DENOISING OF ECG SIGNALS USING WAVELET SHRINKAGE WITH TIME-FREQUENCY DEPENDANT TRESHOLD

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ABSTRACT

A method for ECG denoising based on wavelet shrinkage approach has been investigated. We have proposed a shrinkage threshold which is high for the non-informative wavelet coefficients and low for the informative coefficients. We have set up limitations for the difference between the thresholds of every two adjacent coefficients throughout the scales in order to avoid Gibbs effects.

1 INTRODUCTION

One of the most serious problems in the registration of electrocardiographic (ECG) signals is the parasite interference of muscle active potentials - electromyographic (EMG) signals. Because of EMG wide spectrum it is considered as white noise and hence overlaps the ECG spectrum. This leads to difficulties in determining the ECG signal parameters and making diagnoses.

The ECG contains pulses with different frequencies and amplitudes - high-frequency Q,R,S waves (forming QRS complex) and low-pass P and T waves (Fig.1). Their time-varying character determines ECG as highly nonstationary signal.

The noise presence problem is partially avoided by low-pass (LP) filtering of the signal. This approach improves the SNR but decreases the amplitudes of the high frequency Q, R and S waves, which can be fatal in diagnostics of some diseases.

Another approach for noise removal is based on adaptive filtering [1]. The presence of fast varying nonstationarities requires an adaptive filter with varying impulse response, which is different for each ECG zone. This complicates the calculations and retards the denoising process because of the time, needed for adaptation.

Our goal in the present study has been to suppress the parasite EMG and in the same way to preserve the parameters of the ECG. The reported algorithm is based on Wavelet Shrinkage (WS) denoising approach proposed by Donoho [2]. Our contribution to this technique is the determination of an appropriate threshold for ECG which is different for every transform coefficient depending on its scale and position.

2 WAVELET SHRINKAGE DENOISING

The denoising problem can be formulated as follows: Let have given discrete model signal-noise:

$$y = x + \epsilon, \;\; y, x, \epsilon \in R^N, N = 2^{n_0} \tag{1}$$

where the vector y represents the observed (contaminated by noise) signal, x is the information signal, and ϵ is the noise. The goal is to distinguish the information signal from the noise. In the terms of the transform methods of signal processing this means determination of a coordinate system for signal representation. The correlation between the coordinate vectors and the information signal will be maximal. On the opposite the noise will not be well correlated with the basis. As it is well known the wavelet transform (WT) is near optimal and plays the role of decorrelating transform. So it is an appropriate candidate for denoising transform.

2.1 Wavelet transform

The real dyadic wavelet transform is a decomposition of a real integrable function with a family of real orthonormal basic functions, obtained through translation and dilation of a prototype function called mother wavelet. This decomposition allows signal projections on different scales and resolutions. Key role in the WT algorithmisation play two discrete sequences $h(k)$ and $g(k)$ related with:

$$g(k) = (-1)^{1-k} h(1-k) \tag{2}$$

In signal processing literature $h(k)$ is associated with low-pass filter, whereas $g(k)$ with high-pass filter. They are called quadrature-mirror filter (QMF) pair, because of their frequency characteristics [3]. Using $h(k)$ and $g(k)$ WT is evaluated recursively:

$$v_{j,n} = \sum_k v_{j-1,n} h(k-2n);$$
$$w_{j,n} = \sum_k v_{j-1,n} g(k-2n) \tag{3}$$

Figure 1: Threshold function for scale j

3.1.2 Threshold of the coefficients describing QRS complexes - τ_{qrs}

We consider ECG as a discrete model of signal-noise mixture, according to (1) where x is the noise free ECG signal while ϵ- the noise component (EMG) - is white Gausian with unknown variance. The noise variance estimation is carried out using the wavelet coefficients in scales 1, 2 and 3 the position of which corresponds to the areas outside of QRS complexes and which are completely influenced by the noise:

$$\hat{\sigma} = \sqrt{\sum_{j=1}^{3} \sum_{i=1}^{n+1} \sum_{k=[b_{i-1}/2^j]}^{[e_i/2^j]} \frac{(w(j,k) - \overline{w})^2}{L-1}}, \qquad (5)$$

where n is the number of the QRS complexes in the signal; b_i - sample index, corresponding to the beginning of the i-th QRS complex; e_i - sample index, coresponding to the end of the same QRS complex; $[R]$ - integer part of R;

$$L = \sum_{j=1}^{3} \sum_{i=1}^{n+1} ([e_{i-1}/2^j] - [b_i/2^j]);$$

$$\overline{w} = \frac{1}{L} \sum_{j=1}^{3} \sum_{i=1}^{n+1} \sum_{k=[b_{i-1}/2^j]}^{[e_i/2^j]} w(j,k).$$

Let the information signal x and the noise ϵ with variance σ are known and let $\hat{x}(\tau, \sigma)$ denotes restored (denoised) signal with constant threshold for all wavelet coefficients. The restrored signal distortion in the interval

$$L' = \sum_{k=1}^{n} (e_k - b_k) - 1$$

inside QRS complexes can be computed as follows:

$$D(\tau, \sigma) = \sqrt{\frac{1}{L'} \sum_{i=1}^{n+1} \sum_{j=b_{i-1}}^{e_i} (x_j - \hat{x}_j(\tau, \sigma))^2} \qquad (6)$$

Figure 3: Threshold in the QRS area versus the variance of the noise component

4 RESULTS

In our experiments noise free and normalized ECG signals, sampled with F_s=200 Hz have been mixed with white noise and resulting signals have been denoised using the proposed algorithm. Table 1 shows the mean squared error between restored and original signal for six signals and three noise variances. The results are compared with the denoising results using low- pass filtering with FIR filter of 40 order and cut-off frequency 40 Hz.

