ML DA timing estimation for multiuser FB uplink transmission over static time dispersive channels

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ABSTRACT

We investigate the structure of a maximum likelihood timing estimator for the uplink of a filter bank (FB) based multiuser transmission system used over time dispersive channels. We compute the associated true and modified Cramér-Rao bound and show how it is related to the waveform allocation to the different users.

1 Introduction

Filter bank based multiple access (FB-MA) is an elegant way to describe the classical access methods, like F- (frequency), T- (time) and C- (code) division multiple access [1], but makes it also possible to address other waveforms such as DMT basis functions for example. We consider this formalism to represent the uplink transmission of a multiuser system where the channels are static and time-dispersive, like in powerline-based access networks for instance. The problem under consideration is that of closed-loop network timing synchronization at the level of the line termination (equivalent of the base station in mobile). Timing recovery is a critical operation in all digital communications systems [2]. In the uplink scenario, there are as many timing offsets to compensate as there are users active on the line. The timing error information has to be sent back to the user modems via the downlink.

In the present paper we design a generic data-aided (DA) timing estimator for the maximum likelihood (ML) criterion and compute the associated Cramér-Rao bound (CRB). We basically show how this bound is an extension of the single user bound [3], [4] and how it is influenced by the waveform allocation that is performed to the transmitters.

2 The FB multiple access scheme

2.1 FB modulation

We consider baseband uplink transmission of signals from K_u remote user modems towards the line termination. We suppose that a set of K_r orthogonal signature codes $s_r(m)$ are available to ensure multiple access. Mutually exclusive subsets $\mathcal{C}_k \subseteq \{1, \cdots, K_r\}$ of K_k signatures are allocated to user k with $\sum_{k=1}^{K_u} K_k = K_r$. The signal $x_k^u(t)$ transmitted by user k is given by:

$$x_k^u(t) = \sum_{r\in\mathcal{C}_k} x_r(t) = \sum_{r\in\mathcal{C}_k} \sum_{m=-\infty}^{\infty} I_r(m) g_r(t - mT) \quad (1)$$

where $x_r(t)$ is the signal that corresponds to a given signature waveform $g_r(t)$ modulated by a given stream of real symbols $I_r(m)$ produced at a baud rate $1/T$. The signature waveforms are obtained by associating the signature codes with a continuous-time shaping filter $f(t)$:

$$g_r(t) = \sum_{n=1}^{N} s_r(n)\, f(t - nT_r) \quad (2)$$

where $T_r = T/K_r$ is the chip duration. The filter $f(t)$ is supposed to be of the half root Nyquist type with bandwidth $(1+\alpha)/T_r$. The information symbols $I_r(m)$ are modelled as independent random variables with a uniform discrete distribution function corresponding to some PAM constellation, and a normalized variance σ_I^2. The size of the PAM constellations may be different on the various symbol streams, depending on the signal to noise ratio available at the receiver output.

2.2 Multiuser channel

At the output of the multiple access channel, we recover the sum of the delayed transmitted signals, filtered by the user-specific channels $c_k(t)$, and corrupted by additive noise $n(t)$ with two-sided PSD $N_0/2$:

$$\begin{aligned} r(t) &= \sum_{k=0}^{K_u-1} [x_k^u(t - \tau_k^c) \otimes c_k(t)] + n(t) \quad (3) \\ &= \sum_{m=-\infty}^{+\infty} \sum_{k=0}^{K_u-1} \sum_{r\in\mathcal{C}_k} I_r(m) h_{rk}(t - mT - \tau_k^c) + n(t) \end{aligned}$$

with $h_{rk}(t) = g_r(t) \otimes c_k(t)$ and $\otimes$ denotes convolution. The delays τ_k^c are supposed to be close to their nominal values which correspond to the desired user alignments. The delays are gathered in the $K_u \times 1$ vector $\underline{\tau}_c$. In the nominal mode of operations, obtained at modem startup, the delay vector is supposed to be $\underline{\tau}_0$. The channel timing error is denoted by $\underline{\epsilon}_c = \underline{\tau}_c - \underline{\tau}_0$.

