Blind identification of sparse multipath channels using cyclostationary statistics

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ABSTRACT

Blind identification of a wireless communication channel is an important issue in communication system design. Most existing blind system identification techniques process the unknown information of the system from its output only. However, in many practical situation partial knowledge of the system transfer function is available. By relying on this known information, the performance of channel identification and equalization can be significantly enhanced. In this paper, we introduce a new system identification technique that exploits both the a priori knowledge of the pulse shape filter and the multipath channel propagation model. The approach consists first in processing the cyclo-spectrum of the system output that is shown to be superimposed exponential function of the channel propagation delays and attenuations. Then, the frequency parameters, i.e., channel propagation parameters, are later estimated using the Matrix Pencils (MP) frequency estimation method [7].

1 Introduction

Blind system identification (BSI) is important in applications such as data transmission, reverberation cancellation, seismic deconvolution, and image debluring. Recently, there is a strong interest in the identification problem where the available data can be modeled as outputs of multiple parallel channels which are driven by an unknown input. This problem is largely motivated by blind channel identification and equalization for mobile data communication. In fact, this multichannel problem arises when the output of a data communication channel is sampled at a rate higher than the baud rate or when radio waves coming from the transmitter are sampled by multiple spatially distributed sensors at the receiver [4]. Using fractionally sampled data for blind channel equalization appears to be a new research trend since Sato's work [3]. Since the works by Tong et al [1] and Gardner [2], the world has seen a great research interest in blind identification of single-input multiple-output (SIMO) system using second order statistics (SOS) based methods [5, 6]. The SOS based approach is attractive because it requires much less samples than the traditional higher order statistic (HOS) based approach.

However, most of the SOS based BSI methods are designed to identify the entire discrete channel including the pulse-shaping and receiver filters. In practice, the overall channel is not completely unknown. It is typical that the only unknown part of the channel is the multipath [10, 11]. By using this a priori information, the number of unknown channel parameters is significantly reduced leading to significant performance improvement, especially for the sparse multipath channel [9, 11].

In [11], a modal analysis method for blind system identification has been presented. This method estimates the unknown channel through the estimation of the propagation parameters (i.e., delays and attenuations) using the power spectrum of the observation. We propose here an attractive alternative method that improves on the method in [11] by exploiting more information (i.e., cyclo-spectra) of the observed signal. In the proposed method, we exploit also the particular multipath propagation model to cast the blind channel identification problem into a superimposed exponential parameter estimation problem.

2 Problem Formulation

The continuous-time output from a linear, time-invariant, baseband communication channel driven by PAM/QAM sequence is given by:

$$x(t) = \sum_{l=-\infty}^{\infty} h(t-lT)u(lT) + w(t) \qquad (1)$$

$$u(lT) = \sum_{k} s_k \delta((l-k)T) \qquad (2)$$

where $x(t)$ is the channel output process, $h(t)$ is the channel impulse response, $\{s_k\}$ the sequence of information symbols, $1/T$ the symbol transmission rate, and $w(t)$ an additive noise process. The following conditions are assumed in the paper:

A1. The symbol interval T is exactly known.

A2. $\{s_k\}$ is zero mean, and $E(s_k s_l^*) = \delta(k-l)$, where

$\delta(t)$ is the discrete-time Dirac function.

A3. $w(t)$ is zero mean, white, and uncorrelated with the source signal $\{s_k\}$.

A4. $h(t)$ is the global impulse response that is given by

$$h(t) = p(t) * g(t)$$

where

$$g(t) = \sum_{i=0}^{M-1} \lambda_i \delta(t - \tau_i)$$

is the unknown multipath propagation channel filter and $p(t)$ is the known pulse shape and receipt filter, i.e., $p(t) = p_t(t) * p_r(t)$ where p_t and p_r are the transmit (pulse shape) and receipt filters[1], respectively.

The objective here is to estimate, under the above assumptions, the multipath propagation channel through the estimation of the delay and attenuation parameters $\{\tau_i, \lambda_i\}$.

3 An SOS-Based Method

In this section, we first recall the principle of the matrix pencil (MP) method as shown in [7]. Then, we show how we can apply the MP method to estimate the multipath channel parameters by using the second order cyclo-spectra of the system output.

