TOWARD THE AUTOMATIC SYNTHESIS OF NONLINEAR WAVE DIGITAL MODELS FOR MUSICAL ACOUSTICS

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ABSTRACT

In this paper we propose and describe the principles behind our approach to sound synthesis through nonlinear wave digital modeling. The method is general enough to include a wide variety of nonlinearities that cannot be modeled through classical WDF principles. We also present an automatic synthesis method that, starting from a semantic description of the physical model, generates, validates and initializes an appropriate simulation source code with time-varying parameters.

1 Introduction

The Wave Digital Filter (WDF) theory [1] has reached, over the past two decades, an advanced level of maturity. In fact, a large variety of WDF-based filtering techniques has been developed for a wide range of applications. More recently, however, we witnessed an ever more growing interest in wave digital filters as the research on sound synthesis for acoustic and musical applications turned toward physical modeling [2].

One major problem of physical model synthesis is the treatment of nonlinearities, which are predominant in musical acoustics. They are, in fact, the main responsible of the timbral dynamics of the instruments, therefore they cannot be modeled through simple linearization. Among the numerous solutions available in the literature for implementing nonlinear Wave Digital (WD) structures, the only ones that do not give rise to computability problems are those that directly map Kirchhoff variables (voltage-like and current-like variables) into the WD domain [4] (valid for nonlinear resistors) and the approach based on special two-port adaptors with memory [5, 6] which transform nonlinear dynamic bipoles into instantaneous ones. In this last case, new waves are defined so that the description of the nonlinear element becomes memoryless and the results already formulated for nonlinear resistors [4] can be applied.

A block-based musician-oriented sound synthesis approach based on digital waves requires a great deal of design flexibility. In fact, besides being able to model a wide variety of nonlinear elements, it needs to be capable of handling interfacing problems with pre-existing models obtained through direct measurement of the response of a resonator or designed through different methods. In particular, it would be highly desirable to guarantee the compatibility with Digital WaveGuides [3] (DWG), which are the distributed-parameter counterpart of the ("lumped") WDFs and, therefore, they are particularly suitable for modeling acoustic resonating structures. Finally, it needs to be able to withstand temporal variations of the parameters and needs to be made as automatic as possible.

In this paper we propose and describe the principles behind our approach to sound synthesis through nonlinear wave digital modeling of acoustic instruments. We present an automatic synthesis method that generates, validates and initializes an appropriate WD-based simulation source code, starting from a semantic description of the reference physical model, and is able to incorporate time-varying parameters. The method is general enough to include a wide variety of nonlinearities that cannot be modeled through classical WDF principles.

2 Global Approach

As we know, the WDF approach [1] to digital modeling is a "local" one, in the sense that it puts together a collection of individually discretized elements (resistors, capacitors, inductors, generators, etc.) through a network of instantaneous adaptors, which implements the global constraints (Kirchhoff laws) and specifies the interconnection topology. Our approach to physical modeling is similar, in the sense that it is also based on the interconnection of individually discretized blocks (see Fig. 1), but the class of systems that can play the role of a *block* is wider than in the traditional WDF case, while the interconnection between these macro-blocks is performed through a network of adaptors (oval block in Fig. 1) that are no longer memoryless.

2.1 Adaptation and Instantaneous Decoupling

The class of digital waves that we use for modeling a "port" in the WD domain is of the form

$$\begin{aligned} A(z) &= V(z) + R(z)I(z) \\ B(z) &= V(z) - R(z)I(z) \quad , \end{aligned}$$

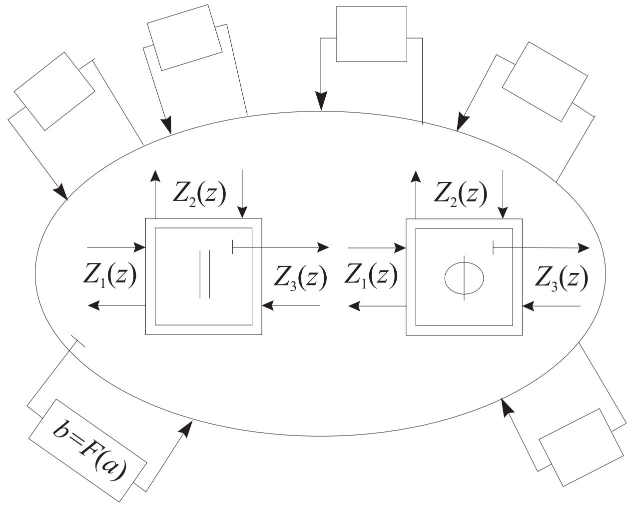

Figure 1: General synthesis scheme of a WD structure: blocks are either filters or instantaneous nonlinearities; adaptors are dynamic.

