A NEW MATRIX METHOD FOR PULSE TRAIN IDENTIFICATION: IMPLEMENTING BY SYSTOLIC ARRAY

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ABSTRACT

In this paper NASH[1] algorithm, a new matrix-based method for identification of radar pulse train, is implemented by systolic array. NASH can be used to identify the PRI for constant, staggered, and jittered signal. Previous matrix-based methods can only identify the first type of signal, i.e. constant PRI.

The complexity of the computation in NASH is more than the previous matrix-based method. To overcome this drawback a systolic array, the best parallel structure for the matrix operation, is designed. The systolic array helps to parallelize the matrix inversion, which is the most time consuming part of NASH algorithm.

Keywords: ESM, Matrix Operation, Systolic array, Deinterleaving, Parallelism.

1. INTRODUCTION

An ESM (Electronic Support Measure) system consists of three sections, namely clustering, deinterleaving and identification. Clustering and deinterleaving means separation of arriving pulses into some pulse trains, associating each one to specific radar [1]. After grouping input pulses, each pulse train must be characterized. Characterization of the radar system, which has sent the pulse train, is usually called identification. The main character that must be specified during identification is PRI (pulse-repetition-interval), which may be constant, staggered or jitter type. A sufficiently large number of samples must be taken for exact identification. Having numerous pulses results in a huge computation in the identification step. On the other hand the ESM system must act as a real time system, so the time of operation is vital. Many different techniques are introduced by researchers for pulse identification [2-4]. The most important goal is the reduction of the total time of identification. Usually pulse time-of-arrival (TOA) is used as the main feature for identification. Two of the most famous techniques for deinterleaving and identification are TOA difference histogram [5] and sequence search [5,6]. Recently a matrix-based technique is introduced for TOA identification, which eliminates the ambiguous parameters such as bin size and thresholding of previous algorithm. Matrix operation and matrix-based method inherently has the ability of parallelism [7]. The proposed method in [7] can only be used to identify the constant PRI.

In this paper we generalize previous method [8], by introducing two new matrices: PM (PRI matrix) and NDT (Normalized Difference TOA). The introduced method, which is called NASH, can be used to identify the staggered and jitter signals, in addition to constant PRI signals. The most time consuming part of NASH is inverting NDT. Systolic array, the most suitable parallel structure for matrix operation, is used to minimize the time of operation and reaching the real time system.

The rest of the paper is organized as follows: In section 2 previous methods of TOA identification is reviewed. In section 3, PM and NDT matrices are introduced. In section 4 the NASH is introduced on basis of inverse of NDT matrix and its computation complexity is compared with other methods. In section 5, systolic structure for implementing NASH is designed. Conclusion is offered in last section.

2. PREVIOUS WORKS

Histogramming and sequence search are two mostly used techniques, which is proposed for pulse identification. [5,6]. In histogramming each TOA is subtracted from every subsequent TOA and a histogram is made on basis of TOA's difference [5,6]. In sequence search algorithm the sequences of identical interval are extracted from input pulses. The latter algorithm is more accurate and reliable at the expense of processing speed [5].

Both techniques have inherent drawbacks. Histogramming needs bin size and thresholding parameters that must be adjusted adaptively. Sequence search needs many computation steps and both techniques inherently cannot be performed on parallel manner.

[1] NASH is first couple of letters of author names.

Recently a matrix-based method is proposed by Ray [7], which has overcome the above drawback. In [7] the matrix of differences of TOA, ΔTOA , is defined as follows:

$$\Delta TOA = \left|TOA(j) - TOA(i)\right| \quad for\ 1 \le i, j \le N \tag{1}$$

Where N denotes the number of pulses. The properties of ΔTOA^{-1} and its relation to PRI of input pulses were discussed. The proposed method in [7] can simplify the identification and also has ability of finding location of missing pulses. Clearly the ability of determining the missing pulses' location is analogues to inability of averaging pulses' features. On the other hand matrix operation can be done in parallel manner, which results in obtaining a real time identifier. The main drawback of proposed method in [7] is the lack of interpreting non-uniform PRI, especially in staggered signals. We will express the RAY's method in next section in detail.

3. PM AND NDT MATRIX

In [7] ΔTOA matrix is considered as product of PRI scalar and a Harmonic Matrix (HM).

$$PRI\ .\ HM = \Delta TOA \tag{2}$$

The harmonic matrix, HM, is a Toeplitz matrix, whose inverse for extremely large value of matrix size would always be tri-diagonal matrix. All main diagonal elements of HM^{-1} are equal to -1 except its two corner elements. For extremely large number of received pulses, N , the two corner elements in main diagonal approach -0.5 . Also for finite value of N the two top-right (TR) and bottom-left (BL) elements are non-zero but for extremely large number of N the TR and BL corner elements of matrix approach zero. Inversion of Equation (2) yields:

$$\frac{1}{PRI}\ .\ HM^{-1} = \Delta TOA^{-1} \tag{3}$$

In Equation (3) PRI is scalar, so the above properties of HM^{-1} will propagate to ΔTOA^{-1} . The main conclusion in [7] expresses that trace of ΔTOA^{-1} is equal to $\frac{(N-1)}{PRI}$. In the staggered signal which the PRI changes periodically; the only result is repeating meaningless numbers on main diagonal of ΔTOA^{-1} with period of staggered degree.

