A bayesian method for GPS signals delay estimation

J. Soubielle, I. Fijalkow, P. Duvaut / J.Y. Delabbaye, A. Bibaut
ETIS / URA-CNRS 2235 Dassault Electronique
ETIS / ENSEA : 6, Avenue du Ponceau
95014 CERGY-PONTOISE Cedex, FRANCE
Tel: +30 1 30 73 66 10; fax: +30 1 30 73 66 27
e-mail: soubiell@ensea.fr

ABSTRACT

In GPS applications, positioning techniques are based on the characteristics of the pseudo-random code autocorrelation function. They don't take into account the eventuality of multipath propagation. One reflected signal may greatly disturb the measures of the usual Early-Late method. A maximum a posteriori (MAP) estimator is proposed in this study, introducing a multipath model and prior laws on both amplitude and delay parameters. This method deals with deconvolution of known functions with simple forms. Performance depends on data observation length but they greatly improve results.

1 Delay estimation

The GPS application consists in estimating the propagation duration of a coded signal between an emitter satellite and the receiver. This delay measure allows to calculate the distance from the user to the satellite, and with at least four estimates from four different emitters, the positioning can be realized [1].

1.1 No-multipath model

If we neglect Doppler effects, the model of the received signal, after demodulation, is for each satellite :

$$s(t) = ac(t-\theta) + w(t) \tag{1}$$

where:

- a: signal amplitude;
- c: Gold code (different for each satellite).
- θ: signal time delay;
- $w(t)$: white gaussian noise.

In this case, the maximum likehood (ML) estimator of θ and the argument maximizing the intercorrelation function of the code $c(t)$ match each other. GPS receivers use this equality to estimate the delay by a "geometric" method called Early-Late (EL) [1].

1.2 Early-Late method

The autocorrelation function $\gamma(t)$ of each Gold code $c_i(t)$ has the characteristic form described on the figure 1.

The sample frequency Fe (2MHz) and the chip (symbol) duration T (977.5ns) ensure the position of three values of the sampled intercorrelation between $s(t)$ and $c(t)$, on the triangular part of $\gamma(t-\theta)$. The delay estimate is then:

$$\widehat{\theta} = k_P.T_e - \alpha.R_{EL}.T_e \tag{2}$$

where:

k_P: the argument maximizing the sampled intercorrelation function and so, a first approximation of the delay θ ;

α: a constant depending on the number of chips and samples;

$R_{EL} = \frac{\gamma[k_P-1]-\gamma[k_P+1]}{\gamma[k_P-1]+\gamma[k_P+1]}$.

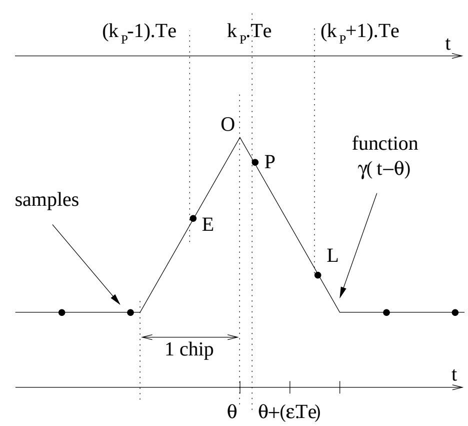

Figure 1: Autocorrelation $\gamma(t)$

2 Specular reflection

Obviously, the (EL) method is based on the validity of the model (1). Unfortunately, a problem frequently encountered in some applications like flying above the sea [2], is multipath propagation that can be represented by a two rays model.

2.1 2-paths model

We will consider in this paragraph the effects of one specular reflected traject. Then, the received signal $s(t)$ may be expressed as:

$$s(t) = a_1 c(t - \theta_1) + a_2 c(t - \theta_2) + w(t) \qquad (3)$$

where:

a_1: direct signal amplitude;
a_2: reflected signal amplitude;
θ_1: direct signal time delay;
θ_2: reflected signal time delay;
$w(t)$: white gaussian noise.