2.3 Receiver front end and multiuser channel matrix

The received signal is filtered by means of an ideal low-pass filter with cutoff $M/2T$ and sampled at the fractional rate $1/T_s = M/T$ where $M \geq K_r(1+\alpha)$ is an integer chosen large enough in order to cover the whole bandwidth of the signal spectrum. We define M polyphase components $r_{\rho,\underline{\epsilon}_c}(m)$ with $\rho \in [0, M-1]$ as follows:

$$\begin{aligned} r_{\rho,\underline{\epsilon}_c}(m) &= r_f(mT + \rho T_s) \\ &= \sum_{m=-\infty}^{+\infty} \sum_{k=0}^{K_u-1} \sum_{r\in\mathcal{C}_k} h_{\rho r,\epsilon_k^c}(m-n) I_r(m) + n_\rho(m) \end{aligned}$$

with $r_f(t) = r(t) \otimes f(t)$. The $h_{\rho r,\epsilon_k^c}(m)$ polyphase component represents the cumulative effect of the signature waveform $g_r(t)$, the channel $c_k(t)$ and the receive filter, sampled at the symbol rate T with a phase $(\rho T_s - \tau_k^c)$. The last term $n_\rho(m)$ is the filtered noise sampled at the symbol rate with a phase ρT_s, that is to say a white gaussian noise sequence with variance $\sigma_n^2 = N_0 M/2T$.

If these equivalent channel polyphase components are limited to a length $L_h + 1 = L_{h1} + L_{h2} + 1$, we may gather the channel parameters into a $(L_h+1)M \times K_r$ matrix as follows:

$$\begin{aligned} \underline{\underline{\mathrm{H}}}_{\underline{\epsilon}_c} &= \begin{bmatrix} \underline{\underline{\mathrm{h}}}_{1,\epsilon_1^c}(-L_{h1}) & \cdots & \underline{\underline{\mathrm{h}}}_{K_u,\epsilon_{K_u}^c}(-L_{h1}) \\ \vdots & & \vdots \\ \underline{\underline{\mathrm{h}}}_{1,\epsilon_1^c}(0) & \cdots & \underline{\underline{\mathrm{h}}}_{K_u,\epsilon_{K_u}^c}(0) \\ \vdots & & \vdots \\ \underline{\underline{\mathrm{h}}}_{1,\epsilon_1^c}(L_{h2}) & \cdots & \underline{\underline{\mathrm{h}}}_{K_u,\epsilon_{K_u}^c}(L_{h2}) \end{bmatrix} \\ &= \begin{bmatrix} \underline{\underline{\mathrm{H}}}_{1,\epsilon_1^c} & \cdots & \underline{\underline{\mathrm{H}}}_{K_u,\epsilon_{K_u}^c} \end{bmatrix} \end{aligned} \tag{4}$$

where each submatrix $\underline{\underline{\mathrm{h}}}_{k,\epsilon_k^c}(m)$ has element (ρ, r) given by $h_{\rho r,\epsilon_k^c}(m)$ with $r \in \mathcal{C}_k$ and $\rho \in [1, \cdots, M]$. The matrices of channel first and second derivatives $\dot{\underline{\underline{\mathrm{H}}}}_{\underline{\epsilon}_c}$ and $\ddot{\underline{\underline{\mathrm{H}}}}_{\underline{\epsilon}_c}$, as well as the corresponding submatrices, are defined in the same fashion except that the polyphase components of $h_{rk}(t)$ are replaced by the polyphase components of $dh_{rk}(t)/dt$ and $d^2h_{rk}(t)/dt^2$, respectively.

