# A NEW TWO-DIMENSIONAL FAST ADAPTIVE FILTER BASED ON THE CHANDRASEKHAR ALGORITHM

*Mounir Sayadi, Farhat Fnaiech, Ahmed Mathlouthi, Abdelkader Chaari and Mohamed Najim*(*)

CEREP, ESSTT, Av. Taha Hussein, 1008, Tunis, Tunisia,
Email : mounir.sayadi@ipeim.rnu.tn , Farhat.Fnaiech@esstt.rnu.tn

(*) Equipe Signal Image, UMR LAP, ENSEIRB, BP.99, 33402, Talence Cedex, France
Email : najim@tsi.u-bordeaux.fr

## ABSTRACT

In this paper, we present a new fast algorithm for two-dimensional (2-D) linear adaptive filtering using the fast Chandrasekhar equations. Using the analogy between the multichannel linear model and the 2-D one, we transform an image into multichannel sequence and we extend the fast Chandrasekhar adaptive multichannel filtering algorithm to the 2-D case i.e. image filtering. The performance of the new 2-D adaptive filter is tested by using this filter to estimate the coefficients of a 2-D Moving Average (2 -D MA) model of an unknown system. Furthermore, an application on adaptive noise cancellation of images is proposed throw a 2-D adaptive noise canceller based on the 2-D Chandrasekhar fast algorithm. Simulation results prove the superiority of the new 2-D Chandrasekhar filter comparing to similar approaches for image model identification.

## 1. INTRODUCTION

In the framework of image filtering, several adaptive algorithms used for the recursive estimation of the 2 -D model coefficients have been proposed in [3][9][10][11]. A 2-D Least Mean Square (2-D LMS) adaptive algorithm was proposed for the first time in [3] and applied to adaptive system identification and image enhancement. Furthermore, a 2-D lattice LMS adaptive algorithm was proposed in [9]- [11]. This algorithm was used in image restoration applications and has given a successful results in terms of signal to noise ratio improvement. A significant amount of research has been reported on developing fast adaptive algorithm for the 2-D filtering. In [8], a 2-D Fast Recursive Least Square (2-D FRLS) transversal algorithm is proposed by Sequira et al. Furthermore, a 2-D Fast Lattice RLS (2-D FLRLS) adaptive algorithm is proposed in [4]. It updates the filter coefficients in growing-order form with a linear computational complexity and uses the geometrical approaches of vector space and orthogonal projection to solve the 2-D prediction problem.

In the other hand, one of the approaches to decrease the computation cost of the adaptive filtering algorithms is the use of the Chandrasekhar factorization techniques. It is a fast alternative to the Kalman filter and can be efficiently applied if the state-space model is time-invariant [5][6]. The strength of this approach derives from the fact that it avoids the resolution of the standard Riccati equation. The derivation of fast adaptive algorithms based on Chandrasekhar fast equations using a state space model was presented in [1] and [2] for MA and ARMA linear filtering. It has been extended to the multichannel linear adaptive filtering in [6] and to the non linear filtering in [7].

In the present paper, we extend the use of the fast Chandrasekhar multichannel adaptive algorithm [6] to image filtering. By transforming a 2D signal to a multichannel sequence, and transforming a 2-D MA linear filter model to a multichannel one, we derive a new fast adaptive algorithm based on the Chandraskhar equations for 2-D linear filtering.

## 2. TRANSFORMATION OF THE 2-D MODEL TO A MULTICHANNEL MODEL

In the case of a quarter-plane support of order ($p$,$q$), (Figure 1), the output $y$ of a 2-D MA (or Finite Impulsional Response FIR) stationary linear filter is given by the following relationship:

$$y_{n,r}=\sum_{i=0}^{p-1}\sum_{j=0}^{q-1}a_{i,j}x_{n-i,r-j}+v_{n,r}\ . \tag{1}$$

$y_{n,r}$ represents the value of a pixel of the image $y$ at line $n$ and column $r$, and $a_{i,j}$ are the 2-D MA transversal filter coefficients. The sequences $\{x_{n,r}\}$ and $\{v_{n,r}\}$ are the 2-D random signal input and the additive noise, respectively.