Moreover, the physical phenomenon of specular reflection guarantees the following inequations:

-attenuation: $a_2 < a_1$ (4)
-time delay: $\theta_2 > \theta_1$ (5)

2.2 Effects of multipath

The characteristics of the function $\gamma(t)$ allow a correct estimation only for delay differences $(\theta_2 - \theta_1)$ larger than 1.5 chips (fig 2). Below this value, the contribution of the reflected signal disturbs consequently the intercorrelation around the direct delay time θ_1 (fig 3) [3].

As a result, the (EL) method can't solve the positioning when the reflected signal is too close from the direct signal (fig 2).

It appears necessary to elaborate a new method that considers the presence of reflected signals in the model.

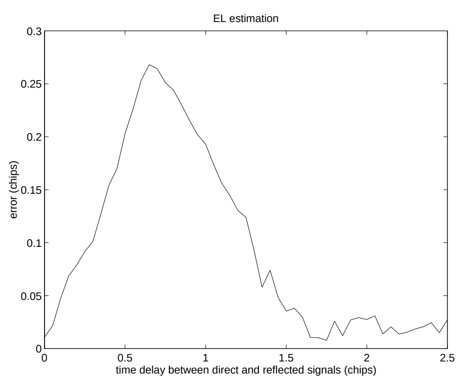

Figure 2: (EL) error vs $(\theta_2 - \theta 1)\epsilon[0, 2.5chips]$

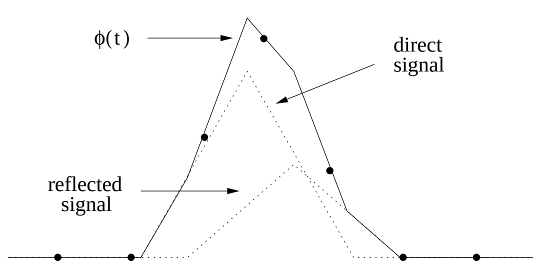

Figure 3: Intercorrelation in presence of a reflected signal: $\phi(t) = a_1\gamma(t - \theta_1) + a_2\gamma(t - \theta_2)$

3 MAP method for delay estimation

In presence of multipath propagation, the intercorrelation function may be viewed as the superposition of a finite number of elementary triangles (fig 1) with unknown parameters (a_i, θ_i).

We propose is to solve the estimation problem by a bayesian MAP approach [4] using some prior laws on the parameters thanks to (3) and (4). Even if θ_1 is the only parameter useful for GPS receiver, all are estimated.

Indeed, a MAP method could be developed on the signals directly. The benefits to still consider the autocorrelation function are to increase the signal to noise ratio and to have simple forms (triangular) to deconvolve.

3.1 Multipath model

In order to consider a more general model, we introduce a multipath model that can represent either K specular reflections or a continuous distribution of reflected energy ($K \rightarrow \infty$), see [2] [5] and fig 5. In the sequel, we will discuss the robustness of the proposed algorithm to the multipath model.

The model can be expressed as:

$$s(t) = \sum_{i=1}^{i=K} a_i c(t - \theta_i) + w(t) \qquad (6)$$

where:

a_1: direct signal amplitude;
θ_1: direct signal time delay;
a_i: i^{th} reflected signal amplitude;
θ_i: i^{th} reflected signal time delay;
$w(t)$: white gaussian noise.

And the intercorrelation $\phi(t)$:

$$\phi(t) = \sum_{i=1}^{i=K} a_i \gamma(t - \theta_i) + b(t) \qquad (7)$$

where b(t) represents the modelling error on the intercorrelation function.

