A NEW WAY TO INVERT THE SLIDING FOURIER TRANSFORM AND ITS APPLICATION TO SIGNAL SEPARATION

Adriana Dapena (1), Christine Serviere (2)

(1) Departamento de Electrónica e Sistemas, Universidade da Coruña
Campus de Elviña s/n, 15.071. A Coruña. SPAIN
Tel: ++34-981-167000, Fax: ++34-981-167160, E-mail: adriana@udc.es
(2) LIS-ENSIEG
BP 46
38402 Saint Martin d'Heres cedex, France
Tel. ++33-476-826-411, E-mail: christine.serviere@lis.inpg.fr

ABSTRACT

In this paper we propose a simple method to invert the Fourier transform computed using zero-padding and sliding windows with delay of one point. Supposing that the window size is less than the number of frequencies, we show that the time-domain signal can be recovered multiplying the frequency-domain signal by a 2×2 matrix. Using this result, we propose a criterion to separate convolutive mixtures of signals in the frequency domain. The proposed approach has a reduced computational cost because only two frequency bins are considered.
Keywords: Frequency-domain approaches, signal separation, convolutive mixtures, Fourier transform.

1. INTRODUCTION

It is well known that the properties of the Fourier transform allow to solve many signal processing problems where time-domain approaches fail. For instance, many authors have recently proposed methods to separate convolutive mixtures of statistically independent signals working in the frequency domain [1, 2, 5, 6, 9, 10, 11]. The idea is to convert the convolutive mixture in several instantaneous mixtures by computing the Fourier transform of the measures taken by several sensors (observations). Subsequently, each individual problem is solved using algorithms proposed to separate instantaneous mixtures (see [3] and references therein) and, finally, the time-domain signals are recovered using the Fourier transform inverse. These frequency-domain approaches are attractive solutions to many problems where temporal approaches fail like, for example, when the mixing system involves non-minimum phase transfer functions [8]. Unfortunately, the practical implementation of the frequency-domain solutions is limited because of the large number of frequencies that must be considered in order to achieve a good performance.

In this paper we introduce a new method to invert the Fourier transform using only two frequency bins. We show that the time-domain signal and the frequency-domain signal are related by a 2×2 matrix when the Fourier transform is computed using zero-padding and sliding windows with delay of one point. As a consequence, the time-domain signal can be recovered multiplying the frequency-domain signal by the inverse of this matrix. Using this result, we propose a method to separate convolutive mixtures with a reduced computational cost because only two frequency bins are considered.

This paper is structured as follows. In Section 2, we present the new method to invert the sliding Fourier transform. This result is used in Section 3 to solve the signal separation problem. In Section 4 we test the performance of the proposed separating system in an industrial application where several motors must be monitored [6, 7]. Finally, Section 5 is devoted to the conclusions.

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2. SLIDING FOURIER TRANSFORM INVERSION

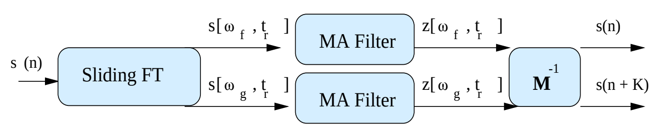

Fig. 1. Scheme to invert the sliding Fourier transform

In this section we propose a method to invert the sliding Fourier transform of a signal $s(t)$. The sliding Fourier transform is obtained by applying the Discrete Fourier Transform (DFT) to moving windows of the signal. Towards this aim, we split $s(t)$ in overlapped windows of K points, i.e., $s(t_r) = [s(t_r), s(t_r + 1), ..., s(t_r + K - 1)]^T$, $t_r = 0, 1, ...$ Subsequently, we compute the L-points DFT ($L > K$, with zero-padding of $L - K$ points) given by

$$s[\omega_k, t_r] = \sum_{m=0}^{K-1} s(t_r + m) e^{-j\omega_k m} \qquad (1)$$

where $\omega_k = 2\pi k/L$ denotes the frequency bin.