For the sake of concision we define the shortened notation: $x(m)_B^A \triangleq \begin{bmatrix} x(m-A) & \cdots & x(m+B) \end{bmatrix}^T$. Looking at a sequence of L_r received segments and taking into account the continuous characteristic of the transmission, we get the following channel equation:

$$\underline{\mathrm{r}}_{\underline{\epsilon}_c}(m)_0^{L_r-1} = \sum_{k=1}^{K_u} \left[\underline{\underline{\mathcal{H}}}_{k,\epsilon_k^c}^{L_r}\right] \underline{\mathrm{I}}_k(m)_{L_{h1}}^{L_r+L_{h2}-1} + \underline{\mathrm{n}}(m)_0^{L_r-1} \tag{5}$$

where $\underline{\mathrm{r}}_{\underline{\epsilon}_c}(m)$ and $\underline{\mathrm{n}}(m)$ are vectors of M polyphase components, $\underline{\mathrm{I}}_k(m)$ is the vector of K_k PAM symbols transmitted by user k at time m, and $\left[\underline{\underline{\mathcal{H}}}_{k,\epsilon_k^c}^{L_r}\right]$ is a $ML_r \times K_k(L_r + L_h)$ matrix structured as follows:

$$\begin{bmatrix} \underline{\underline{\mathrm{h}}}_{k,\epsilon_k^c}(L_{h2}) & \cdots & \underline{\underline{\mathrm{h}}}_{k,\epsilon_k^c}(-L_{h1}) & & \underline{\underline{0}} \\ & \ddots & & \ddots & \\ \underline{\underline{0}} & & \underline{\underline{\mathrm{h}}}_{k,\epsilon_k^c}(L_{h2}) & \cdots & \underline{\underline{\mathrm{h}}}_{k,\epsilon_k^c}(-L_{h1}) \end{bmatrix} \tag{6}$$

This matrix is obtained by stacking L_r cyclic shifts of the first row of $M \times K_k$ blocks.

2.4 Symbol energy and mean square bandwidth

The energy of the received symbols is given by:

$$E_{rk} = \sigma_I^2 \int_{-\infty}^{+\infty} |h_{rk}(t)|^2 \, dt = \frac{\sigma_I^2}{2\pi} \int_{-\infty}^{+\infty} |H_{rk}(\omega)|^2 \, d\omega \tag{7}$$

This energy depends both on the spectral characteristics of the signature and on the effect of the considered channel on that signature. The total symbol energy received from user k is

$$E_k = \sum_{r\in\mathcal{C}_k} E_{rk} = \sigma_I^2 \frac{T}{M} \mathrm{tr}\left(\underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}^T \underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}\right) \tag{8}$$

We define the mean square bandwidth of the signal received from user k as follows:

$$\begin{aligned} W_k^2 &= \frac{\dfrac{\sigma_I^2}{2\pi} \displaystyle\int_{-\infty}^{+\infty} \sum_{r\in\mathcal{C}_k} \omega^2 |H_{rk}(\omega)|^2 \, d\omega}{\dfrac{\sigma_I^2}{2\pi} \displaystyle\int_{-\infty}^{+\infty} \sum_{r\in\mathcal{C}_k} |H_{rk}(\omega)|^2 \, d\omega} \\ &= -\frac{\mathrm{tr}\left(\ddot{\underline{\underline{\mathrm{H}}}}_{k,\epsilon_k^c}^T \underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}\right)}{\mathrm{tr}\left(\underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}^T \underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}\right)} = \frac{\mathrm{tr}\left(\dot{\underline{\underline{\mathrm{H}}}}_{k,\epsilon_k^c}^T \dot{\underline{\underline{\mathrm{H}}}}_{k,\epsilon_k^c}\right)}{\mathrm{tr}\left(\underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}^T \underline{\underline{\mathrm{H}}}_{k,\epsilon_k^c}\right)} \end{aligned} \tag{9}$$

The above quantities E_k and W_k^2 do not depend on the timing delays ϵ_k^c even if they appear in the definitions.