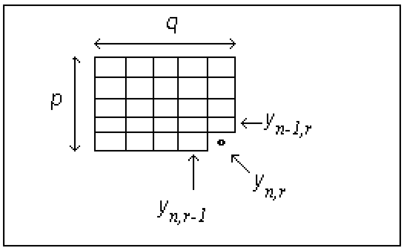


**Figure 1:** A Quarter-plane 2-D model

Let consider $y$ a square image of size ($L\times L$). To transform the 2-D signal $\{y_{n,r}\}$ into a multichannel one of $M$ channels, we propose to scan the rows of the 2 -D signal by a set $M$ mono-dimensional (1-D) sequences defined as follow:

$$\{u^{(1)}\}=[y_{1,1},y_{1,2},\ldots,y_{1,L},y_{2,1},y_{2,2},\ldots,\ldots y_{L-M+1,1},\ldots,y_{L-M+1,L},]$$

$$\{u^{(2)}\}=[y_{2,1},y_{2,2},\ldots,y_{2,L},y_{3,1},y_{3,2},\ldots,\ldots y_{L-M+2,1},\ldots,y_{L-M+2,L},]$$

...

$\{u^{(M)}\}=\lfloor y_{M,1},y_{M,2},\ldots,y_{M,L},y_{M+1,1},y_{M+1,2},\ldots,\ldots y_{L,1},\ldots,y_{L,L},\rfloor.$

Thus, each sequence $u^{(i)}$ can denote the input signal of the $i^{th}$ channel of a multichannel linear FIR filter.

Let now transform the 2-D linear filter given by equation (1) to the following multichannel filter [6]:

$$y_k=\sum_{i=1}^{M}\sum_{j=1}^{N_i} b_j^{(i)} u_{k-j}^{(i)}+v_k \qquad (2)$$

In this model, a linear scaning index $k$ is introduced as $k=r+L.(n-1)$. The total number of channels $M$ is chosen equal to $p$. The index $i$ is the channel order, $u^{(i)}$ denotes the input signal of the $i^{th}$ channel, and $N_i$ is the dimension of the transversal channels chosen equal to $q$. The coefficient vector for the $i^{th}$ channel is given by $\underline{b}^{(i)}=\left\lfloor b_{N_i}^{(i)} b_{N_i-1}^{(i)}\ldots . b_1^{(i)}\right\rfloor^t$ .

A multichannel filter coefficient $b_i^{(j)}$ have to estimate to the 2-D MA filter coefficients $a_{i,j}$ .

An equivalent state-space model of the multichannel filter can be easily written as: [2][6]

$$\begin{cases}\underline{\Theta}_k = D\underline{\Theta}_{k-1} \\ y_k = \underline{Z}^t\,\underline{\Theta}_k + n_k\end{cases}, \quad k=1,2,\ldots,m . \qquad (3)$$

Here, $\underline{Z}$ is an extended data vector of dimension $p = Mm + n - M$ , where $(n = N_1+\ldots+N_M)$ is the total number of coefficients and $m$ is the index corresponding to the last input samples available,

$\underline{Z} = \left[(\underline{u}^{(1)})^t\,(\underline{u}^{(2)})^t\,(\underline{u}^{(3)})^t \ldots\ldots (\underline{u}^{(M)})^t\right]^t$, with

$\underline{u}^{(i)} = \left[u_{-N_i+1}^{(i)}\ldots\ldots u_0^{(i)} u_1^{(i)}\ldots . u_{m-1}^{(i)}\right]^t$ .

