ON THE LINEARLY CONSTRAINED BLIND MULTICHANNEL EQUALIZATION[1]

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ABSTRACT

Working at baud rate regime (one sample per symbol), it is well known the equivalence between the linear prediction problem and a proper linearly constrained power minimization criterium [1]. However, the success of this criterium as a blind equalization technique is limited to the minimum phase channel condition. On the other hand, if it is assumed a multichannel model (several samples per symbol or several sensors), it has been shown, under certain hypotheses, the minimum phase character of the multivariate transfer function; this property is in fact which allows one of the main approaches of multichannel blind equalization as a multivariate linear prediction problem [2]. Our main goal is to introduce a family of adaptive algorithms dealing with the formulation of the blind equalization task as a linearly constrained cost function in order to generalize the baud rate case results: a detailed analysis of the mentioned cost functions is included and also supported by several computer simulations.

1. INTRODUCTION

Blind equalization is a fundamental signal processing technique that retrieves the unknown transmitted sequence by analyzing only the characteristics of the channel output. Conventional equalization schemes are suitable when just one sample per symbol is achieved; in a generic case, information on Higher Order Statistics is required for properly deconvolution. Probably, one of the first schemes dealing with the blind equalization task was developed in [1] in the seventies: that work pointed out that a minimum phase channel may be inverted by the minimization of the received sequence power if the first coefficient of the one-sided (infinite) equalizer is fixed. Also in that paper is remarked the relationship between this formulation and the classical linear prediction problem. However, the minimum phase assumption looks very restrictive if we are considering a baud rate system as it was usually done. From an intuitive point of view, the first tap anchoring, provides the preservation (for minimum phase channels only) of the actual symbol and therefore only ISI is minimized.

On the other hand, it has been shown that multichannel equalization overcomes the main drawbacks of conventional blind equalization techniques exploiting the redundancies of SIMO (single input-multiple output). Many techniques have been developed in the recent years for estimating the unknown channel (identification) or recovering the transmitted sequence (equalization).

If we concentrate in equalization, one of the main approaches has shown that the multichannel moving average process is also a finite order autorregresive process: this is really a remarkable fact since system input characteristics can be estimated by applying multivariate linear prediction on the system output [2]. This formulation, allows us a new interpretation of the multichannel deconvolution techniques extending the result mentioned previously of classical blind equalization: the equivalence between a linear prediction scheme and a linearly constrained power minimization algorithm. Two main advantages must be pointed out for the multivariate case: the minimum phase characteristic of the multivariate transfer function (if FIR subchannels do not share common zeros) and the fact that FIR channels can be also exactly inverted by mutivariate FIR equalizers.

More indeed, another approach described in the literature was proposed in [3]: this contribution is based on a deterministic model for the input signal and on the exploitation of cross relations between a channel output pair. The basic idea behind this approach is to exploit different instantiations of the same input signal by multiple FIR channels. Let observe that an stochastic cross-relation minimization will force the cancellation of the input signal because of the SIMO structure: however, if we recall the fact that a linearly constrained scheme is proposed in order to preserve the actual symbol, the stochastic cross-correlation will cancel only the ISI; therefore, tap anchoring allows again the possibility of exploiting cross information between subchannels.

Our proposal intends to follow the formulation and analysis of a family of blind multivariate equalizers, exploiting the fact that tap anchoring (in this case, a finite set of coefficients of the predictor matrix) assures the preservation of the actual symbol. This strategy provides a proper decomposition of the deconvolved multivariate signal in two terms: one of them as a scalar version of the actual symbol whose equalizer matrix coefficient is fixed, meanwhile the other is a linear combination of previous transmitted symbols affected by the free and unknown coefficients of the predictor. The preservation of the actual symbol provides a simple criterium for properly

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deconvolution: the minimization of a linear combination of powers for every virtual channel and maybe also the extension to the minimization of cross powers between couples of virtual channels. This criterium can be easily improved by incorporating some salient features of the signal, e.g. its discrete format or constant modulus character. In the sequel they will labeled as LCMBE (Linearly constrained Multichannel Blind Equalizers).