3.1 Matrix Pencil Method

Consider a noise free exponential data sequence which can be described by:

$$x(t) = \sum_{i=0}^{M-1} a_i e^{jf_i t}, \qquad 0 \le t \le N-1$$

where a_i and f_i are the amplitude and frequency of the i-th sinusoids, respectively (with $f_i \neq f_j$ for $i \neq j$). The matrix pencil approach relies on the following model inherent in the exponential signals [7]: Define

$$\begin{aligned} \mathbf{x}(t) &= [x(t), \cdots, x(N-L+t-1)]^T \\ \mathbf{X}_0 &= [\mathbf{x}(L-1), \cdots, \mathbf{x}(0)] \\ \mathbf{X}_1 &= [\mathbf{x}(L), \cdots, \mathbf{x}(1)] \end{aligned}$$

where "T'' denotes the transpose and L is a chosen parameter called the pencil parameter (it satisfies $M \le L \le N-M$). It is shown in [7] that

$$\begin{bmatrix} \mathbf{X}_0 \\ \mathbf{X}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_L\mathbf{A} \\ \mathbf{Z}_L\mathbf{A}\mathbf{Z} \end{bmatrix} \mathbf{Z}_R \tag{3}$$

where $\mathbf{Z}_L$ and $\mathbf{Z}_R$ are full rank matrices, $\mathbf{A} = \mathrm{diag}(a_0, \cdots, a_{M-1})$, and $\mathbf{Z} = \mathrm{diag}(e^{jf_0}, \cdots, e^{jf_{M-1}})$. From (3), we can see that e^{jf_i}, $i = 0, \cdots, M-1$ are the rank reducing number of the matrix pencil $\mathbf{X}_1 - z\mathbf{X}_0$. To estimate the frequency parameter (i.e., $\mathbf{Z}$), we can proceed as follows:

- Estimate $\mathbf{E}$ the matrix given by the M [2] dominant singular (left) eigenvectors of $\mathbf{X} \stackrel{\text{def}}{=} [\mathbf{X}_0^T \ \mathbf{X}_1^T]^T$. We have

$$\mathrm{range}(\mathbf{E}) = \mathrm{range}(\mathbf{X}) = \mathrm{range}(\begin{bmatrix} \mathbf{Z}_L\mathbf{A} \\ \mathbf{Z}_L\mathbf{A}\mathbf{Z} \end{bmatrix})$$

  and thus, there exists a non-singular matrix $\mathbf{T}$ satisfying

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_0 \\ \mathbf{E}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_L\mathbf{A} \\ \mathbf{Z}_L\mathbf{A}\mathbf{Z} \end{bmatrix} \mathbf{T}$$

- Estimate $e^{jf_0}, \cdots, e^{jf_{M-1}}$ as the eigenvalues of

$$\mathbf{U} = \mathbf{E}_0^{\#}\mathbf{E}_1 = \mathbf{T}^{-1}\mathbf{Z}\mathbf{T}$$

  where $\mathbf{E}_0^{\#}$ denotes the pseudo-inverse of $\mathbf{E}_0$. Then, estimate the amplitude parameters using a least-squares fitting approach.

3.2 Cyclo-spectra of the system output

The actual channel output $x(t)$ as in (1) is cyclostationary instead of stationary. It can be verified that for a stationary channel input s_k (assumption A2) and stationary noise $w(t)$ (assumption A3) we have

$$\begin{aligned} R_x(t_1, t_2) &\stackrel{\text{def}}{=} E(x(t_1)x^*(t_2)) \\ &= \sum_k h(t_1 - kT)h^*(t_2 - kT) + R_w(t_1 - t_2) \\ &= R_x(t_1 + T, t_2 + T) \end{aligned}$$

where $R_w(t_1 - t_2) = \sigma^2\delta(t_1 - t_2)$ is the covariance function of the noise. For simplicity, we will assume that σ^2 is known or a priori estimated. As we can see, $x(t)$ is a second order cyclostationary process with fundamental period T. Its cyclic autocorrelation function is defined by:

$$\begin{aligned} R_x^{\alpha_k}(\tau) &\stackrel{\text{def}}{=} \frac{1}{T}\int_{-T/2}^{T/2} R_x(t, t-\tau)e^{-j2\pi\alpha_k t}dt \qquad (4) \\ &= \frac{1}{T}\int_{-\infty}^{\infty} h(t)h^*(t-\tau)e^{-j2\pi\alpha_k t}dt + \frac{\sigma^2}{T}\delta(k)\delta(\tau) \end{aligned}$$

where $\alpha_k = k/T$ for a given integer k. Correspondingly, the second order cyclo-spectrum (also known as spectral correlation density) is given by

$$\begin{aligned} S_x^{\alpha_k}(\omega) &\stackrel{\text{def}}{=} \int_{-\infty}^{\infty} R_x^{\alpha_k}(\tau)e^{-j2\pi\omega\tau}d\tau \\ &= \frac{1}{T}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(t)h^*(t-\tau)e^{j2\pi\omega(t-\tau)} \\ &\quad e^{-j2\pi(\omega+\alpha_k)t}dtd\tau + \sigma^2\delta(k) \\ &= \frac{1}{T}H^*(\omega)H(\omega+\alpha_k) + \delta(k)\frac{\sigma^2}{T} \qquad (5) \end{aligned}$$

[1]The receipt filter is generally matched with the transmit filter so that $p_r(t) = p_t^*(-t)$.