	Mean squared error; WS denoising with D4 wavelet		
signal	$\sigma_{noise} = 0.1$	$\sigma_{noise} = 0.2$	$\sigma_{noise} = 0.3$
n0174	0.05	0.08	0.11
n0157	0.05	0.09	0.13
n0137	0.05	0.09	0.14
n0106	0.06	0.10	0.14
n0147	0.07	0.10	0.13
n0154	0.05	0.09	0.13
	Mean squared error; LP filtering denoising		
signal	$\sigma_{noise} = 0.1$	$\sigma_{noise} = 0.2$	$\sigma_{noise} = 0.3$
n0174	0.09	0.11	0.14
n0157	0.09	0.11	0.14
n0137	0.06	0.09	0.12
n0106	0.06	0.09	0.12
n0147	0.09	0.12	0.14
n0154	0.06	0.09	0.12

Table 1: Comparison of the results of WS denoising and LP filtering denoising

In Fig.4 we have shown a clear ECG, the result of the superposition of the same ECG with white noise with variance 0.2, the denoised signal and the difference between the clear ECG and the denoised signal.

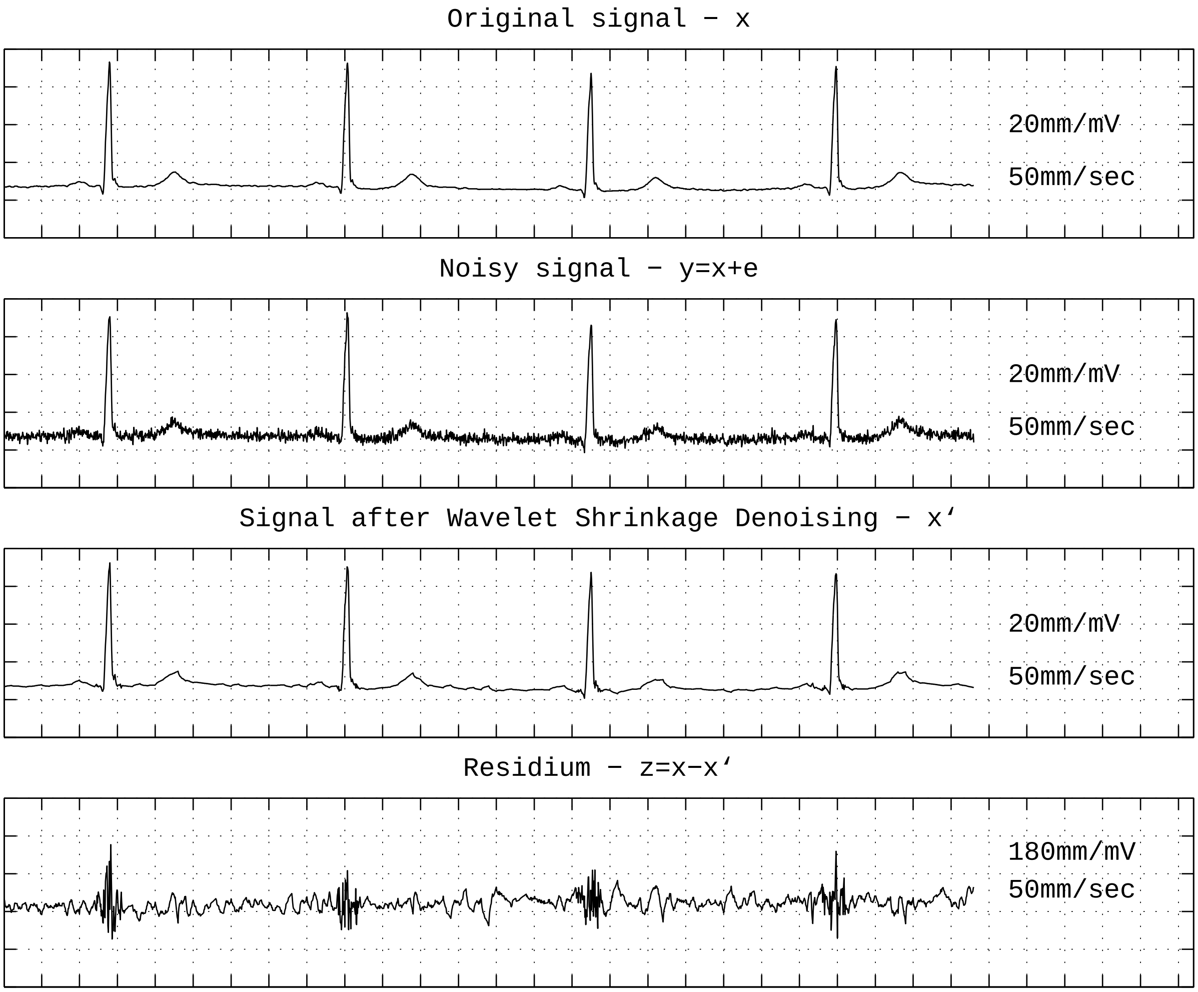

Figure 4: Denoising results

5 CONCLUSIONS

Our approach is based on suggestion, that the wavelet coefficients, representing different ECG zones, are noise influenced in different ways. This suggestion determines the choice of a threshold, which is different for the different wavelet coefficients (resp. for different ECG zones).

The results of denoising show that proposed technique allows suppression of the parasite EMG and in the same way preservation of the parameters of the ECG signals. It can be improved further by formulating non-linear dependence of $\tau_{sq}(j,k)$ as well as by investigation the dependence $\tau_{D_{min}}(\sigma)$ for different ECG leads for normal and pathological ECGs.

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