The matrix $D$ is equal to
$$D=\begin{bmatrix} 0 & 0 & 0 & & & 0 \\ 1 & 0 & 0 & & & 0 \\ 0 & 1 & 0 & 0 & & \\ .. & .. & & ... & 0 & 0 \\ 0 & & & 1 & 0 & 0 \\ 0 & 0 & & 0 & 1 & 0 \end{bmatrix}, \qquad (4)$$

and $\underline{\Theta}_k$ is a parameter vector of augmented dimension $p$:

$$\underline{\Theta}_k = \left[0_{k-1}^t\,(\underline{b}^{(1)})^t\,0_{m-1}^t\,(\underline{b}^{(2)})^t\,0_{m-1}^t\ldots\ldots(\underline{b}^{(M)})^t\,0_{m-k}^t\right]^t, \qquad (5)$$

where $0_i$ is a zero vector of size $i$. The model (3) represents a time-invariant state-space system since the matrix $D$ and the vector $\underline{Z}$ are constant.

## 3. THE FAST CHANDRASEKHAR ALGORITHM

Taking into account the state-space model (3), the estimate of the parameter vector $\underline{\Theta}_k$ may be performed by the standard Kalman filter which yields:

$$\hat{\underline{\Theta}}_{k+1/k} = D\hat{\underline{\Theta}}_{k/k-1} + \underline{K}_k\,(R_k)^{-1}(y_k - \underline{Z}^t\,\hat{\underline{\Theta}}_{k/k-1}) \qquad k=1,2,\ldots,m$$

where the Kalman gain $\underline{K}_k$ and the innovations covariance $R_k$ are computed by :

$$R_k=\underline{Z}^t P_{k/k-1}\underline{Z}+s_v^2 \qquad (6)$$

$$\underline{K}_k = DP_{k/k-1}\underline{Z} \qquad (7)$$

$$P_{k+1/k} = DP_{k/k-1}D^t - \underline{K}_k\,(R_k)^{-1}\,\underline{K}_k^t . \qquad (8)$$

The Riccati difference equation may be obtained from equation (8) by replacing the Kalman gain and the innovation covariance by their respective expressions (6) and (7). In [1][2] and [6], the Chandrasekhar factorization technique is applied to decrease the computational complexity of the Kalman filter. The term $P_{k/k-1}$ is replaced by a factorized form of the covariance matrix increments $d\,P_k = P_{k/k-1} - P_{k-1/k-2}=(L_{k-1}M_{k-1}L_{k-1}^t)$.

Accordingly, a multichannel Chandrasekhar fast algorithm of reduced dimension were proposed in [6]. We give a summary of the steps of this algorithm in Table 1. The computational complexity of the reduced algorithm depends on $\alpha.n$ rather then $n^2$. ($\alpha$ is the rank of the increment $d\,P_2 = P_{2/1} - P_{1/0}$ ).

For more details about the derivation of the algorithm, the reader is referred to [6].