3.2 Method

With this model and with vectors $\Phi = [\phi_n]$ of $\phi(t)$ samples, Θ=$[\theta_i]$ and $A = [a_i]$: we propose to estimate jointly all parameters by:

$$(\hat{\Theta}, \hat{A}, \hat{K}) = \underset{(\Theta, A, K)}{argmax} \; [p(\Theta, A, K|\Phi)] \qquad (8)$$

With the hypothesis that amplitude and delay parameters are independant, this a posteriori probability can be also written thanks the Baye's law:

$$p(\Theta, A, K|\Phi) = \frac{p(\Phi|\Theta, A, K)p(\Theta|K)p(A|K)p(K)}{p(\Phi)} \qquad (9)$$

And if we don't affect prior information to K, choosing an uniform law on the range $[1, K_{max}]$, the equation (8) is equivalent to:

$$(\hat{\Theta}, \hat{A}, \hat{K}) = \underset{(\Theta, A, K)}{argmin} \; [Q_K(\Theta, A) + H_{1,K}(A) + H_{2,K}(\Theta)] \qquad (10)$$

where:

$$\begin{cases} Q_K(\Theta, A) &= -ln[p(\Phi|\Theta, A, K)] \\ H_{1,K}(A) &= -ln[p(A|K)] \\ H_{2,K}(\Theta) &= -ln[p(\Theta|K)] \end{cases} \quad (11)$$

and because of lack of knowledge on b(t), we consider the Least Square error (known to be optimal in the white gaussian case):

$$Q_K(\Theta, A) = \frac{1}{2\sigma^2} \sum_n \left| \phi_n - \sum_{i=1}^{i=K} a_i \gamma(nT_e - \theta_i) \right|^2 \quad (12)$$

Introducing the cost function $J_k(\Theta, A)$ (13) and separating the estimation of discret value parameter K and value parameters (Θ, A), the estimation is realized in (K_{max}+1) steps (14):

$$J_k(\Theta, A) = Q_{K=k}(\Theta, A) + H_{1,K=k}(A) + H_{2,K=k}(\Theta) \quad (13)$$

$$\begin{cases} a) \quad \text{for } k \in [1, K_{max}]: \\ \qquad (\hat{\Theta}_k, \hat{A}_k) = \underset{(\Theta, A)}{argmin} \quad [J_k(\Theta, A)] \\ b) \qquad \hat{K} = \underset{k}{argmin} \quad [J_k(\hat{\Theta}_k, \hat{A}_k)] \end{cases} \quad (14)$$

Finally, the solution to (8) is the triplet ($\hat{\Theta}_{\hat{K}}$,$\hat{A}_{\hat{K}}$,$\hat{K}$).

3.3 Prior laws

The parameters (a_i) are supposed to be respectively mutually independent and to follow a Gamma law:

$$p(A|K) = \prod_{i=1}^{K} p(a_i) \quad (15)$$

$$p(a_i) = \frac{1}{\Gamma(\alpha)} a_i^{\alpha-1} exp^{-a_i} \quad (16)$$

The same hypotheses are chosen for parameters (θ_i-θ_{i-1}) for ($i > 2$). For θ_1, which is the time delay of direct signal, we choose an uniform density of probability.

$$p(\Theta|K) = p(\theta_1) \prod_{i=2}^{K} p(\theta_i - \theta_{i-1}) \propto \prod_{i=2}^{K} p(\theta_i - \theta_{i-1}) \quad (17)$$

$$p(\theta_i - \theta_{i-1}) = \frac{1}{\Gamma(\beta)} (\frac{\theta_i - \theta_{i-1}}{T})^{\beta-1} exp^{-(\frac{\theta_i - \theta_{i-1}}{T})} \quad (18)$$

Since we don't mind reflected signals whose delays are more than 1.5 chips later than the direct signal, β has been chosen to favor small differences between the θ_i (figure 4.b). On the contrary, α authorize more variations around the value 1 (figure 4.a).