It is important to note that this transform introduces redundancy into the frequency-domain signal. For instance, note that $s[\omega_f, t_r]$ and $s[\omega_f, t_r + 1]$ differ in two samples, $s(t_r)$ and $s(t_r + K)$. This redundancy is exploited by the system shown

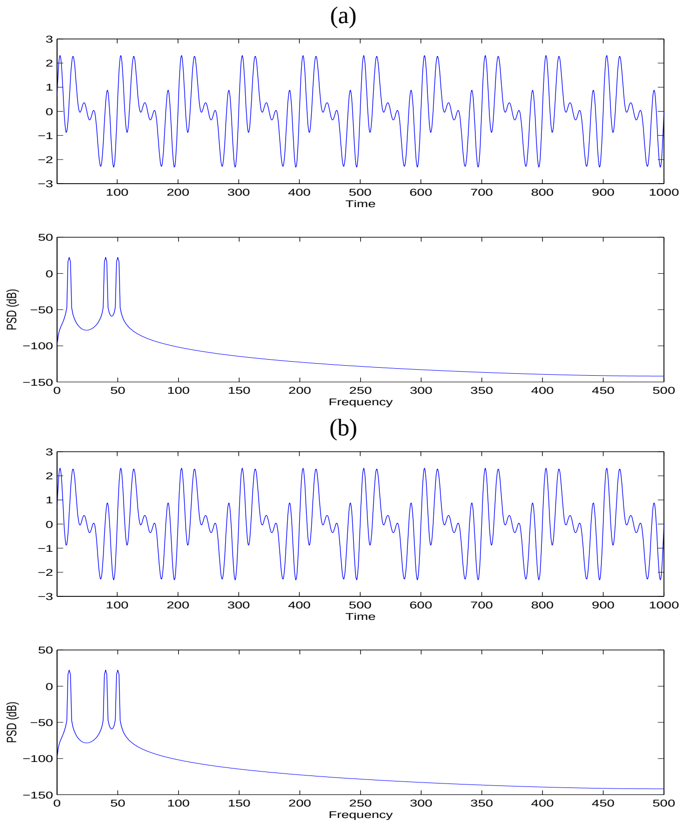

Fig. 2. Example: original signal and its Power Spectral Density; (b) recovered signal and its Power Spectral Density

in Figure 1 to recover the time-domain signal using only two frequency bins, ω_f and ω_g. First, we filter the $s[\omega_f, t_r]$ and $s[\omega_g, t_r]$ versus the index t_r using

$$\begin{aligned} z[\omega_f, t_r] &= s[\omega_f, t_r] - s[\omega_f, t_r+1]e^{-j2\pi\frac{f}{L}} \\ z[\omega_g, t_r] &= s[\omega_g, t_r] - s[\omega_g, t_r+1]e^{-j2\pi\frac{g}{L}} \end{aligned} \quad (2)$$

Since the Fourier transform has been computed over sliding windows with delay of one point, the frequency-domain signals at t_r and at $t_r + 1$ are overlapped in K points and we can write

$$\begin{aligned} s[\omega_f, t_r+1] &= \sum_{m=0}^{K-1} s(t_r+m)e^{-j2\pi\frac{fm}{L}} \\ &= s[\omega_f, t_r] - s(t_r) + s(t_r+K)e^{-j2\pi\frac{f(K-1)}{L}} \\ s[\omega_g, t_r+1] &= \sum_{m=0}^{K-1} s(t_r+m)e^{-j2\pi\frac{gm}{L}} \\ &= s[\omega_g, t_r] - s(t_r) + s(t_r+K)e^{-j2\pi\frac{g(K-1)}{L}} \end{aligned} \quad (3)$$

Substituting (3) in (2), we see that the filtered outputs only depend on the time instant $s(t_r)$ and $s(t_r + K)$

$$\begin{aligned} z[\omega_f, t_r] &= s(t_r) - s(t_r+K)e^{-j2\pi\frac{fK}{L}} \\ z[\omega_g, t_r] &= s(t_r) - s(t_r+K)e^{-j2\pi\frac{gK}{L}} \end{aligned} \quad (4)$$

In a compact form, we can write

$$\mathbf{z}[\omega] = \mathbf{M}\mathbf{s}(t), \ \ i = 1, .., N \quad (5)$$

where $\mathbf{z}[\omega, t_r] = [z[\omega_f, t_r], z[\omega_g, t_r]]^T$, $\mathbf{s}(t) = [s(t_r), s(t_r + K)]^T$, and

$$\mathbf{M} = \begin{bmatrix} 1 & -e^{-j2\pi\frac{fK}{L}} \\ 1 & -e^{-j2\pi\frac{gK}{L}} \end{bmatrix} \quad (6)$$

Since we have computed the DFT with zero-padding of $L - K$ points ($L > K$), the condition $f \neq g + pL/K$ (where p is an integer number) guarantees that $\mathbf{M}$ is an invertible matrix and the signals can be recovered using

$$\mathbf{s}(t) = \mathbf{M}^{-1}\mathbf{z}[\omega, t_r] \quad (7)$$

Finally, the component corresponding to $s(t_r)$ is taken.