The lack of resulting and interpreting main diagonal elements of ΔTOA^{-1} in [7] originated from invalidation of HM for staggered signals.

Here we change the scenario by defining PRI Matrix, PM, which in a special case (of uniform PRI) will be the same as HM.

Let PM (PRI Matrix) be defined as a diagonal matrix with size N (the number of received pulses), whose diagonal elements will be equal to PRI's of received signals. If the signal was staggered the repeated sets of PRI's appear on the diagonal of PM. For constant PRI signal, all diagonal elements of PM will be equal to PRI value. Equation (4) shows PM for staggered signal, with staggered degree of S .

$$[PM]_{N\times N} = \begin{bmatrix} T_1 & 0 & 0 & \cdots\cdots & 0 & 0 & 0 & 0 \\ 0 & T_2 & 0 & \cdots\cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & T_3 & \cdots\cdots & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots\cdots & T_s & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots\cdots & 0 & T_1 & 0 & 0 \\ 0 & 0 & 0 & \cdots\cdots & 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & \cdots\cdots & 0 & 0 & 0 & T_3 \end{bmatrix} \tag{4}$$

In constant PRI signals equation (2) must be improved as follows:

$$PM \times HM = \Delta TOA \tag{5}$$

It must be noted that for staggered signal the HM must also be changed. To generalize Equation (5), a general harmonic matrix must be defined so that its multiplication with PM makes ΔTOA . Therefore:

$$PM \times GHM = \Delta TOA \tag{6}$$

where GHM denotes general harmonic matrix.

The difference time of arrival matrix, ΔTOA , is defined as follows:

$$[\Delta TOA]_{N\times N} = \begin{bmatrix} 0 & t_2 - t_1 & t_3 - t_1 & t_4 - t_1 & \cdots\cdots & t_{N-1} - t_1 & t_N - t_1 \\ t_2 - t_1 & 0 & t_3 - t_2 & t_4 - t_2 & \cdots\cdots & t_{N-1} - t_2 & t_N - t_2 \\ t_3 - t_1 & t_3 - t_2 & 0 & t_4 - t_3 & \cdots\cdots & t_{N-1} - t_3 & t_N - t_3 \\ \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\ t_N - t_1 & t_N - t_2 & t_N - t_3 & t_N - t_4 & \cdots\cdots & t_N - t_{N-1} & 0 \end{bmatrix} \tag{7}$$

in which $t_{k+1} - t_k = T_k$. According to Equation (6) and (7), the harmonic matrix must be a function of PRI's. Obviously GHM can be found by normalizing each row of ΔTOA by its upper co-diagonal elements. Since GHM is the normalized ΔTOA , in the rest of this paper we call it NDT . Hence:

$$[NDT]_{N\times N} = \begin{bmatrix} 0 & \frac{T_1}{T_1} & \frac{T_1+T_2}{T_1} & \cdots\cdots & \frac{T_1+T_2+\ldots+T_{N-2}}{T_1} & \frac{T_1+T_2+\ldots+T_{N-1}}{T_1} \\ \frac{T_1}{T_2} & 0 & \frac{T_2}{T_2} & \cdots\cdots & \frac{T_2+T_3+\ldots+T_{N-2}}{T_2} & \frac{T_2+T_3+\ldots+T_{N-1}}{T_2} \\ \frac{T_1+T_2}{T_3} & \frac{T_2}{T_3} & 0 & \cdots\cdots & \frac{T_3+T_4+\ldots+T_{N-2}}{T_3} & \frac{T_3+T_4+\ldots+T_{N-1}}{T_3} \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots\cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \frac{T_1+T_2+\ldots+T_{N-1}}{T_N} & \frac{T_2+T_3+\ldots+T_{N-1}}{T_N} & \frac{T_3+T_4+\ldots+T_{N-1}}{T_N} & \cdots\cdots & \frac{T_{N-1}}{T_N} & 0 \end{bmatrix} \tag{8}$$

Obviously in a special case where all T_i 's are the same (constant PRI signals), the NDT will be the same as harmonic matrix (HM) in [7].