2. PROBLEM STATEMENT

The scenario we have considered is a linear digital modulation (PAM/QAM/PSK) over a linear time invariant channel with additive white Gaussian noise (AWGN): therefore the received signal is given by:

$$y(t) = \sum_k s(k)\, h(t - kT) + w(t) \qquad (1)$$

where $s(k)$ is an independent and identically distributed (i.i.d.) sequence of symbols and $w(t)$ is the noise (AWGN), T is the baud duration, and $h(t)$ is the equivalent impulse response of the transmit filter, channel and receive filter. Our reception strategy consist in either oversampling the received signal by an integer factor q>1, considering an antenna array (q antennas) or either both of them. Any case lead us to a cyclostationary scalar-valued signal model or a stationary vector-valued signal. Therefore, we can consider the received (vector-valued) signal as the noisy output of a qx1 time invariant polynomial transfer function $\mathbf{h}(z)$ driven by the scalar sequence $s(n)$. Assuming a causal channel and a time span limited to (M+1) symbols duration, equation (1) can be written as follows:

$$\begin{aligned} \mathbf{y}(n) &= [\mathbf{h}(z)]\, s(n) + \mathbf{w}(n) = \\ &= \left[\mathbf{h}_1(z);\, \mathbf{h}_2(z) \quad \cdots; \mathbf{h}_q(z)\right] s(n) + \mathbf{w}(n) = \\ &= \mathbf{H}(z)\, s(n) + \mathbf{w}(n) \end{aligned} \qquad (2)$$

where $\mathbf{H}(z)$ is the multivariate transfer function. The main result, developed is several papers [4-5], establishes the minimum phase character of $\mathbf{H}(z)$ providing the equalization task as a multivariate linear prediction scheme.

Let us discuss our proposal. Considering the previous formulation now neglecting the noise effect, we can express in the discrete time domain as follows:

$$\mathbf{y}(n) = \mathbf{Hs}(n) \qquad (3)$$

where matrix $\mathbf{H}$ is full column rank and block Toeplitz (i.e. the different subchannels do not share common zeros as is usually assumed).

Defining the innovation vector $\mathbf{i}(n)$ as the conventional linear prediction problem (considering an order P linear prediction filter), the deconvolved sequence yields:

$$\mathbf{i}(n) = \mathbf{y}(n) + \sum_{k=1}^{P} \mathbf{B}(k)\ \ \mathbf{y}(n-k) \qquad (4)$$

where $\mathbf{B}(k)$ are the (qxq) matrix coefficients of the prediction error filter to be identified.

Let us observe that this relationship can be considered as a multivariate linearly constrained filtering:

$$\mathbf{i}(n) = \begin{bmatrix}\mathbf{B}(0) & \mathbf{B}(1) & \cdots & \mathbf{B}(P)\end{bmatrix} \begin{bmatrix}\mathbf{y}(n) \\ \mathbf{y}(n-1) \\ \vdots \\ \mathbf{y}(n-P)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_0 & \mathbf{A}_1\end{bmatrix} \begin{bmatrix}\mathbf{y}(n) \\ \mathbf{y}(n-1) \\ \vdots \\ \mathbf{y}(n-P)\end{bmatrix} \qquad (5)$$

linearly constrained because the tap anchoring $\mathbf{A}_0$=$\mathbf{B}(0)$=$\mathbf{I}_{qxq}$ (a proper identity matrix) meanwhile $\mathbf{A}_1$ =[$\mathbf{B}(1)$ $\mathbf{B}(2)$ ... $\mathbf{B}(P)$] represents the free coefficients.

Let us suppose, for simplicity, a bivariate system (the extension is straightforward for a generic scenario as we will show later on) whose impulse responses are denoted as follows:

$$\begin{aligned} &\text{Channel a: } \mathbf{h}_a = \begin{bmatrix}h_a(0) & h_a(1) & \cdots & h_a(M)\end{bmatrix}^t = \begin{bmatrix}h_{a0} & \mathbf{h}_{a1}^t\end{bmatrix} \\ &\text{Channel b: } \mathbf{h}_b = \begin{bmatrix}h_b(0) & h_b(1) & \cdots & h_b(M)\end{bmatrix}^t = \begin{bmatrix}h_{b0} & \mathbf{h}_{b1}^t\end{bmatrix} \end{aligned} \qquad (6)$$

decomposed in the first sample and the remainder for convenience. Let us also, decompose the filtering matrix $\mathbf{H}$ in the following way:

$$\mathbf{H} = \begin{bmatrix} h_{a0} & \mathbf{h}_{a1}^t \\ h_{b0} & \mathbf{h}_{b1}^t \\ \mathbf{0} & \mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_0 & \mathbf{H}_1^t \\ \mathbf{0} & \mathbf{H}_2 \end{bmatrix} \qquad (7)$$

