OPTIMUM SIGNALING FOR MIMO-OFDM SYSTEMS WITH INTERFERENCE*

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ABSTRACT

Optimum signaling is studied for multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems with multiple links which mutually interfere. Numerical results show that using the optimum approach for cases without interference does not always achieve system capacity. In some cases, the overall system performance is improved by forcing users to use different subcarriers or to transmit using less antennas or both.

1 INTRODUCTION

Multiple input and multiple output (MIMO) systems have been shown to have great potential in providing high spectral efficiency in wireless communications [1]. The optimum approach for cases without interference [1] sends an independent data stream from each possible antenna. However, recent studies [2] have shown that cochannel interference in MIMO systems can degrade the overall capacity of a cellular system significantly. It was further demonstrated [3] [4] that decreasing the number of streams in MIMO systems gives better overall system performance for certain interference environments.

While the majority of research has been on single carrier MIMO (SC-MIMO), MIMO-OFDM (orthogonal frequency division multiplexing) [5] [6] is beginning to receive attention. It can sometimes remove the need for equalization. In this paper, we study optimum signaling for MIMO-OFDM systems in the presence of cochannel interference. We assume that the transmitter has no knowledge about the instantaneous channel state information, while the receiver has perfect information. Compared with SC-MIMO, MIMO-OFDM provides an additional dimension, the frequency dimension, for resource allocation.

The remainder of this paper is organized as follows. Section 2 introduces the MIMO-OFDM system model and formulates the optimum signaling problem from a mutual information point of view. Section 3 and Section 4 investigate optimum signaling for both an isolated link and a system with multiple links. In Section 5, simulation results give the performance of a simplified system to explain the importance of choosing the appropriate signaling scheme. Conclusions are given in Section 6.

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2 PROBLEM FORMULATION

Consider a MIMO-OFDM system with L users, where each user employs N_t transmit antennas and N_r receive antennas, and suffers from cochannel interference from the other $L-1$ users. Let K be the total number of subcarriers. Assuming the OFDM parameters are chosen so there is no inter-channel or inter-block interference, the received complex baseband signal vector ($N_r \times 1$) corresponding to the kth subcarrier of user ℓ is given by

$$\mathbf{Y}_\ell(k) = \sqrt{\rho_\ell}\mathbf{H}_{\ell,\ell}(k)\mathbf{X}_\ell(k) + \sum_{j=1,j\neq\ell}^{L} \sqrt{\eta_{\ell,j}}\mathbf{H}_{\ell,j}(k)\mathbf{X}_j(k) + \mathbf{N}_\ell(k), \quad (1)$$

where $\mathbf{H}_{\ell,j}(k)$ ($N_r \times N_t$) denotes the kth subcarrier's normalized channel frequency response matrix between the transmit antennas of user j and the receive antennas of user ℓ, and $\mathbf{N}_\ell(k)$ ($N_r \times 1$) is the complex Gaussian noise vector with independent components with zero mean and unit variance. The transmitted signal $\mathbf{X}_j(k)$ ($N_t \times 1$) is normalized such that the covariance matrix $\mathbf{Q}_j(k) = E\{\mathbf{X}_j(k)\mathbf{X}_j(k)^H\}$ satisfies $\frac{1}{K}\sum_{k=1}^{K} trace(\mathbf{Q}_j(k)) = 1$. The signal-to-noise ratio (SNR) of user ℓ is ρ_ℓ, and $\eta_{\ell,j}$ is the interference-to-noise ratio (INR) of the interference generated by user j which is received by user ℓ's receiver.