4. NASH IDENTIFICATION METHOD

As expressed in previous section the properties of ΔTOA^{-1} is due to the properties of the inverse of harmonic matrix, which in general case is NDT^{-1} . Since this general matrix also contains information of pulses' arrival time, it may be used for identification of input signals. In the rest of this section we discuss the main properties of NDT and propose an algorithm for extracting PRI's of staggered signals. Equation (8), NDT matrix expression, can be rewritten as follows:

$$NDT_{ij} = \begin{cases} \sum_{k=i}^{j-1} R_{k,i} & i < j \\ 0 & i = j \\ \sum_{k=j}^{i-1} R_{k,i-1} & i > j \end{cases} \tag{9}$$

where $R_{ij} = T_i / T_j$. Obviously $R_{ii} = 1$, so upper co-diagonal elements of NDT are equal to 1.

Computing NDT^{-1} results in the following properties:

1. NDT^{-1} is tri-diagonal matrix except for two elements of bottom-left (BL), top-right (TR) corner. If size of matrix, N, is extremely large, these two corner elements approach zero.
2. All main diagonal elements are negative and all of co-diagonal elements are positive.
3. All elements of the lower co-diagonal are equal to 0.5.
4. Summation of non-zero elements in each column is equal to zero (for extremely large value of N). For finite value of N, sum of two other elements in those columns is equal to non-zero corner (BL or TR).
5. Elements of the main diagonal (except the two corner elements) and upper co-diagonal are repeated periodically with period of S, the staggered degree.

According to the above properties, one can write the general form NDT^{-1} as follows:

$$\begin{bmatrix} x_1 & -x_2-0.5 & 0 & 0 & \cdots & x_{TR} \\ 0.5 & x_2 & -x_3-0.5 & 0 & \cdots & 0 \\ 0 & 0.5 & x_3 & -x_4-0.5 & \cdots & 0 \\ 0 & 0 & 0.5 & x_4 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \\ \vdots & \vdots & \vdots & \vdots & x_{N-1} & x_{TR}-x_N \\ x_1+0.5 & 0 & 0 & \cdots & 0.5 & x_N \end{bmatrix} \tag{10}$$

Equations (9) and (10) can be used to compute PRIs of input signals versus NDT^{-1} elements. Constant values in main lower co-diagonal of NDT^{-1}, and relation between upper co-diagonal NDT and zero value of main diagonal of NDT, will simplify this computation and results in a recursive formula.

Multiplying NDT by NDT^{-1} obviously makes a unity matrix, so:

$$\begin{aligned} &0.5R_{11} + (x_1+0.5)\sum_{k=1}^{N-1} R_{k1} = 1 \\ &R_{i-1,i}(-x_i-0.5) + (1\times 0.5) = 1 \quad \forall\, i \geq 2 \end{aligned} \tag{11}$$

Since $R_{11} = 1$, $R_{k1} = \dfrac{T_k}{T_1}$ and $R_{i-1,i} = \dfrac{T_{i-1}}{T_i}$:

$$\begin{aligned} &0.5 + \frac{(x_1+0.5)}{T_1}\sum_{k=1}^{N-1} T_k = 1 \\ &\frac{T_{i-1}}{T_i}(-x_i-0.5) + 0.5 = 1 \qquad \forall i \geq 2 \end{aligned} \tag{12}$$

Also it must be noted that $\sum_{k=1}^{N-1} T_k$ is equal to time of measuring or total interval time of receiving N pulses which we show by T_{Total}. Rearranging equation (12) yields:

$$\begin{aligned} &T_1 = (2x_1+1)T_{Total} \\ &T_i = (-2x_i-1)\,T_{i-1} \qquad for\ i \geq 2 \end{aligned} \tag{13}$$

The set of Equation (14) can be used recursively for finding PRI's. Note that the staggered degree can be found when $T_{S+1} = T_1$. Therefore the NASH algorithm steps are as follows:

1. Computing NDT
2. Finding diagonal elements of NDT^{-1}
3. Calculating T_1 from first element of NDT^{-1} diagonal (Equation 13)
4. Calculating T_i's recursively from T_{i-1} and NDT^{-1} diagonal elements (Equation 13)
5. Comparing T_i with T_1 and finding staggered degree, S (smallest i which $T_{i+1} = T_1$).
6. Calculating average of T_i's from $\overline{T_i} = \dfrac{N}{S}\sum_{k=1}^{N/S} T_{ks+i}$

So NASH has the following advantages comparing previous matrix-based method:

1. NASH can detect missing pulses similar to the method introduced in [7] (the related elements in the main diagonal and co-diagonal would change and its effect appears in computed T_i regarding $\overline{T}_i$.)
2. NASH computes the average of PRI's so it can be used for noisy PRI's.
3. NASH can be applied for staggered signals, which is lacking in [7]. Also it can be used for jittered and uniform PRI's like other methods.

