner product in (14). The sharing of FPGA multipliers is a time-share process where the multipliers servicing ZF process are background time-multiplexed to also service the demands of the MF constraint. Since we have shown that the LC-LMS-E maintains SNR integrity this joint process solution offers an alternative to the standard dual filter cascade design, and therefore, enables the RRC MF preceding the FSE in the demodulator architecture to be removed.

Because the equations responsible for MF processing, (14) and (15), are mirror images of those which perform the ZF, (8) and (9), the workload required to operate the LC-LMS-E is approximately twice that of the unconstrained LMS-E, an acceptable increase if both the MF and FSE are of comparable lengths. However, when the length of the FSE becomes far greater than that of the RRC MF the benefit in conserving the MF's FPGA real estate is diminished, as a large number of computations are required to drive the LC-LMS-E. This, unfortunately, is the inherit trade-off between hardware conservation and increased computational complexity normally encountered when joint process algorithms are developed to improve system efficiency. A solution to this dilemma has been successfully developed [6] and involves performing a time-domain digital windowing of each complex constraint sinusoid with the intent to weight the samples farthest from the midpoint of the total duty cycle with negligible importance so that they need not contribute to the inner product in (14).[†]

[3] The total integrated noise power beyond the –3 dB frequency is –3 dBw.

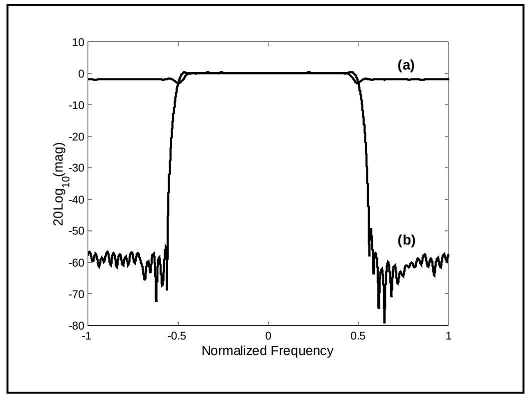

Figure 5. Spectrum of the LMS-E (a), LC-LMS-E (b).

CONCLUSIONS AND FUTURE RESEARCH

In this paper a novel approach to the control of the out-of-band spectral response of a fractional-spaced digital equalizer has been presented. A second paper, to follow this one, will expand on the operation and performance of the structure presented here [5]. It will discuss the selection criteria for the number and location of LC-LMSA constraint frequencies. It will also address LC-LMS-E performance in the presence of channel multi-path and CW tonal interference.

Future papers will also present current results obtained when the constrained update is applied to other adaptive inverse channel modelers such as the decision-feedback equalizer and the blind equalizer operating under the direction of the constant modulus algorithm (CMA). Attempts will also be made to generate the MF with the FSE via initialization of the equalizer with the RRC filter taps.[†]

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[6] F. Bologna and f. harris, "Workload Reduction of the LC-LMS Joint Zero-Forcing and Matched Filter Adaptive Equalizer via Time-domain Windowing of the Complex Constraint Sinusoids", Paper Submitted to *IEEE Trans. on Signal Processing*.

[†] Constraint windowing, decision-feedback, blind CMA, and initialization solutions are all encompassed under **United States Patent #60/293,423 --** "*A Joint Zero-Forcing and Matched Filter Adaptive Digital Equalizer*".