For simplicity, we assume all transmitted signals are Gaussian distributed, which is typically the case for capacity optimum signaling for MIMO problems. From (1), the covariance matrix of interference-plus-noise for the kth subcarrier of user ℓ becomes $\mathbf{R}_\ell(k) = \mathbf{I}_{N_r} + \sum_{j=1,j\neq\ell}^{L} \eta_{\ell,j}\mathbf{H}_{\ell,j}(k)\mathbf{Q}_j(k)\mathbf{H}_{\ell,j}(k)^H$. By applying $\mathbf{R}_\ell(k)^{-1/2}$ to (1) to whiten the interference-plus-noise , we obtain [1] the instantaneous mutual information for user ℓ as

$$\begin{aligned} MI_\ell &= \frac{1}{K}\sum_{k=1}^{K} \log_2 \Big\{ \det \Big(\mathbf{I}_{N_r} + \rho_\ell (\mathbf{R}_\ell(k)^{-1/2}\mathbf{H}_{\ell,\ell}(k)) \\ &\qquad \mathbf{Q}_\ell(k)(\mathbf{R}_\ell(k)^{-1/2}\mathbf{H}_{\ell,\ell}(k))^H \Big) \Big\} \\ &= \frac{1}{K}\sum_{k=1}^{K} \log_2 \{ \det (\mathbf{I}_{N_r} + \rho_\ell \mathbf{H}_{\ell,\ell}(k) \\ &\qquad \mathbf{Q}_\ell(k)\mathbf{H}_{\ell,\ell}(k)^H\mathbf{R}_\ell(k)^{-1}) \} . \end{aligned} \quad (2)$$

Therefore, the instantaneous mutual information of the overall L-user system is

$$MI = \frac{1}{K}\sum_{\ell=1}^{L}\sum_{k=1}^{K}\log_2\left\{\det\left(\mathbf{I}_{N_r} + \rho_\ell \mathbf{H}_{\ell,\ell}(k)\right.\right.$$
$$\left.\left.\mathbf{Q}_\ell(k)\mathbf{H}_{\ell,\ell}(k)^H\mathbf{R}_\ell(k)^{-1}\right)\right\}. \quad (3)$$

In this paper, we study the optimum signaling problems for two cases.

(1) We consider an isolated link, where we assume the user of interest has no control over the signaling schemes of the interferers and the covariance matrices of the interferers are fixed. Assuming the user of interest is user 1, our goal is to find the optimum matrices $\mathbf{Q}_1(k), k = 1, \cdots, K$ which maximize the single link ergodic mutual information $E\{MI_1\}$ under the power constraint $\frac{1}{K}\sum_{k=1}^{K} trace(\mathbf{Q}_1(k)) = 1$.

(2) We are interested in the system performance as opposed to the single-user performance, where we need to find the optimum matrices $\mathbf{Q}_\ell(k), k = 1, \cdots, K, \ell = 1, \cdots, L$ which maximize the system ergodic mutual information $E\{MI\}$ under the constraint $\frac{1}{K}\sum_{k=1}^{K} trace(\mathbf{Q}_\ell(k)) = 1, \ell = 1, \cdots, L$.

Since $\mathbf{Q}_j(k)$ is non-negative definite, we have $\mathbf{Q}_j(k) = \mathbf{U}_j(k)\mathbf{D}_j(k)\mathbf{U}_j(k)^H$, where $\mathbf{U}_j(k)$ is unitary and $\mathbf{D}_j(k)$ is non-negative diagonal. Substituting this into (2) and (3) and considering that $\mathbf{H}_{\ell,j}(k)\mathbf{U}_j(k)$ has the same distribution as $\mathbf{H}_{\ell,j}(k)$ [1], the ergodic mutual information $E\{MI_\ell\}$ and $E\{MI\}$ remain the same if we replace $\mathbf{Q}_\ell(k)$ with $\mathbf{D}_\ell(k)$ for $k = 1, \cdots, K, \ell = 1, \cdots, L$. Therefore, we only need to consider diagonal non-negative $\mathbf{Q}_\ell(k)$ in our optimization problems. We call the number of non-zero entries in $\mathbf{Q}_\ell(k)$ the number of streams, as per [3].