5. SYSTOLIC ARRAY DESIGN

In this section the systolic array, which is the best parallel structure for matrix operation [8], is used for matrix inversion, the third step of NASH. Although systolic array can be used for other steps of NASH, the third step is the most time consuming part of NASH.

Figure 1 shows the structure of the systolic array for $N = 4$. The size of matrix, N, which is equal to number of pulses, must be clearly more than 4, but the selected value for N in figure 1, is chosen for simplicity of figure and the systolic structure can certainly be expanded to the suitable size.

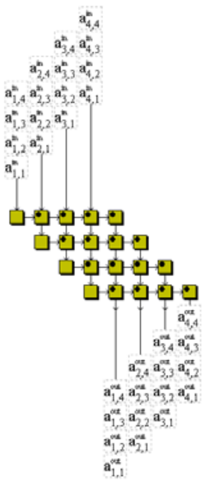

Figure1: Systolic array for matrix inversion for $N=4$

The speedup for the systolic structure of figure1 is equal to $N^3/(5N-2)\approx 0.2N^2$. Table 1 shows the computation complexity of Nash algorithm. The total computation steps are approximately equal to $1.5N^2$. So implementing NASH by systolic reduces computation complexity of NASH that was NASH drawback comparing [7]. Also NASH can compute the average and the ratio of PRI in the staggered signals, which is the lacking of the previous method. Comparing non-matrix-based methods, which usually use basic difference histogramming, Nash has significantly lower computational complexity. Their complexity is approximately PRI equal to $0.5N^3$ for uniform PRI and $0.5SN^3$ for staggered signals, where N and S denote the number of pulses and staggered degree, respectively [9].

Table 1. NASH algorithm Computation complexity

Step	*Operation needs*
Determining ΔTOA	$0.5N(N-1)$
Computing NDT	$N^2-(2N-1)$
Reversing NDT	$5N-2$
Computing T_i *'s*	$2N$
Computing $\overline{T_i}$ *'s*	$N+S$
Total Operation	$1.5N^2+5.5N+S-2 \approx 1.5N^2$

6. CONCLUSION

Pulse train identification is a time-consuming job in ESM systems, considering the real-time operation of such systems. So much research has been carried out to find more rapid algorithm for pulse identification. In this paper we offer a new matrix-based algorithm by defining PRI matrix (PM) and a general harmonic matrix (NDT), with their product making the difference time of arrival matrix ΔTOA. The properties of NDT^{-1} were discussed and the NASH algorithm is introduced. NASH can be applied to identify the staggered signals as well as constant PRI signals while previous matrix-based method can only be used to identify the constant PRI signal. Systolic array is the most used parallel structure for matrix operation. To reduce the computational time, the systolic array is used to implement NASH.

An area for further study is investigating the effects of noise on the system. One can compute the error caused by noise on the pulses as a function of the level of noise in the system. Another area for future work is denoting the other part of NASH by systolic arrays.

REFERENCES

[1] M.W.Maier, "Processing Throughput Estimation for Radar Intercept Receivers", IEEE Trans. on Aerospace and Electronic Systems, Vol. 34, No. 1, pp.84-92, Jan 1998.

[2] K.Nishiguchi, M.Kobayashi, "Improved Algorithm for Estimating Pulse Repetition Intervals", IEEE Trans. on Aerospace and Electronic Systems, Vol. 36, no. 2, pp.407-421, April 2000.

[3] Y.Zhou, J.P.V.Lee, "Combining Clustering Techniques and Information Theoretic Criteria Based Approach for Emitter Number Detection in ESM Applications." IEEE Int'l Conference on Signals, Systems, and Computers, pp. 474-478,1999.

[4] T.Conroy, J.B.Moore, "On The Estimation of Interleaved Pulse Train Phases", 5th Int'l Symp. on Signal Processing and Its Applications ISSPA99, Australia, PP.223-226, August 1999.

[5] H.K.Mardia, "New Techniques for the Deinterleaving of Repetitive Sequences", IEE Proc., Vol. 136, No.4, pp.149-154, Aug. 1989.

[6] D.J.Milojevic, B.M.Popovic, "Improved Algorithm for the Deinterleaving of Radar Pulses", IEE Proceedings, Vol.139, No.1, pp.98-104, Feb. 1992.

[7] P.S.Ray, "A novel pulse TOA Analysis Technique for Radar Identification", IEEE Trans. on Aerospace and Electronic Systems, Vol. 34, No. 3, pp.716-721, July 1998.

[8] DeCegama, A., *The Technology of Parallel Processing*, Prentice Hall, 1993.

[9] A. Naseri, Systolic Array Application in Clustering and Deinterleaving of Pulse Train, 2nd Report of Ph.D. Project IUST Dec. 2001.