where $R(z)$ is a reference transfer function [5]. With this choice, the class of nonlinearities that can be modeled in the WD domain is, in fact, that of all algebraic bipoles of the form

$$p = g\,(q)\ ,\quad P(z) = H_v(z)V(z)\ ,\quad Q(z) = H_i(z)I(z)\quad ,$$

where p and q are related to v and i, respectively, through a finite difference equation. The above choice of digital waves allows us to model a wide class of nonlinear dynamical elements, such as nonlinear reactances or, more generally, linear circuits containing a *lumped* nonlinearity. The *memory* of the nonlinear element is, in fact, incorporated in the interconnection block. The adaptors of the interconnection block, in fact, cannot be memoryless as they are characterized by reflection filters instead of reflection coefficients. Macro-blocks can thus be either filters or instantaneous nonlinearities.

Even with this more general definition of the digital waves, we can define adaptation conditions for any linear bipole by selecting the reference transfer function in such a way as to eliminate the instantaneous input/output connection in its WD implementation (instantaneous adaptation). An "adapted" bipole will thus be modeled in the WD domain as $B(z) = z^{-1}K(z)A(z)$, where the delayed reflection filter $K(z)$ can also be identically zero.

In general, the possibility of "extracting" a delay element from an adapted bipole does not hold true for an adapted port of a multi-port element. In fact, the port adaptation of a multi-port element only implies that no *local* instantaneous reflection can occur, while nothing can be said about instantaneous reflections through outer paths. If it is true that a delay can actually be extracted from a port, then we will talk about *decoupling*, in order to distinguish this concept from that of port adaptation. Only in the case of the bipole is a decoupled port also an adapted port.

The concept of *decoupling* is important as it allows us to break down large WD structures into smaller substructures that can be synthesized and initialized individually. If N portions of a WD structure that are connected together through a decoupling N-port block ($N \geq 2$), which is a multi-port element that exhibits at least $N-1$ decoupling ports, then such portions are said to be decoupled, as they do not instantaneously interact with each other. Such structures can, quite clearly, be synthesized and initialized individually. The concept of *decoupling* is also important as it allows us to model WD structures containing more than one nonlinearity. We know, in fact, that only one of all the ports of the interconnection block (oval block of Fig. 1) can be adapted, therefore only one nonlinearity can be connected to it. Through decoupling N-port blocks, however, we can connect together N interconnection blocks, each of which is allowed one nonlinear element.

Decoupling blocks are quite frequent in musical acoustics. In fact, reverberating structures are often implemented as a network of delay lines (digital waveguides), which are decoupling two-port elements. We will later see that the WD structure of an acoustic piano is based on this decoupling principle. In this case, in fact, a number of hammers are connected, through a waveguide digital model of a string, to the same (decoupling) resonating structure (soundboard).

In conclusion, the global structure of a WD implementation of a physical model can be seen as a number of decoupled interconnection blocks such as those of Fig. 1, whose aim is to connect together either linear macro-blocks or instantaneous nonlinear blocks. The presence of decoupling ports, allows us to approach the synthesis/initialization problem in a block-wise fashion. For example, if an interconnection block is connected to a set of adapted macro-blocks of the form $B(z) = z^{-1}K(z)A(z)$, then we can separate the synthesis/initialization of the macro-blocks of the form $K(z)$ from that of the interconnection block. A similar reasoning holds for two decoupled portions of the global WD structure.