3 Optimum Signaling for an Isolated Link

It was proved [1] that for a SC-MIMO system without interference, the capacity is achieved when the signal covariance matrix is a multiple of the identity matrix. Here we show that the same conclusion holds for an isolated link in a SC-MIMO system ($K = 1$) with interference. From (2), the ergodic mutual information of user 1 is given by $\Phi_{SC}(\mathbf{Q}_1(1)) = E\{\log_2\{\det(\mathbf{I}_{N_r} + \rho_1\mathbf{H}_{1,1}(1)\mathbf{Q}_1(1)\mathbf{H}_{1,1}(1)^H\mathbf{R}_1(1)^{-1})\}\}$, where $\mathbf{R}_1(1) = \mathbf{I}_{N_r} + \sum_{j=2}^{L}\eta_{1,j}\mathbf{H}_{1,j}(1)\mathbf{Q}_j(1)\mathbf{H}_{1,j}(1)^H$ for $\mathbf{Q}_j(1), j = 2, \cdots, L$ fixed. By recognizing that $\Phi_{SC}(\mathbf{Q}_1(1))$ is a concave function of $\mathbf{Q}_1(1)$, we know that $\Phi_{SC}(\mathbf{Q}_1(1))$ is maximized by a $\mathbf{Q}_1(1)$ of the form $\alpha\mathbf{I}_{N_t}$, with α a scalar, by using similar arguments as in [1].

Now consider an isolated link in a MIMO-OFDM system. Based on the result we just discussed for the SC-MIMO system, $\mathbf{Q}_1(k)$ must be of the form $\alpha_k\mathbf{I}_{N_t}$ for $k = 1, \cdots, K$ in order to maximize $E\{MI_1\}$. However, the optimum way to distribute the power between subcarriers depends on the interference environment for each subcarrier. If all subcarriers experience identical interference environments, in particular, if $\mathbf{Q}_j(1) = \mathbf{Q}_j(2) = \cdots = \mathbf{Q}_j(K), j = 2, \cdots, L$, we claim that the optimum covariance matrices are $\mathbf{Q}_1(k) = \frac{1}{N_t}\mathbf{I}_{N_t}, k = 1, \cdots, K$, or equivalently, the desired user should always distribute the power evenly among all subcarriers and employ a multiple of the identity matrix for signaling in each subcarrier in order to achieve capacity. To be more specific, we rewrite $E\{MI_1\}$ from (2) as

$$\Phi_1(\mathbf{Q}_1) = E\left\{\frac{1}{K}\log_2\left\{\det\left(\mathbf{I}_{KN_r} + \rho_1\mathbf{H}_{1,1}\mathbf{Q}_1\mathbf{H}_{1,1}^H\mathbf{R}_1^{-1}\right)\right\}\right\}, \quad (4)$$

where $\mathbf{H}_{1,1} = \text{diag}\{\mathbf{H}_{1,1}(1), \cdots, \mathbf{H}_{1,1}(K)\}$, $\mathbf{Q}_1 = \text{diag}\{\mathbf{Q}_1(1), \cdots, \mathbf{Q}_1(K)\}$, and $\mathbf{R}_1 = \text{diag}\{\mathbf{R}_1(1), \cdots, \mathbf{R}_1(K)\}$. For any given $\mathbf{Q}_1(k) = \alpha_k\mathbf{I}_{N_t}, k = 1, \cdots, K$, let $\Pi(\mathbf{Q}_1(1), \cdots, \mathbf{Q}_1(K))$ denote a permutation of $(\mathbf{Q}_1(1), \cdots, \mathbf{Q}_1(K))$ and $\Pi(\mathbf{Q}_1) = \text{diag}\{\Pi(\mathbf{Q}_1(1), \cdots, \mathbf{Q}_1(K))\}$. Since $\mathbf{H}_{1,1}(1), \cdots, \mathbf{H}_{1,1}(K)$ have the same distribution and so do $\mathbf{R}_1(1), \cdots, \mathbf{R}_1(K)$, we find $\Phi_1(\mathbf{Q}_1) = \Phi_1(\Pi(\mathbf{Q}_1))$. Due to the fact that $\log\det$ is concave on the set of positive definite matrices, $E\{MI_1\}$ is concave on $\mathbf{Q}_1$. Define $\bar{\mathbf{Q}}_1 = \frac{1}{K!}\sum_\Pi \Pi(\mathbf{Q}_1)$, then we have $\Phi_1(\bar{\mathbf{Q}}_1) \geq \Phi_1(\mathbf{Q}_1)$, where $\bar{\mathbf{Q}}_1$ is a multiple of identity matrix. Therefore, $E\{MI_1\}$ is maximized when $\mathbf{Q}_1 = \frac{1}{N_t}\mathbf{I}_{KN_t}$, which means that $\mathbf{Q}_1(k) = \frac{1}{N_t}\mathbf{I}_{N_t}, k = 1, \cdots, K$. An isolated link without interference is just a special case where $\mathbf{Q}_j(k) = \mathbf{0}, j = 2, \cdots, L$.