As a result, the expressions of the functions $H_{1,k}(\Theta)$ and $H_{2,k}(A)$ are:

$$H_{1,k}(A) = (1-\alpha) \sum_{i=1}^{k} ln[a_i] + \sum_{i=1}^{k} a_i + k \ ln[\Gamma(\alpha)] \quad (19)$$

$$\begin{cases} \text{if } (k \geq 2): & H_{2,k}(\Theta) = (1-\beta) \sum_{i=2}^{k} ln[\frac{(\theta_i - \theta_{i-1})}{T}] \\ & \qquad + \sum_{i=2}^{k} \frac{(\theta_i - \theta_{i-1})}{T} + k \ ln[\Gamma(\beta)] \\ \text{if } (k = 1): & H_{2,1}(\Theta) = ln[\Gamma(\beta)] \end{cases} \quad (20)$$

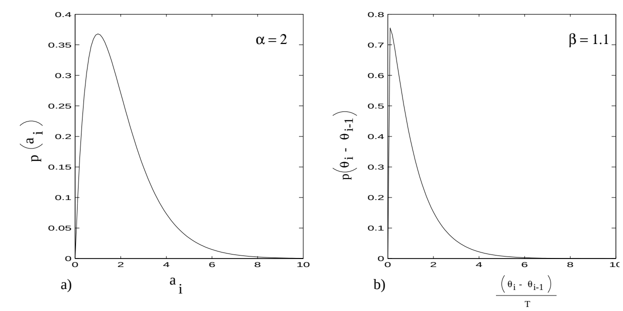

Figure 4: prior laws

4 Performances

4.1 Simulation settings

The signal $s(t)$ was generated as shown on equation (6) with parameters θ_i and a_i determined respectively by an uniform and an exponential laws such that the power distribution versus delay time match different experimental results [5] [2] which have been realized.

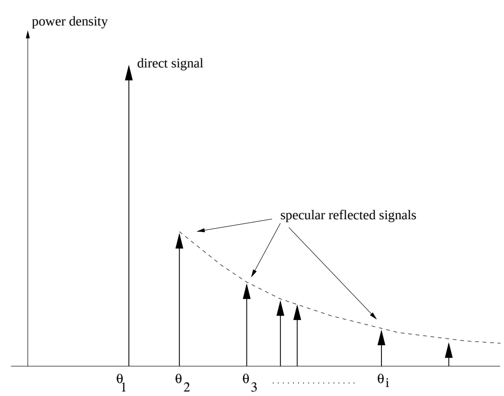

Figure 5: example of delays and amplitudes distribution

All delays are on a range of 1.5 chips (2.2). The entire power of reflected signals is between $[-4dB, -6dB]$ lower the direct signal power.

Finally, parameters (θ_i,$a_{i=1}$) are taken constant during all the observation (n_p periods of code) whereas ($a_{i>1}$) only during 1ms (code period).

Whatever the observation duration (n_p ms) of signal $s(t)$ ($Fe = 2MHz$) is used, only the N ($N = 2000$) first samples of intercorrelation $y(t)$ are estimated by the algorithm. But the interest to consider a larger observation is to increase the signal to noise ratio (SNR).

4.2 2-paths model

The MAP method has been tested in presence of one specular reflection.

Figures 6.a) and 6.b) show the improvement using the maximum a posteriori method (MAP) compared to Early-Late Method (EL) in presence of only one reflected signal.Both methods are compared to the Cramer-Rao bound (CR) (6.b) despite the EL and MAP estimators are biased. Moreover, this CR bound has been calculated with known amplitudes.

The MAP algorithm turns out to be more efficient than the EL estimation not only in terms of variance but also for the bias which decreases by a factor 4.

Finally, the algorithm always detects, even for low SNR, the right number of reflected signals.