| **Equation** | **Dimension** |
|---|---|
| Transform the 2-D signal $x_{n,r}$ into a multichannel one: | |
| $\{u^{(1)}\}=\lfloor x_{1,1},\ldots,x_{1,L},x_{2,1},\ldots,\ldots x_{L-M+1,1},\ldots,x_{L-M+1,L},\rfloor$ | |
| $\{u^{(2)}\}=\lfloor x_{2,1},\ldots,x_{2,L},x_{3,1},\ldots x_{L-M+2,1},\ldots,x_{L-M+2,L},\rfloor$ | |
| …. | |
| $\{u^{(M)}\}=\lfloor x_{M,1},\ldots,x_{M,L},x_{M+1},\ldots,\ldots . x_{L,1},\ldots,x_{L,L},\rfloor$ | |
| $\underline{V}_k=\sum_{i=1}^{M}\left(S_k^{(i)}\right)^t\begin{bmatrix}\underline{U}_{k-1}^{(i)}\\ u_k^{(i)}\end{bmatrix}$ | $(\alpha\times1)$ |
| where $\underline{U}_{k-1}^{(i)} = \left[u_{k-N_i}^{(i)}\ldots\ldots\ldots u_{k-1}^{(i)}\right]^t$ . | |
| $\underline{W}_k = M_k\underline{V}_k$ | $(\alpha\times1)$ |
| $M_{k+1} = M_k + \underline{W}_k\,(R_k)^{-1}\underline{W}_k^t$ | $(\alpha\times\alpha)$ |
| $R_{k+1} = R_k + \underline{V}_k^t\,\underline{W}_k$ | $(1\times1)$ |
| for $i=1..M$ : | |
| $\underline{K}_{k+1}^{(i)} = \begin{bmatrix}0\\ \underline{C}_{k+1}^{(i)}\end{bmatrix} = \begin{bmatrix}\underline{C}_k^{(i)}\\ 0\end{bmatrix} + S_k^{(i)}\underline{W}_k$ | $((N_i+1)\times1)$ |
| for $i=1..M$ : | |
| $S_{k+1}^{(i)} = S_k^{(i)} - \underline{K}_{k+1}^{(i)}(R_{k+1})^{-1}\underline{V}_k^t$ | $((N_i+1)\times\alpha)$ |
| Prediction error : | |
| $e_k = y_k - \sum_{i=1}^{M}(\underline{U}_{k-1}^{(i)})^t\,\hat{\underline{b}}_{k/k-1}^{(i)}$ | $(1\times1)$ |
| Estimated coefficient vectors : | |
| for $i=1..M$ : $\hat{\underline{b}}_{k+1/k}^{(i)}=\hat{\underline{b}}_{k/k-1}^{(i)}+\underline{C}_k^{(i)}(R_k)^{-1}e_k$ | $(N_i\times1)$ |

Table 1: Steps of the fast Chandrasekhar multichannel algorithm.

The algorithm is initialized with the following relations: for $i=1..M$, $\underline{K}_1^i = 0_{N_i+1}$ and $R_1 = s_v^2$.

$0_k^i$ is an $(i,k)$ matrix whose elements are zeros. The rank α, is equal to $2.M$.

By defining a pinning vector of length $p$: $\underline{r}_i^p = \left[0_{i-1}^t \; 1 \; 0_{p-i}^t\right]^t$, the initialization of $M_1$ and $S_1$ are:

$$M_1 = \begin{bmatrix} -1 & 0 & 0 & .. & 0 & 0 \\ 0 & 1 & 0 & .. & & 0 \\ 0 & 0 & -1 & .. & & 0 \\ .. & & & ... & & \\ 0 & 0 & & & -1 & 0 \\ 0 & 0 & 0 & .. & 0 & 1 \end{bmatrix},$$

$$S_1^{(1)} = \left[ s_1 \underline{r}_1^{N_1+1} \middle| s_1 \underline{r}_{N_1+1}^{N_1+1} \middle| 0_{N_1+1} \middle| ......... , \right.$$
$$\left. ................................... \middle| 0_{N_1+1} \right]$$

$$S_1^{(2)} = \left[ 0_{N_2+1} \middle| 0_{N_2+1} \middle| s_2 \underline{r}_1^{N_2+1} \middle| s_2 \underline{r}_{N_2+1}^{N_2+1} \middle| 0_{N_2+1} \middle| ......... , \right.$$
$$\left. ........... \middle| 0_{N_2+1} \right]$$

.
.
.

$$S_1^{(M)} = \left[ 0_{N_M+1} \middle| 0_{N_M+1} \middle| 0_{N_M+1} \middle| 0_{N_M+1} \middle| ........... \right.$$
$$\left. ....... \middle| s_M \underline{r}_1^{N_M+1} \middle| s_M \underline{r}_{N_M+1}^{N_M+1} \right]$$

$s_i^2$ is the variance of the $i^{\text{th}}$ channel coefficients. The initial values of the channel coefficient vectors are set to zero, namely, for $i=1..M$, $\hat{\underline{b}}_{1/0}^{(i)} = 0_{N_i}$.