4 Optimum Signaling for an Overall System

The optimum signaling for SC-MIMO systems with interference was discussed in [3] [4]. It was shown that, when the interference is strong or the signal is weak, transmitting fewer streams than the maximum is necessary to reach the system capacity. Results presented in this section give insights on what the optimum signaling schemes are for MIMO-OFDM under different conditions.

Due to the considerable complexity of $E\{MI\}$ from (3), only results for $N_t = N_r = 2$ are given. For simplicity, we assume $\rho_\ell = \rho, \forall\ell$ and $\eta_{\ell j} = \eta, \forall\ell, j$. We consider on-off schemes, where each user can transmit $n_s \leq N_t$ streams in each subcarrier, and power is evenly distributed among the subcarriers used and the streams used. Since $N_t = 2$, each user could use 0, 1, or 2 streams for each subcarrier. An exhaustive search is conducted among all possibilities to find the optimum one for some simple cases. Each possible scheme is represented by a stream allocation matrix ($L \times K$) $\mathbf{S} = [s_{\ell k}]$, where $s_{\ell k}$ denotes the number of streams used for the kth subcarrier and user ℓ. Under the condition that $\rho_\ell = \rho$ and $\eta_{\ell j} = \eta$, due to the symmetry between users and between subcarriers, exchanging of rows or columns in $\mathbf{S}$ does not change the system ergodic mutual information $E\{MI\}$.

The optimum stream allocation matrices, for different number of users and subcarriers, are shown in Fig. 1 on an SNR-INR plane. In each case, the SNR-INR plane is divided into several regions, where each region has a corresponding optimum stream allocation matrix. For example, let us consider the simplest case where $L = 2$ and $K = 2$ (Fig. 1 (A)). The whole plane is divided into two regions. For the region above the curve, the optimum stream allocation matrix