Let us remark the attractive expression for the deconvolved sequence $\mathbf{i}(n)$ after the linearly constrained multivariate linear filtering by the prediction matrix $\mathbf{A}$.

$$\mathbf{i} = \mathbf{AHs} = \begin{bmatrix}\mathbf{I}_{2x2} & \mathbf{A}_1\end{bmatrix} \begin{bmatrix} \mathbf{h}_0 & \mathbf{H}_1^t \\ \mathbf{0} & \mathbf{H}_2 \end{bmatrix} \begin{bmatrix} s_0 \\ \mathbf{s}_1 \end{bmatrix} = \mathbf{h}_0\, s_0 + \left(\mathbf{H}_1^t + \mathbf{A}_1 \mathbf{H}_2\right)\ \mathbf{s}_1 \qquad (8)$$

where s_0 denotes the actual symbol and $\mathbf{s}_1$ represents a vector collecting the samples of previous symbols. Equation (8) shows that the tap anchoring ($\mathbf{I}_{2x2}$) really preserves the actual symbol meanwhile the second term represents the ISI to be canceled.

Figure 1 shows the block diagram of the mutichannel transmission and the multivariate linearly constrained equalizer; the optimization criterium will be derived in the next section.

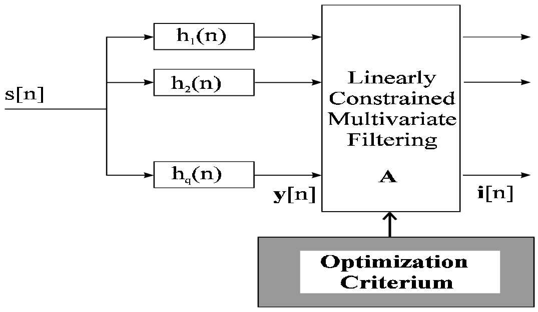

Fig.1. Block diagram of a linearly constrained multivariate filtering

3. THE LCMBE (LINEARLY CONSTRAINED MULTICHANNEL BLIND EQUALIZERS)

The main goal of our proposal is the derivation of a family of optimization criteria for the deconvolution of a mutivariate system exploiting the fact remarked in equation (8): the decorrelation between the desired symbol and the ISI. Let us proceed in the following way, describing several optimization criteria:

3.1 Description of criterium I.

As we mentioned in the introduction, the univariate linear prediction method is equivalent to a linearly constrained power minimization problem. The extension to the multivariate case is straightforward in terms of the trace operator (*tr*):

$$J_1 = \min_{\mathbf{A}} E\left\{ tr\left(\mathbf{i}(n)\ \mathbf{i}^H(n)\right)\right\} = \min_{\mathbf{A}}\left\{ tr\left(\mathbf{A}\mathbf{R}_{\mathbf{Y}}\mathbf{A}\right)\right\} \qquad (9)$$

Constrained to: $\mathbf{A}(0) = \mathbf{I}_{qxq}$

It can be shown that the analysis of the stationary points of equation (9) lead us to the well known result:

$$\frac{\partial J_1}{\partial \mathbf{A}} = \mathbf{H}_2\mathbf{H}_1 + \mathbf{H}_2\mathbf{H}_2^H\mathbf{A}_1 = \mathbf{0} \ \rightarrow \mathbf{A}_1 = -\left(\mathbf{H}_2\mathbf{H}_2^H\mathbf{H}_2\mathbf{H}_2^H\right)^{-1}\mathbf{H}_2\mathbf{H}_2^H\mathbf{H}_2\mathbf{H}_1 \qquad (10)$$

showing the solution of the multivariate Yule-Walker equations (usually eq. (10) is expressed in terms of the Moore-Penrose pseudo-inverse $^{\#}$):

$$\left(\mathbf{H}_2\mathbf{H}_2^H\right)^{\#} = \left(\mathbf{H}_2\mathbf{H}_2^H\mathbf{H}_2\mathbf{H}_2^H\right)^{-1}\mathbf{H}_2\mathbf{H}_2^H \qquad (11)$$