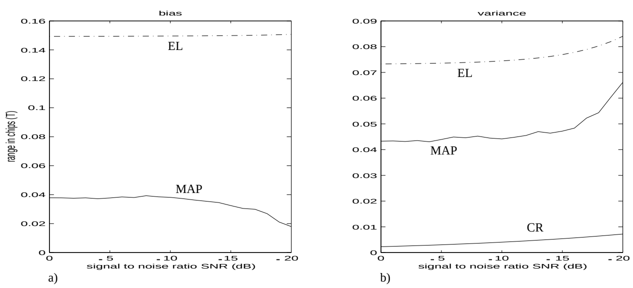

Figure 6: Bias and variance of EL and MAP (specular reflection, $n_p = 10$)

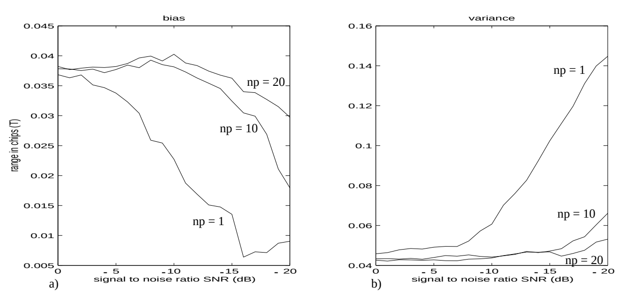

Figure 7: MAP bias and variance for different estimation duration (n_p ms)

It can be noticed the surprising effect of bias increases with n_p (fig 7.a) whereas as expected the variance decreases(fig 7.b). But the maximum value of bias (0.04 chips) is still very low compared to figure 2.

Nevertheless, these figures show that it isn't worth increasing too much the observation duration (n_p ms) : performances are quite identical for 10 and 20 ms.

4.3 Multipath model

Figures 8.a and 8.b were generated with delays and amplitudes distribution of figure 5 with $K = 15$ but the algorithm minimum was found for $k = 2$ (direct path + one specular reflection). In this case, the results of the estimator $\hat{\theta}_1$ prove that the MAP estimator is equivalent to EL method in terms of variance (which isn't so high). But, in terms of bias, MAP algorithm give a result five times better rather than the EL estimator.

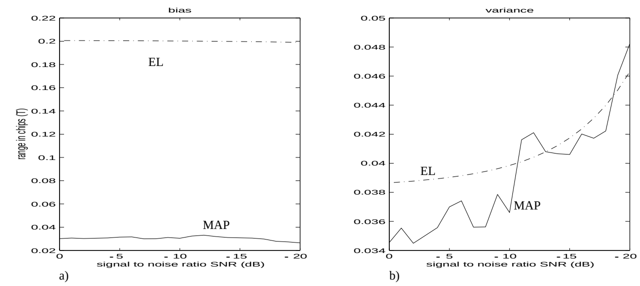

Figure 8: Approximation of multipath by a 2-paths model

5 Conclusion

We have shown that the MAP method can realize better positioning with respect to the EL algorithm in presence of specular reflection. Such improvements could have been done thanks to a priori laws given to delays and amplitudes of reflected signals whose characterization is nowadays the subject of numbered studies.

Moreover, it would be difficult to detect all reflected signals in the case of multipath propagation and to estimate all parameters. As we demonstrated, the algorithm is still robust to the presence of modelling error.

References

[1] Elliot D. Kaplan. *Understanding GPS : principles and applications.* Artech House Publishers, 1996.

[2] J-C. Auber, A. Bibaut, S. Morlet, and P. Seure. Analyse théorique et expérimentale des trajets multiples sur mer en réception GPS . IFN, 1995.

[3] J-C. Auber, A. Bibaut, S. Morlet, and P. Seure. How multipath adversely affects an airborne GPS receiver and how to deal with it. DSNS, 1996.

[4] Ali Mohammad-Djafari. Une méthode bayésienne pour la localisation et la séparation de sources de formes connues. GRETSI, 1997.

[5] Delphine Doris-Blais. Modélisation de récepteur GPS . Institut National Polytechnique de Toulouse, thèse 1997.