[2]The number of sinusoids M can be estimated by using the recently developed LS detection method [8].

where the channel frequency response is given by

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi\omega t} dt$$

Under assumption A4, we have

$$\begin{aligned} H(\omega) &= P(\omega)G(\omega) \\ G(\omega) &= \sum_{i=1}^{M} \lambda_i e^{-j2\pi\tau_i\omega} \end{aligned} \qquad (6)$$

$G(\omega)$ and $P(\omega)$ are the Fourier transforms of $g(t)$ and $p(t)$ respectively. Since $P(\omega)$ is known, we can compute

$$\begin{aligned} F(l) &\stackrel{\text{def}}{=} \frac{T(S_x^{\alpha_0}(l\Delta\omega) - \sigma^2)}{|P(l\Delta\omega)|^2} \\ &= |G(l\Delta\omega)|^2 \\ &= \sum_{0\le i,j\le M-1} \lambda_i \lambda_j^* e^{-j2\pi(\tau_i - \tau_j)l\Delta\omega} \end{aligned} \qquad (7)$$

where $\Delta\omega$ is a chosen frequency step satisfying $\max_{i,j} |(\tau_i - \tau_j)\Delta\omega| < 1$. And for each cyclo-frequency $\alpha_k = k/T$

$$\begin{aligned} S(k) &\stackrel{\text{def}}{=} \frac{T(S_x^{\alpha_k}(0) - \delta(k)\sigma^2)}{\sqrt{F(0)}P^*(0)P(k/T)} \\ &= G^*(0)G(k/T) \\ &= \frac{G^*(0)}{|G(0)|} \sum_{i=0}^{M-1} \lambda_i e^{-j2k\pi\tau_i/T} \end{aligned} \qquad (8)$$

As we can see, $S(k)$ is a superimposed exponential signal with frequencies $2\pi\tau_i/T$ and amplitudes $\frac{G^*(0)}{|G(0)|}\lambda_i$, $i = 0, \cdots, M-1$. Therefore, we can first estimate the delays (up to integer multiples of T) and the attenuations (up to a common unitary scalar, i.e., $\frac{G^*(0)}{|G(0)|}$) from the sequence $S(k)$. Later, we can fully estimate the delays, i.e., estimate the integer multiples of T, by using the sequence $F(l)$.

Remark: By using the second order statistics information, i.e., the cyclo-spectra $G^*(\omega)G(\omega + k/T)$, we can at best estimate the unknown channel attenuations up to a common unitary scalar and the unknown channel delays up to a common integer multiple of T since

$$\tilde{G}^*(\omega)\tilde{G}(\omega + k/T) = G^*(\omega)G(\omega + k/T)$$

for any $\tilde{G}(\omega) = c\sum_{i=0}^{M-1} \lambda_i e^{-j2\pi(\tau_i + nT)\omega}$ where n is a given integer and $|c| = 1$.

We take advantage of this indeterminacy to assume, without loss of generality, that the delay correponding to the strongest coefficient λ_{i_0} ($|\lambda_{i_0}| = \max_i |\lambda_i|$) satisfies $0 \le \tau_{i_0} < 1$.

3.3 Implementation

In summary, we have the following algorithm:

- Choose $p > M$ and let the sampling interval be $\Delta = T/p$. The oversampled discrete signals are

  $$x_i = x(i\Delta), \quad h_i = h(i\Delta), \text{ and } \quad w_i = w(i\Delta)$$

  x_i is a discrete time cyclostationary process with period p.

- Estimate the correlation function of x_i, i.e $R_x(n + m, n) = E(x_{n+m}x_n^*)$, $n = 0, \cdots, p-1$ as

  $$\hat{R}_x(n+m, n) = \frac{1}{T} \sum_k x_{n+m+kp} x_{n+kp}^*$$

  Then, for $k = -p+1, \cdots, p-1$, estimate the cyclic autocorrelation functions of x_i as

  $$\hat{R}_x^{\alpha_k}(m) = \sum_{n=0}^{p-1} \hat{R}_x(n+m, n) e^{-j2\pi n\alpha_k}, \quad \alpha_k = k/p$$

- Choose $\Delta\omega$ small enough and estimate the power spectrum as

  $$\hat{S}_x^{\alpha_0}(l\Delta\omega) = \sum_m \hat{R}_x^{\alpha_0}(m) e^{-j2\pi lm\Delta\omega}$$

  Then, estimate the sequence $F(l)$ as defined in (7).