2.2 Synthesis of the Interconnection Block

The synthesis of the interconnection block can be approached in an iterative automatic fashion by following a *tableau*-based method, specifically designed for WD structures. Let us first consider the dynamic adaptor of Fig. 2, characterized by the three reference transfer functions $Z_1(z)$, $Z_2(z)$ and $Z_3(z)$. Let $\mathbf{C}(z) = \begin{bmatrix} A_1 & A_2 & A_3 & B_1 & B_2 & B_3 \end{bmatrix}^T$ be the vector of the Z-transforms of the input and output signals of the adaptor. The Kirchhoff constraints associated to this adaptor can readily be expressed in the form

$\mathbf{S}(z)\mathbf{C}(z) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, where

$$\mathbf{S}(z) = \mathbf{S}^{(0)} + \sum_{n=1}^{N} z^{-n}\mathbf{S}^{(n)} \quad , \qquad (1)$$

is the 6×3 matrix that characterizes the adaptor in the Z-transform domain. Decomposing $\mathbf{S}(z)$ as in eq. (1), corresponds to extracting the memory from the adaptor, as shown in Fig. 3, which allows us to automatize the generation of the the state-update matrix of the adaptor and simplifies the initialization problem.

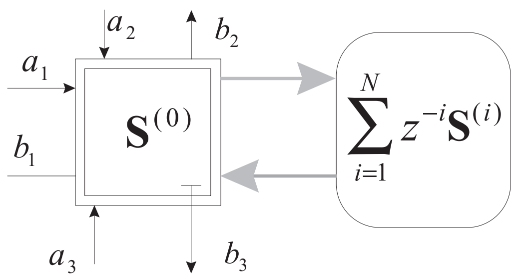

Figure 2: Memory extraction from a dynamic adaptor.

An interconnection block, in turn, can be seen as a more complex type of dynamic adaptor. In order to derive its state-update equations, we proceed iteratively by first block-wise constructing the matrix $\mathbf{S}(z)$, then by decomposing it like in eq. (1) and, finally, deriving the global state-update matrix.

2.3 Initialization

Unlike traditional wave digital filters, the interconnection block of our WD structures needs to be properly initialized as all the memories need be assigned an initial value. This operation can become critical for WD models of mechanical systems because initial conditions of mechanical systems usually concern the reciprocal position and contact conditions of mechanical elements.

The above procedure for determining the state-update equation can be seen as a direct form of the synthesis problem, in the sense that output signals are computed from input signals and memory content. The initialization, on the other hand, can be seen as an *inverse* problem, as the memory content must be derived from output signals and input signals.

Although the global WD structure is nonlinear, the nonlinearity is "lumped", which greatly simplifies the initialization problem. In fact, this operation can be performed through the computation of a fixed-point (nonlinearity inversion), plus a matrix inversion.

2.4 Time-Varying Models

Changing any model parameters in a WD structure requires all the other parameters to be updated as well. In fact, all parameters are bound to satisfy some global adaptation conditions. Temporal variations of the nonlinearities are dealt with by employing special WD two-port elements that are able to perform a variety of transformation on the nonlinear characteristics (non-homogeneous scaling, rotation, etc.). Time-varying port/block impedance changes, on the other hand, are dealt with through a global re-computation of all model parameters on the behalf of a process that works in parallel with the simulator. This operation requires the re-mapping of the nonlinearities as well. It is important to remember that large parameter changes must be carefully dealt with in order not to affect the global energy in an uncontrollable fashion.

3 Implementation

As already said above, the synthesis of the whole WD structure can be broken down into the individual construction of the linear macro-blocks, of the nonlinear maps and of the interconnection blocks. Some methods are already available for synthesizing linear macro-blocks [3], therefore the automatic synthesis procedure is based on the assumption that such elements are already available in the form of a collection of pre-synthesized structures. In its current state, the system that we developed is able to automatically construct WD structures based on standard WDF adaptors and new dynamic adaptors chosen from a reasonably wide family. Currently, this family includes WD mutators [5] and other types of adaptors developed for modeling some typical nonlinear elements of the classical nonlinear circuit theory (both resistive and reactive). The available linear macro-blocks belong to the family of the DWGs [3], while the nonlinear maps are currently pointwise described in the Kirchhoff domain and then automatically converted in a piecewise continuous WD map. The parameters can be modified "on the fly" in order to make the structure time-varying. A parallel process deals with the problem of re-computation of all WD parameters, depending on their changes expressed in the Kirchhoff domain.