In order to illustrate the performance of the proposed method, Figure 2 (a) shows a time-domain signal and its Power Spectral Density (PSD). It can see that the important components appear at $10Hz$, $40Hz$ and $50Hz$. We have computed the sliding Fourier transform of $L = 256$ frequencies over windows of $K = 64$ samples, and we have selected the frequency bins $f = 10$ and $g = 11$. Figure 2 (b) shows the recovered signal and its PSD. It is apparent that the original signal have been perfectly recovered.

3. SIGNAL SEPARATION

Using the result presented above, we will propose a criterion to separate statistically independent signals in the frequency domain. The method has a reduced computational cost because only two frequency bins are used. First, we will introduce the classical model used in signal separation. Let $\mathbf{s}(n) = [s_1(n), ..., s_N(n)]^T$ be the vector of N signals whose exact probability density functions are unknown. We assume that the signals are real-valued, non-Gaussian distributed and statistically independent. The observation vector $\mathbf{x}(n) = [x_1(n), ..., x_N(n)]^T$ provides a convolutive combination of the N signals, i.e.,

$$\mathbf{x}(n) = \sum_{k=-\infty}^{\infty} \mathbf{A}(k)\mathbf{s}(n-k) \quad (8)$$

where $\mathbf{A}(k)$ is an unknown $N \times N$ matrix representing the mixing system. From the properties of the Fourier transform, we can assume that the observations at each frequency bin are instantaneous mixtures of the signals, i.e.,

$$\begin{aligned} \mathbf{x}[\omega_f, t_r] &= \mathbf{A}[\omega_f]\mathbf{s}[\omega_f, t_r] \\ \mathbf{x}[\omega_g, t_r] &= \mathbf{A}[\omega_g]\mathbf{s}[\omega_g, t_r] \end{aligned} \quad (9)$$

where the DFT has been applied over overlapped moving windows (with delay of one point) and using zero-padding of $L - K$ points. Here L is the number of frequencies and K is the window length. Note that in the frequency-domain, the observations have the following form

$$\begin{aligned} x_l[\omega_f, t_r] &= \sum_{m=0}^{K-1} x_l(t_r+m)e^{-j\omega_f m} \\ x_l[\omega_g, t_r] &= \sum_{m=0}^{K-1} x_l(t_r+m)e^{-j\omega_g m} \end{aligned} \quad (10)$$

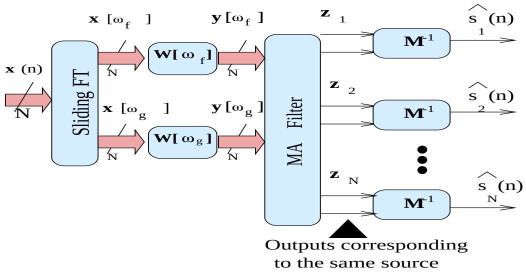

Fig. 3. Separating system

where $\omega_f = 2\pi f/L$ and $\omega_g = 2\pi g/L$ denote the selected frequencies.

In order to separate the signals we propose to use the system shown in Figure 3. In the frequency-domain, the instantaneous mixtures given in expression (9) are independently separated using systems with coefficients $\mathbf{W}[\omega_f]$ and $\mathbf{W}[\omega_g]$. The outputs of the separating systems are computed as follows

$$\mathbf{y}[\omega_f, t_r] = \mathbf{W}[\omega_f]\mathbf{x}[\omega_f, t_r]$$
$$\mathbf{y}[\omega_g, t_r] = \mathbf{W}[\omega_g]\mathbf{x}[\omega_g, t_r] \qquad (11)$$

The aim in signal separation is to obtain the separating matrices such as all the signals are recovered. For instance, if the mixing matrices $\mathbf{A}[\omega_f]$ and $\mathbf{A}[\omega_g]$ in (9) are invertible and known, we can recover the signals at each frequency using $\mathbf{W}[\omega_f] = \mathbf{A}^{-1}[\omega_f]$ and $\mathbf{W}[\omega_g] = \mathbf{A}^{-1}[\omega_g]$. In other case, when the mixing systems are unknown, the matrices $\mathbf{W}[\omega_f]$ and $\mathbf{W}[\omega_g]$ can be obtained using unsupervised algorithms proposed to separate instantaneous mixtures (see [3] and references therein).