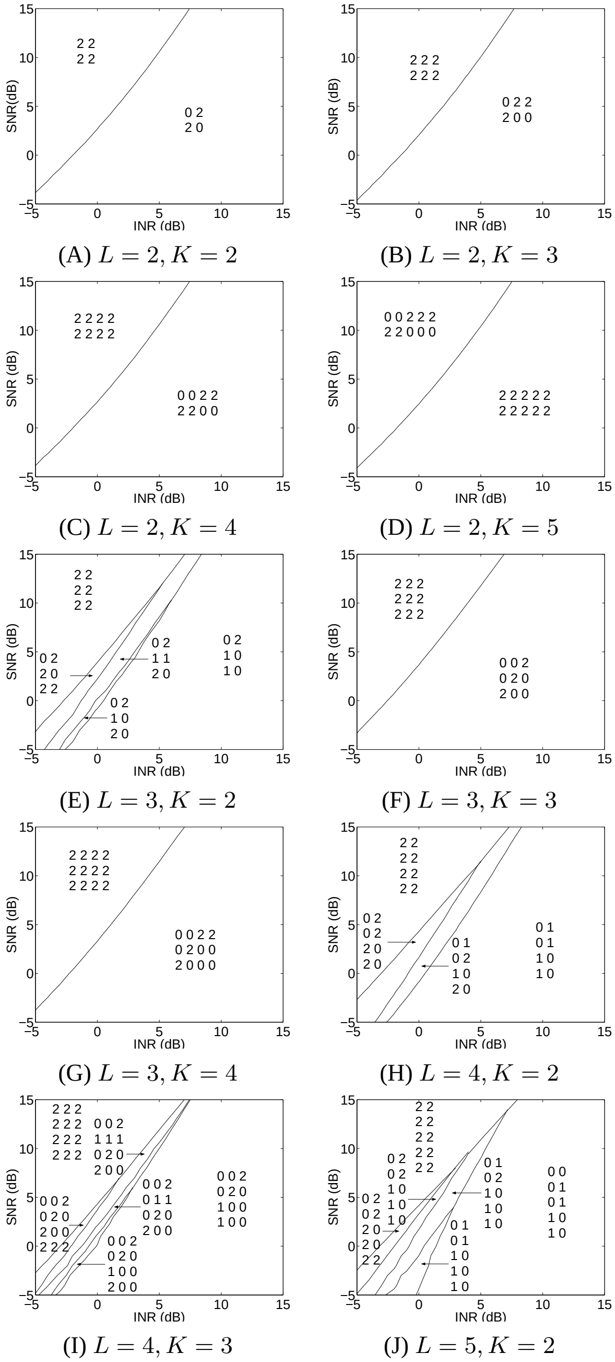

Figure 1: Optimum stream allocation matrices for different number of users (L) and subcarriers (K)

is $\mathbf{S}_1 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, which corresponds to the covariance matrices $\mathbf{Q}_1(1) = \mathbf{Q}_1(2) = \mathbf{Q}_2(1) = \mathbf{Q}_2(2) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. For the SNR and INR values which fall in the region below the curve, however, it is the best to use the stream allocation matrix $\mathbf{S}_2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, which corresponds to $\mathbf{Q}_1(1) = \mathbf{Q}_2(2) = \mathbf{0}$ and $\mathbf{Q}_1(2) = \mathbf{Q}_2(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Here the two users are using different subcarriers. To see why this occurs, note that in this case each user is actually operating under a condition without interference from the other. Therefore, when the INR becomes large, the interference will degrade the performance severely if two users operate in the same subcarrier, while the performance when using different subcarriers is not affected by the large interference.

By examining the numerical results in Fig. 1, we have the following observations and conjectures:

(1) For sufficiently strong signal or sufficiently weak interference, the optimum signaling scheme is that all users transmit the maximum number of streams for all subcarriers. This holds for any number of users and subcarriers.

(2) When $L \leq K$ (Fig. 1 (A) (B) (C) (D) (F) (G)), the plane is always divided into two regions. We conjecture that for any $L \leq K$, there are only two possible optimum schemes. When SNR and INR belong to the region above the dividing curve, it is optimum for all users to use all transmit antennas for all subcarriers; otherwise, it is optimum for different users to use different subcarriers (the number of subcarriers should be distributed to all users as evenly as possible) and maximum number of streams are employed for any active subcarrier.

(3) When $L > K$ (Fig. 1 (E) (H) (I) (J)), things are a little more complicated. A common point is: when SNR is sufficiently small or INR is sufficiently large, the optimum scheme first tries to allocate the users to subcarriers as evenly as possible. Then for any subcarrier, if there is only one user, this user should use the maximum number of streams; if there are two users or more, only two users will be active at a time and both of them are transmitting only one stream. Note that some users may be shut off completely when $L > 2K$. This seems unfair to these users, but fairness can be restored by allocating to each user the resources for an equal percentage of time. Also note that when $L > K$, there are some transition regions between the two extreme cases. In these transition regions, the optimum schemes either shut off some subcarriers for some users or decrease the number of streams in some subcarriers or both.

Therefore, for MIMO-OFDM systems with cochannel interference, it is not always best to use all transmit antennas for all subcarriers when considering overall system performance. When the signal strength is small or the interference strength is large, having users use different subcarriers as frequently as possible gives better performance. If some users are still forced to use the same subcarrier, less streams should be sent by each user or some users should even be shut off or both. This is different from what was observed in SC-MIMO [3] [4], because MIMO-OFDM has more flexibility to allocate resources due to the additional frequency dimension. As we can see, the frequency dimension plays an important role in MIMO-OFDM which is different from the spatial dimension. Finally, all these numerical results are for the case of 2 transmit and 2 receive antennas. But we expect similar phenomena will be observed for larger number of transmit and receive antennas.