## 4. SIMULATION RESULTS

### Experiment 1: 2-D MA model identification

The purpose of this example is to illustrate the performance of the proposed 2-D Chandrasekhar fast adaptive algorithm for a linear 2-D MA model identification following the block diagram of Figure 2. The desired 2-D MA filter output is :

$$s_{n,r} = -0.29.x_{n-2,r-2} + 0.68.x_{n-2,r-1} - 0.37.x_{n-2,r} + 0.69.x_{n-1,r-2} - 1.68.x_{n-1,r-1} + 1.1.x_{n-1,r} - 0.38.x_{n,r-2} + 1.12.x_{n,r-1} + v_{n,r}$$

The sequences $\{x_{n,r}\}$ and $\{v_{n,r}\}$ are the 2-D random signal input and the additive gaussian noise of variance $s_v^2$, respectively.

The signal-to-noise ratio is defined as :

$\text{SNR (dB)} = 10 \log \frac{s_s^2}{s_v^2}$ where $s_s^2$ is the variance of the desired filter output.

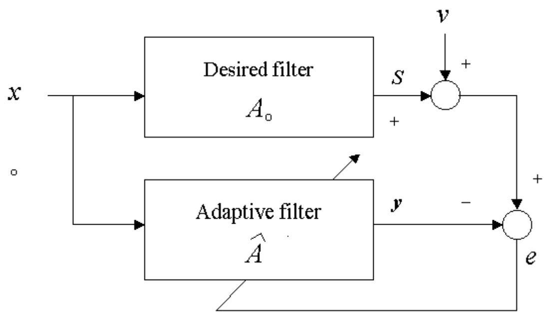


**Figure 2:** Block diagram of the MA model adaptive identification

We apply the new 2-D fast Chandrasekhar adaptive filter with 3 channels to estimate the desired 2-D filter coefficients.

The performance criterion chosen is the norm of the coefficient error vectors defined as:

$\text{Er (dB)} = 10.\log \frac{\sum_{i=1}^{3}\sum_{j=1}^{3}(\hat{b}_j^{(i)} - A_{0ji})^2}{\sum_{i=1}^{3}\sum_{j=1}^{3}(A_{0ji})^2}$, where $A_{0ji}$ are the 2-D coefficient of the desired filter of index $j,i$.

For a desired image $S$ of 20×20 pixels, Figure 3 illustrates the evolution of the coefficient error norm Er (dB) when the SNR is equal to 20 dB. We subplot in Figure 3 the coefficient error norms of the 2-D normalized LMS algorithm [3] and the 2-D FLRLS algorithm [4]. The coefficient error norm of the new Chandrasekhar algorithm is smaller than the one of both other algorithms.

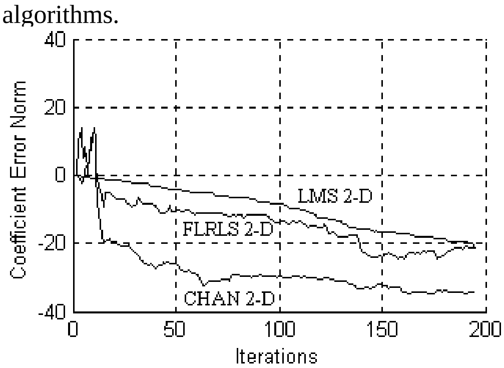


**Figure 3:** Norm of coefficient-error vector for the 2-D fast Chandrasekhar, 2-D LMS and 2-D FLRLS algorithms.

### Experiment 2: 2-D Adaptive noise cancellation

The purpose of this experiment is to illustrate the performance of the proposed 2-D Chandrasekhar fast adaptive algorithm in additive noise cancellation using the block diagram of Widrow's adaptive noise canceller [12] (Figure 4).