It is important to note that when the signals are separated, each output has the following form

$$\begin{aligned} y_i[\omega_f, t_r] &= s_i[\omega_f, t_r] \\ &= \sum_{m=0}^{K-1} s_i(t_r+m)e^{-j2\pi\frac{fm}{L}} \\ y_i[\omega_g, t_r] &= s_i[\omega_g, t_r] \\ &= \sum_{m=0}^{K-1} s_i(t_r+m)e^{-j2\pi\frac{gm}{L}} \\ & \quad i = 1, ..., N \end{aligned} \qquad (12)$$

In order to obtain the time-domain signals, we filter $y_i[\omega_f, t_r]$ and $y_i[\omega_g, t_r]$ versus the index t_r using

$$\begin{aligned} z_i[\omega_f, t_r] &= y_i[\omega_f, t_r] - y_i[\omega_f, t_r+1]e^{-j2\pi\frac{f}{L}} \\ z_i[\omega_g, t_r] &= y_i[\omega_g, t_r] - y_i[\omega_g, t_r+1]e^{-j2\pi\frac{g}{L}} \\ & \quad i = 1, ..., N \end{aligned} \qquad (13)$$

Note that these filtered outputs have the same form like expression (2), i.e.,

$$\begin{aligned} z_i[\omega_f, t_r] &= s_i[\omega_f, t_r] - s_i[\omega_f, t_r+1]e^{-j2\pi\frac{f}{L}} \\ z_i[\omega_g, t_r] &= s_i[\omega_g, t_r] - s_i[\omega_g, t_r+1]e^{-j2\pi\frac{g}{L}} \\ & \quad i = 1, ..., N \end{aligned} \qquad (14)$$

As a consequence, we obtain

$$\begin{aligned} z_i[\omega_f, t_r] &= s_i(t_r) - s_i(t_r+K)e^{-j2\pi\frac{fK}{L}} \\ z_i[\omega_g, t_r] &= s_i(t_r) - s_i(t_r+K)e^{-j2\pi\frac{gK}{L}} \\ & \quad i = 1, ..., N \end{aligned} \qquad (15)$$

By the reasoning in Section 2, we find that the relationship between the vector containing the filtered outputs, $\mathbf{z}_i[\omega, t_r] = [z_i[\omega_f, t_r], z_i[\omega_g, t_r]]^T$, and the vector corresponding to the i-th signal, $\mathbf{s}_i(t) = [s_i(t_r), s_i(t_r+K)]^T$, is the following

$$\mathbf{z}_i[\omega, t_r] = \mathbf{M}\mathbf{s}_i(t), \;\; i = 1, .., N \qquad (16)$$

where $\mathbf{M}$ is the 2×2 matrix given in expression (6). Recall that the condition $f \neq g + pL/K$ (where p is an integer number) guarantees that $\mathbf{M}$ is an invertible matrix and the signals can be recovered using

$$\mathbf{s}_i(t) = \mathbf{M}^{-1}\mathbf{z}_i[\omega, t_r], \;\; i = 1, ..., N \qquad (17)$$

Finally, the component corresponding to $s_i(t_r)$ is taken.

4. ROTATING MACHINES MONITORING

There exists a great interest in applying signal separation methods for monitoring mechanical system [6, 7]. The idea is to recoverer the signatures of several motors without having to stop them. From the signatures, it is possible to obtain important information about the motors such as, for example, the existence of faults.

In our experiment, we have considered two DC motors whose signatures are showed in Figure 4 (a). The motor $\#1$ has a rotation speed of 48.5 Hz. This motor is fed by a single phase wiring (rectified) which presents 100 Hz for fundamental frequency plus harmonics. Motor $\#2$ turns at 31.5 Hz and is fed by two phase wiring which present 100 and 200 Hz. Each motor is fitted out with two single row roller bearings and drives a main shaft equipped with two self aligning roller bearings. Roller bearings induce several defect frequencies: motor $\#1$ presents a fault at 207 Hz and motor $\#2$ presents three faults at 134 Hz, 179 Hz and 210 Hz. Details of the test bench can be consulted in [6]. We have used $20,000$ samples of the temporal signals recorded to 2 kHz. The signals have been passed thought filters $a_{ij}(z) = (a_{ij} + z^{-1})/(1 + a_{ij}z^{-1})$ where the coefficients a_{ij} have been randomly generated.