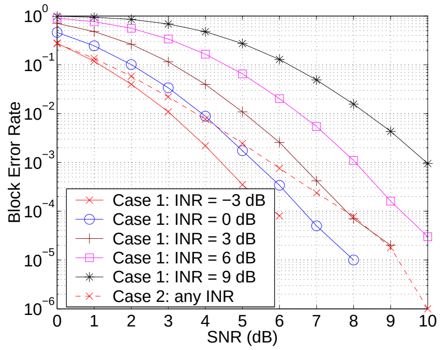

Figure 2: Block error rate for two signaling schemes.

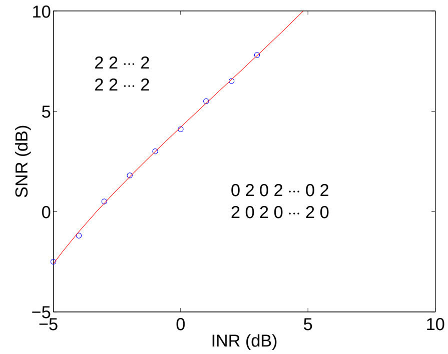

Figure 3: Signaling scheme selection for different SNRs and INRs.

5 Simulation Results

It has already been shown that, from the mutual information point of view, choosing the appropriate covariance matrices for the transmitted signals plays an important role in order to achieve high capacity. However, capacity assumes infinite constellation size and coding with infinite complexity. In this section, the simulation results of a simplified MIMO-OFDM system are provided to show these results are meaningful for finite complexity cases with simple modulation and coding.

Again, we consider a 2-user MIMO-OFDM system as modeled in Section 2, where each user has 2 transmit and 2 receive antennas. System parameters employed are as follows: A total bandwidth of 20 MHz is divided into 64 subchannels, resulting in a block length of 3.2 μs[1]. An exponential channel delay profile is assumed with rms and maximum delay of 0.2 μs and 0.8 μs, respectively.

Based on the results in Fig. 1 (A) (B) (C) (D), we consider the block error rate of the system when two different signaling schemes are used. One approach assumes both users are transmitting the maximum number of streams for all subcarriers, while the other approach assumes the two users are using different subcarriers and transmitting the maximum number of streams for the subcarriers used. In both cases, the average data rate is fixed to be 1 bit/subcarrier, and QPSK is employed for modulation. At the transmitter side, V-BLAST [7] is employed. For case 1, where $\mathbf{S}_1 = \begin{bmatrix} 2 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 \end{bmatrix}$, the 1/4-rate 16-state convolutional code with the connection polynomial of (52,56,66,76) is used for each antenna, while for case 2, where $\mathbf{S}_2 = \begin{bmatrix} 0 & 2 & 0 & 2 & \cdots & 0 & 2 \\ 2 & 0 & 2 & 0 & \cdots & 2 & 0 \end{bmatrix}$, the 1/2-rate 16-state convolutional code with the connection polynomial of (46, 72) is used. A very small interleaver was employed. At the receiver side, pre-whitening is conducted prior to Viterbi decoding, and successive interference cancellation is adopted to avoid the huge complexity of joint optimum detection. Block error rate versus SNR is shown in Fig. 2 for different INRs. As expected, case 1 (both users are transmitting the maximum number of streams) does not always give better performance than case 2 (two users are using different subcarriers). This agrees well with the capacity result in Section 4.

In Fig. 3, we divide the SNR-INR plane into two regions where one scheme performs better than the other. For SNRs and INRs between 0 and 10 dB, which are of interest in wireless communications, it is preferable for two users to use different subcarriers in most cases. Moreover, if SNR and INR can be estimated, we can use Fig. 3 to determine which scheme to choose.

6 Conclusion

We showed that for a MIMO-OFDM system, the overall system performance can be improved in some cases by forcing users to use different subcarriers or by decreasing the number of streams used for some subcarriers. This suggests the potential of an adaptive technique which determines the subcarriers to be used and the number of streams to be transmitted jointly for all users based on the interference environment.

[1] These parameters are from 802.11a wireless local area network standard. However, here we assume two users are transmitting signals all the time instead of modeling the random network traffic.

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