- Estimate the cyclo-spectra $S_x^{\alpha_k}(\omega)$ at the frequency $\omega = 0$

  $$\hat{S}_x^{\alpha_k}(0) = \sum_m \hat{R}_x^{\alpha_k}(m)$$

  Then, estimate the superimposed exponential sequence $S(k)$ as defined in (8).

- Apply the MP method to $S(k)$ to estimate the delay and attenuation parameters, i.e., $\{\hat{\tau}_i, \hat{\lambda}_i\}$, up to the indeterminacies shown above.

- Apply the MP method to $F(l)$ to estimate the $M(M-1)/2$ parameters $\{\tilde{\tau}_i - \tilde{\tau}_j\}_{0\le i<j\le M-1}$ and their corresponding coefficients $\{\tilde{\lambda}_i\tilde{\lambda}_j^*\}$.

- Select from the $M(M-1)/2$ parameters those corresponding to $\tau_i - \tau_{i_0}$, $i \ne i_0$ as follows: for each $i \ne i_0$ we correspond the element in the set $X = \{\tilde{\tau}_i - \tilde{\tau}_j\}$ that satisfies:

  $$\min_{x\in X} |\hat{\lambda}_i \hat{\lambda}_{i_0}^* - x|$$

  assuming implicitly that $\lambda_i \ne \lambda_j$ for $i \ne j$.

  Then, estimate the delays as

  $$\tau_i \approx \begin{cases} (\tilde{\tau}_i - \tilde{\tau}_{i_0}) + \hat{\tau}_{i_0} & \text{if } i \ne i_0 \\ \hat{\tau}_{i_0} & \text{if } i = i_0 \end{cases}$$

3.4 Some Comments

W give here some important remarks on the above BSI approach:

- Table 1 shows an example of sparse multipath channel used by ATTC [11]. As we can see, the channel transfer function is finite but corresponds to a high degree polynomial function (in this example, the degree d is over 106) with very few non-zero coefficients. The block-Sylvester matrix corresponding to the polyphase channel transfer functions (see [4] for more details) is a sparse matrix that is ill conditioned due to close common roots phenomenon. This is the reason why subspace methods provide very poor performance estimation in this context [11]. Moreover, subspace techniques are computationally unattractive in this case due to the large dimension of the block-Sylvester matrix given by $pW \times (W + d)$, $W > d$ where d is the polynomial degree and p is the oversampling rate.

  Table 1: A sparse multipath channel used by ATTC

Path	Delay	Phase	Atten.
1	0 T (0 μs)	288^o	20 dB
2	9.68 T (1.8 μs)	180^o	0 dB
3	10.49 T (1.95 μs)	0^o	20 dB
4	19.37 T (3.6 μs)	72^o	10 dB
5	40.35 T (7.5 μs)	144^o	14 dB
6	106.52 T (19.8 μs)	216^o	18 dB

  On the other hand, the proposed method provides good performance estimation in this context (in this example, only 6 delay and attenuation parameters have to be estimated). Another unique aspect of this approach is its ability to estimate the multipath transfer function even if the polyphase channels share common zeros.

- In [11], a modal analysis method to BSI has been proposed. Roughly, the idea of this method is to estimate the frequency and amplitude parameters of the function

  $$|G(\omega)|^2 = \sum_{i,j} \lambda_i \lambda_j^* e^{-j2\pi(\tau_i - \tau_j)\omega}$$

  that are given by $\tau_i - \tau_j$ and $\lambda_i \lambda_j^*$, respectively. Then, in a second step, it uses a non linear inverse transform to estimate the propagation parameters, i.e., delays and attenuations. The main drawbacks of this method are **(i)** the amplitudes $\lambda_i \lambda_j^*$ can be very small leading to poor estimation accuracy of their corresponding frequency components, e.g., $|\lambda_1 \lambda_3| = -40dB$ in the example given by Table 1, **(ii)** the non linear inverse transform is computationally expensive and is not robust in the situation of low or moderate SNR or short sample size.

  In comparison with this approach, our method is computationally much simpler, and is expected to be more robust to noise and finite sample effects.

4 Conclusion

We presented a knowledge based modal analysis approach for blind system identification. The proposed method exploits both the a priori knowledge of the pulse shape filter and the multipath channel propagation model. Thanks to this side information, the number of estimation parameters is significantly reduced leading to a significant performance improvement. This improvement is particularly important in the case of sparse multipath channel where the standard SOS-based approaches provide poor estimation accuracy. In comparison with the method presented in [11], our approach is computationally simpler and more robust to noise and finite sample effects.

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