4 An Example of Application

As an example of application of the above procedure, a model of the hammer-string interaction in an acoustic piano is presented. The implementation, entirely built with WDF and WDG elements, includes a mechanical model of the hammer and its interaction with a physical model of a string (which includes stiffness and distributed losses) connected to a soundboard. The physical model of the piano and its WD implementation are shown in Fig. 3. Notice that the model of the string acts as a decoupling element, and this allows the soundboard to be connected to several WD hammers. Starting from an appropriate semantic description of the building blocks of the system and their topology of interconnection and interaction with the user, the system automatically built simulation source code (in C language), which produced the results shown in Fig. 4.

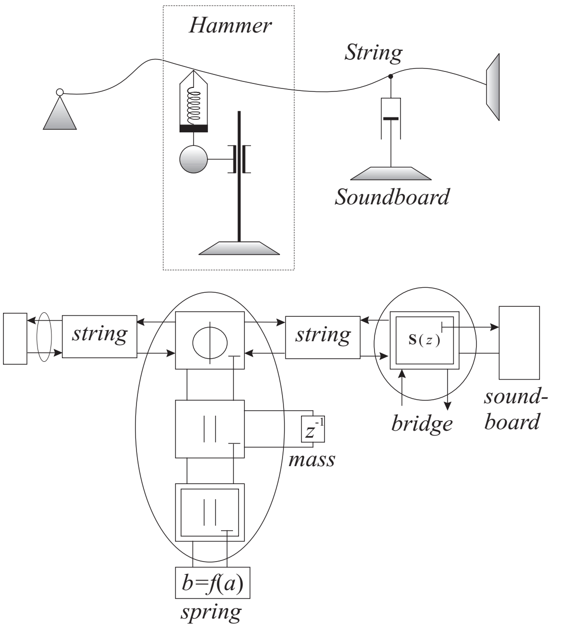

Figure 3: Physical model of a piano and its WD implementation.

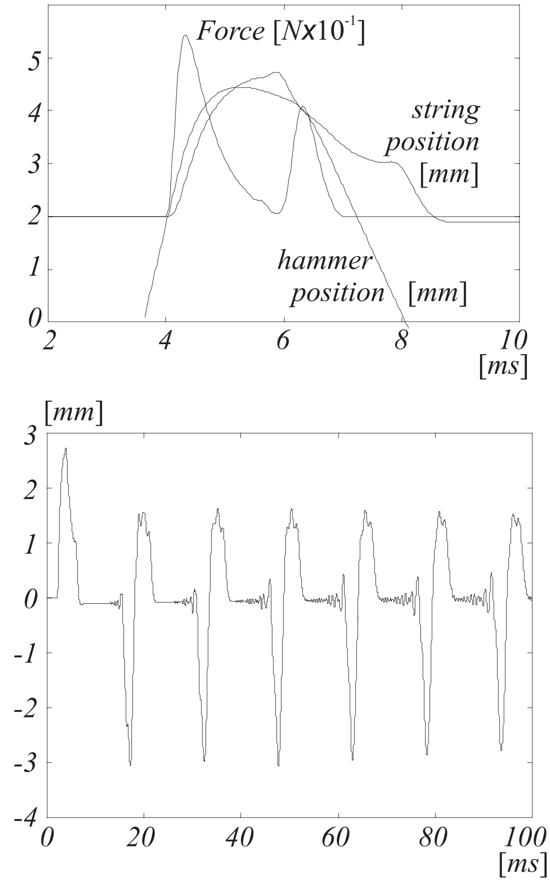

Figure 4: Simulation results for the WD model of the piano interaction.

5 Conclusions

In this paper we illustrated our approach to the problem of the automatic synthesis of nonlinear wave digital structures for physical modeling of in musical acoustics. In particular, we illustrated our block-based approach based on the local synthesis of instantaneously decoupled substructures, and we showed how to separate the synthesis of the individual macro-blocks from that of the topology of interconnection. We also proposed a method for automatically initializing a WD structure, starting from the initial conditions of the physical model. Our implementation proved effective for the automatic and modular synthesis of a wide class of physical structures encountered in musical acoustics.

We are currently working on the synthesis of structures containing multi-port non-decouplable nonlinearities.

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