3 ML-DA timing estimation

3.1 Derivation of the ML-DA estimator

The log-likelihood function of the received signal is:

$$\Lambda_l\left(\underline{\mathrm{r}} \,|\, \underline{\mathrm{I}}, \underline{\epsilon}\right) = \kappa - \frac{T}{N_0 M} \left| \underline{\mathrm{r}}_{\underline{\epsilon}_c} - \sum_{k=1}^{K_u} \left[\underline{\underline{\mathcal{H}}}_{k,\epsilon_k}^{L_r}\right] \underline{\mathrm{I}}_k \right|^2 \tag{10}$$

where κ is a constant. The ML-DA estimate of the delays is the vector $\hat{\underline{\epsilon}}$ that maximizes this expression. In other words, the first derivative of the log-likelihood function with respect to each timing parameter:

$$\frac{\partial \Lambda_l \left[\underline{\mathrm{r}} \,|\, \underline{\mathrm{I}}, \underline{\epsilon}\right]}{\partial \epsilon_k} = \frac{2T}{N_0 M} \left[\underline{\mathrm{I}}_k^T \left[\underline{\underline{\dot{\mathcal{H}}}}_{k,\epsilon_k}^{L_r} \right]^T \left(\underline{\mathrm{r}}_{\underline{\epsilon}_c} - \sum_{k'=1}^{K_u} \left[\underline{\underline{\mathcal{H}}}_{k',\epsilon_{k'}}^{L_r} \right] \underline{\mathrm{I}}_{k'} \right) \right] \quad (11)$$

is zero for $\epsilon_k = \hat{\epsilon}_k$ and $k \in [1, K_u]$. This estimator is obviously unbiased as $E\left\{\partial \Lambda_l / \partial \underline{\epsilon}\right\} = 0$ for $\underline{\epsilon} = \underline{\epsilon}_c$. The timing estimation error is defined as $\underline{\Delta}_\epsilon = \hat{\underline{\epsilon}} - \underline{\epsilon}_c$.

3.2 True Cramér-Rao lower bound (CRB)

The Fisher information matrix associated with this log-likelihood function is the $K_u \times K_u$ symmetric and positive definite matrix whose elements are defined by

$$\begin{aligned} J_{kk'}(\underline{\epsilon}_c, \underline{\mathrm{I}}) &= -\mathrm{E}_n \left\{ \frac{\partial^2 \Lambda_l \left[\underline{\mathrm{r}} \,|\, \underline{\mathrm{I}}, \underline{\epsilon}\right]}{\partial \epsilon_k \partial \epsilon_{k'}} \right\}_{\underline{\epsilon} = \underline{\epsilon}_c} \quad (12) \\ &= \frac{2T}{N_0 M} \left(\underline{\mathrm{I}}_k^T \, \underline{\underline{\Phi}}_{kk'} \, \underline{\mathrm{I}}_{k'}^T \right) \end{aligned}$$

where $\underline{\underline{\Phi}}_{kk'} \triangleq \left[\underline{\underline{\dot{\mathcal{H}}}}_{k,\epsilon_k^c}^{L_r} \right]^T \left[\underline{\underline{\dot{\mathcal{H}}}}_{k',\epsilon_{k'}^c}^{L_r} \right]$ and the expectation is taken with respect to the additive noise. From the Cramér-Rao theorem, we know that

$$\mathrm{E}_n \left\{ \underline{\Delta}_\epsilon \underline{\Delta}_\epsilon^T \right\} - \underline{\underline{\mathrm{J}}}(\underline{\epsilon}_c, \underline{\mathrm{I}})^{-1} \geq \underline{\underline{0}} \quad (13)$$

where $\geq \underline{\underline{0}}$ is interpreted as meaning that the matrix is positive semidefinite. The lower bound on the timing error variance is dependent on the specific sequence of information symbols involved in the received signal segment [5]. In a tracking mode of operation, the timing parameters are continuously estimated and the lower bound on the average timing error variance becomes:

$$\mathrm{var}(\hat{\epsilon}_k) \geq \mathrm{E}_I \left\{ \left[\underline{\underline{\mathrm{J}}}(\underline{\epsilon}_c, \underline{\mathrm{I}})^{-1} \right]_{kk} \right\} \quad (14)$$