The relationship between the additive noise $y$ and the reference noiss $x$ is given by:

$$y_{n,r} = 0.1.x_{n-2,r-2} + 0.2.x_{n-2,r-1} + 0.5.x_{n-2,r} + 0.2.x_{n-1,r-2} - 0.5.x_{n-1,r-1} + 0.4.x_{n-1,r} - 0.3.x_{n,r-2} + 0.2.x_{n,r-1}$$

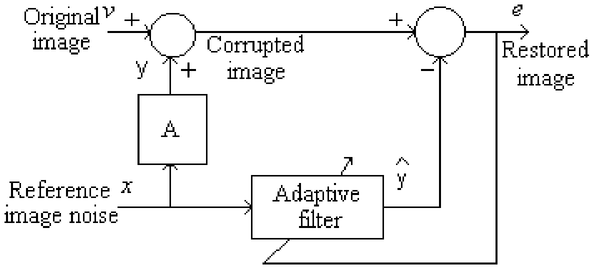


**Figure 4:** Block diagram of the Widrow's adaptive noise canceller.

The signal-to-noise ratio of the corrupted image is defined as: $10\log\frac{\mathrm{var}(y+v)}{\mathrm{var}(y)}$ and the SNR of the restored image is defined as $10\log\frac{\mathrm{var}(e)}{\mathrm{var}(e-v)}$, where var($x$) notes the variance value of the signal $x$.

Figure 5 shows the original image $v$ of the bridge ($256\times256$ pixels) and the corrupted image with a gaussian white noise (SNR 4.2dB). The restored image using the 2-D normalized LMS filter has an SNR 9.3dB, while the restored image using the new 2-D Chandrasekhar filter provides a high SNR of 20.9dB. The gain is about 16.7 dB, which proves the superiority of the proposed Chandrasekhar filter for image filtering.

## 5. CONCLUDING REMARKS

In this paper, we have presented a new fast algorithm for two-dimensional (2-D) linear adaptive filtering using the Chandrasekhar equations. This algorithm is based on the analogy between the multichannel linear model and the 2-D one. The new 2-D adaptive filter provides an satisfactory performance in 2D MA Model identification and in 2D adaptive noise cancellation. More works have to be done to generalize this algorithm to the case of 2-D Auto-regressive (AR) filtering and to use it for practical application such as texture characterization.

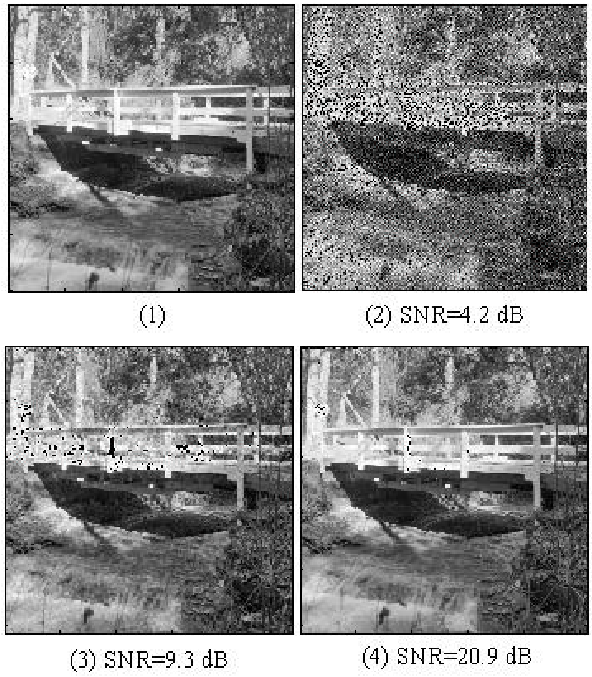


**Figure 5:** (1) Original image, (2) Corrupted image (SNR 4.2dB) (3) Restored image by the 2-D LMS filter (SNR 9.3dB) (4) Restored image by the 2-D Chandrasekhar filter (SNR 20.9dB).

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