In the separating system, we have used $K = 100$ and $L = 1024$ and we have selected $f = 12$ and $g = 13$ because the signals have similar powers in these frequency bins. In order to separate each instantaneous mixture, we have considered the theoretical solution (SFDA-TS) where the inverse of each true mixing matrix is used and a blind solution where the separating matrices are obtained using the JADE (Joint Approximate Diagonalization of Eigen-matrices) algorithm proposed in [4]. The performance has been measured in terms of the MSE (Mean Square Error) between the original and the recovered signatures. Table 1 shows the MSE (averaged over 10 independent realizations) obtained using SFDA-TS and SFDA-JADE. From these results we can conclude that both SFDA-TS and SFDA-JADE present a good behavior.

Figure 4 also shows the Power Spectral Density (PSD) of the observations (part (b)) and of the recovered signals (part (c)) for a simulation where the obtained MSE has been of -63.5646 dB for SFDA-TS and of -45.0820 dB for SFDA-JADE. Each PSD has been normalized by its maximum value. In Figure 4 (c), we can see that the rotating frequency plus harmonics of the two motors have been recovered. However, like in the original signature of the motor #2, only the rotating frequency and the first harmonic can be easily identified. The feeding frequencies (at 100 and 200 Hz) present in both signals have been also recovered. Concerning the bearing frequencies, the fault in 207 Hz is easily associated to motor #1 and the faults at 134 Hz and 179 Hz are associated to motor #2. It is difficult to associate the fault at 210 Hz but it also occurs in the original signatures.

5. CONCLUSION

We have proposed a method to invert the sliding Fourier transform. The basic idea is to find a 2×2 matrix which relates the time-domain signal and the frequency domain-signal. This matrix exists if the Fourier transform of L frequencies is computed over windows of K points with $K < L$. Using this result, we have proposed a method to separate statistically independent signals using only two frequency bins. In addition, we have presented simulation results that show the good performance of the proposed separating system in an industrial application where several motors are monitored.

	SFDA-TS	SFDA-JADE
Rotating machines	-67.1731 dB	-45.3598 dB

Table 1. MSE obtained in the simulations

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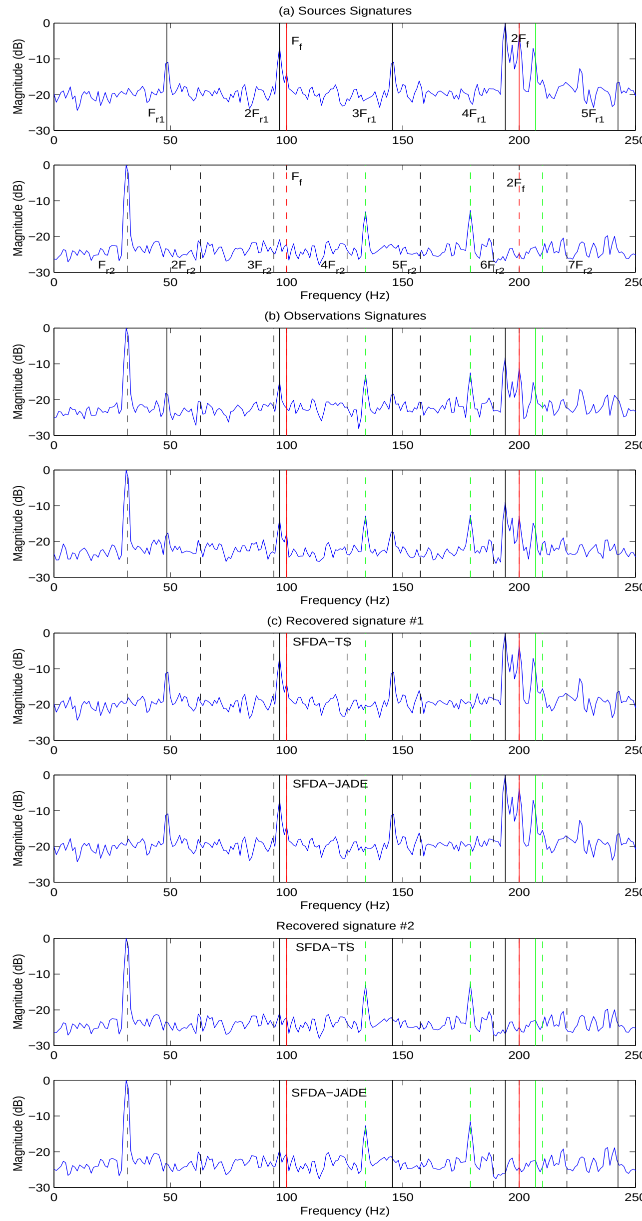

Fig. 4. Experimental results obtained for rotating machines monitoring: (a) PSD of the signals; (b) PSD of the observations; (c) PSD of the outputs obtained with SFDA-TS and SFDA-JADE.