3.3 Modified Cramér-Rao lower bound

The right-hand side of (14) is not easy to compute. Some simplifications are made possible by using the law of large numbers, if the length L_r of the observation interval (and the length of the corresponding sequences of symbols) is long. In that case, the diagonal elements of the Fisher information matrix are tightly distributed around a large mean, while the non-diagonal elements are tightly distributed around a zero mean:

$$\begin{aligned} J_{kk} &\sim \mathcal{N}\left[\left(L_r \sigma_I^2 \right) m_{kk}, \left(\sqrt{L_r} \sigma_I^2 \right) s_{kk} \right] \\ J_{kk'} &\sim \mathcal{N}\left[0, \left(\sqrt{L_r} \sigma_I^2 \right) s_{kk'} \right] \quad (15) \end{aligned}$$

with

$$\begin{aligned} m_{kk} &= \frac{1}{L_r} \mathrm{tr}\left(\underline{\underline{\Phi}}_{kk} \right) \\ s_{kk}^2 &= \frac{1}{L_r} \left[\sum_i \rho_i^2 \left(\underline{\underline{\Phi}}_{kk} \right)_{ii}^2 + \sum_i \sum_{j>i} \left(2 \underline{\underline{\Phi}}_{kk} \right)_{ij}^2 \right] \\ s_{kk'}^2 &= \frac{1}{L_r} \left[\sum_i \sum_j \left(\underline{\underline{\Phi}}_{kk'} \right)_{ij}^2 \right] \quad (16) \end{aligned}$$

where $\rho_i^2 = \mathrm{E}\left\{ I^4 \right\} / \sigma_I^4 - 1 < 4/5$ depends on the PAM constellation used on the i^{th} symbol stream. The quantities defined in (16) are independent of the observation window size L_r (at least by neglecting the side-effect due to continuous transmission).

To get a tractable expression of the CRB, we have to observe two inequalities:

- The symmetric positive definite Fisher information matrix, has the property that:

$$\left[\underline{\underline{\mathrm{J}}}^{-1} \right]_{kk} \geq \frac{1}{J_{kk}} \quad \forall \, \underline{\mathrm{I}} \quad (17)$$

  Inequality (17) becomes an equality if and only if the Fisher information matrix is diagonal. In other words, considering it as an equality is equivalent to neglecting the performance degradation due to the joint estimation process. For a long observation window, the next approximation for the matrix inversion is valid:

$$\left[\underline{\underline{\mathrm{J}}}^{-1} \right]_{kk} \approx \frac{1}{J_{kk}} \left(1 + \sum_{k' \neq k} \frac{J_{kk'}^2}{J_{kk} \, J_{k'k'}} \right) \quad (18)$$

  and the average inverse of the Fisher information matrix becomes:

$$\mathrm{E}_I \left\{ \left[\underline{\underline{\mathrm{J}}}^{-1} \right]_{kk} \right\} \approx \mathrm{E}_I \left\{ \frac{1}{J_{kk}} \right\} \left(1 + \frac{1}{L_r} \sum_{k' \neq k} \frac{s_{kk'}^2}{m_{kk} m_{k'k'}} \right) \quad (19)$$

  The last factor appears as a correction term which decreases linearly with the size of the observation window.

- The application of Jensen's inequality to the convex function $f(x) = 1/x$ gives:

$$\mathrm{E}_I \left\{ \frac{1}{J_{kk}} \right\} \geq \frac{1}{\mathrm{E}_I \left\{ J_{kk} \right\}} \quad (20)$$

  As J_{kk} is concentrated near its mean, a second order Taylor expansion of its inverse can be used to provide the following approximation:

$$\mathrm{E}_I \left\{ \frac{1}{J_{kk}} \right\} \approx \frac{1}{\mathrm{E}_I \left\{ J_{kk} \right\}} \left(1 + \frac{1}{L_r} \frac{s_{kk}^2}{m_{kk}^2} \right) \quad (21)$$

The last factor appears again as a correction term which decreases linearly with the size of the observation window.

In the light of relations (17) and (20), we obtain a modified (looser) lower bound on the timing error variance:

$$\mathrm{var}(\hat{\epsilon}_k) > \left[\mathrm{E}_I \left\{\underline{\underline{\mathrm{J}}}\left(\underline{\epsilon}_c, \underline{\mathrm{I}}\right)\right\}\right]^{-1}_{kk} = \frac{1}{\mathrm{E}_I \left\{J_{kk}\right\}} \tag{22}$$

From a careful examination of the $\left[\underline{\underline{\dot{\mathcal{H}}}}^{L_r}_{k,\epsilon^c_k}\right]$ matrix structure shown in (6), we can check that

$$m_{kk} = \frac{1}{L_r}\mathrm{tr}\left(\underline{\underline{\Phi}}_{kk}\right) = \mathrm{tr}\left(\underline{\underline{\dot{\mathrm{H}}}}^T_{k,\epsilon^c_k}\, \underline{\underline{\dot{\mathrm{H}}}}_{k,\epsilon^c_k}\right) \tag{23}$$

Finally, the proposed modified bound is:

$$\mathrm{var}(\hat{\epsilon}_k) > \left[\frac{2E_k}{N_0}\right]^{-1} \frac{1}{L_r\, W_k^2}. \tag{24}$$

Notice that this modified bound is independent on the true value to be estimated $\underline{\epsilon}_c$. The proposed bound corresponds to the modified Cramér-Rao bound (MCRB) introduced for vector parameter estimation by [6]. The MCRB for a given user is the same as in the single user scenario, that is to say it only depends on the average matched filter bound (MFB) $2E_k/N_0$ and the mean square bandwidth (MSB) W_k^2 of the considered user. However, both quantities are dependent on the individual channels and the waveforms allocated to the different users. In a FB system, the waveform allocation has thus a strong impact on the multiuser timing estimator performance: it should be matched to the different channels in such a way that the average MFB-MSB product is maximized for each user. This is a matter for optimization.

3.4 Timing estimation vs. timing sensitivity

The timing estimator performance has an large impact on the symbol detection process. As a matter of fact, the variance of the timing estimator (which decreases with the MSB) has to be put in perspective with the sensitivity of the symbol detector to timing errors (which increases with the MSB)[7].

Let us consider a simple AWGN scenario with a matched filter bank receiver. The symbol energy on the waveforms allocated to user k is $E_{k0} = E_k/K_k$. The variance of the symbol error $e_r = \hat{I}_r - I_r$ at the output of the r^{th} receiver output is given by:

$$\sigma^2_{e_r}\left(\underline{\epsilon}\right) = \sigma_I^2 \left[\left(\frac{2E_{k0}}{N_0}\right)^{-1} + \frac{1}{2}\underline{\epsilon}^T \underline{\underline{\ddot{\mathrm{X}}}}_r \underline{\epsilon}\right] \tag{25}$$

where $\underline{\underline{\ddot{\mathrm{X}}}}_r$ is the multiuser timing sensitivity for waveform r, which depends on the waveform allocation. It can be shown as an extension of [7], that (i) for a paraunitary FB and (ii) when the excess bandwidth α is zero, we have:

$$\frac{1}{2}\sum_{k'=1}^{K_u}\left(\frac{E_{k0}}{E_{k'0}}\right)\left[\underline{\underline{\ddot{\mathrm{X}}}}_r\right]_{k'k'} = W_r^2 \tag{26}$$

Using (24), (25) and (26), it can be shown that for a balanced waveform allocation (same $K_k W_k^2$ product for each user), the average symbol error variance for user k is lower bounded as follows:

$$\frac{1}{K_k}\sum_{r\in\mathcal{C}_k}\mathrm{E}_{\underline{\epsilon}}\left\{\sigma^2_{e_r}\left(\underline{\epsilon}\right)\right\} > \sigma_I^2\left(\frac{2E_{k0}}{N_0}\right)^{-1}\left[1 + \frac{1}{L_r K_r}\left(\frac{K_r}{K_k}\right)\right] \tag{27}$$

$L_r K_r$ gives the length of the timing observation window (in terms of chip durations) and $K_r/K_k > 1$ gives the penalty due to the multiuser estimation process.

4 Conclusion

A fundamental lower bound on the multiuser DA timing estimation variance was computed in a filter-bank based uplink transmission system. With long symbol sequences, it was shown to be a direct extension of the single user lower bound.

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