The Hessian matrix of (9) shows the quadratic character of the cost function:

$$\frac{\partial^2 J_1}{\partial \mathbf{A}\mathbf{A}^H} = \begin{pmatrix} \mathbf{H}_2\mathbf{H}_2^H & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2\mathbf{H}_2^H \end{pmatrix} \qquad (12)$$

which is clearly definite positive due to the Toeplitz nature of matrix $\mathbf{H}_2\mathbf{H}_2^H$.

3.2 Description of criterium II.

Likewise, let us point out that, although the proposed criterium J_1 performs essentially as the linear prediction method, it can also be exploited the decorrelation between different subchannels: note that imposing the decorrelation between subchannels (by couples therefore forcing a second order algorithm) the ISI will be eliminated since the actual symbol is preserved by the tap anchoring; in fact, in order to guarantee the real character of the cost function using only second order statistics, we have considered the following cost function:

$$J_2 = \min_{\mathbf{A}} \sum_{i=1}^{q-1}\sum_{j>i}^{q} 2\Re\left\{E\left[i_i i_j^*\right]\right\} = \min_{\mathbf{A}} \sum_{i=1}^{q-1}\sum_{j>i}^{q} E\left[i_i i_j^*\right] + E\left[i_j i_i^*\right] \qquad (13)$$

Constrained to: $\mathbf{A}(0) = \mathbf{I}_{qxq}$

We have shown that this function is also quadratic achieving the same gradient and Hessian as J_1 (see equations 10 and 12). Let us remark that this criterium will behave better in <u>white noisy environments</u> due to the noise decorrelation between samples.

3.3 Description of criterium III.

Combining both criteria J_1 and J_2, we propose a linear combination of equations 9 and 13 as follows:

$$J_3 = \min_{\mathbf{A}}\left((1-\alpha)\ E\left\{tr\left(\mathbf{i}(n)\mathbf{i}^H(n)\right)\right\} + \alpha \sum_{i=1}^{q-1}\sum_{j>i}^{q} 2\Re\left\{E\left[i_i i_j^*\right]\right\}\right) =$$

$$= \min_{\mathbf{A}}\left((1-\alpha)\sum_{i=1}^{q} E\left[\left|i_i\right|^2\right] + \alpha \sum_{i=1}^{q-1}\sum_{j>i}^{q} 2\Re\left\{E\left[i_i i_j^*\right]\right\}\right) \qquad (14)$$

Constrained to: $\mathbf{A}(0) = \mathbf{I}_{qxq}$

where operator $\Re$ means real part and parameter α provides a different weight for both criteria (the selected value can be determined by experimentation or maybe by an adaptive updating).

3.4 Description of criterium IV.

The later criterium can be improved by exploiting the constant modulus nature of the signals by introducing a linearly constrained CMA-like cost functions. This criterium is very popular in conventional blind equalization because it usually achieves a satisfactory performance; in the multivariate case, its interest is increased by the fact of its globally convergent behavior [6]. Exploiting the 'constant modulus' feature of data transmission, the algorithm convergence speeds-up:

$$J_4 = \min_{\mathbf{A}}\left((1-\alpha)\sum_{i=1}^{q} E\left\{\left|\left|i_i\right|^2 - R\right|^2\right\} + \alpha \sum_{i=1}^{q-1}\sum_{j>i}^{q} 2\Re\left\{E\left[i_i i_j^*\right]\right\}\right) \qquad (15)$$

where R is the Godard constant.

The analysis of stationary points is quite more complicated in this case due to the nonlinearity involved in the constant modulus cost function. However, it can be argued that equation 15 is really in fact the linear combination of two globally convex cost functions: the first term is globally convex because it is well known that a linear constrained globally convex function is also globally convex and second term convexity is shown in equation (12).

3.5 Description of criterium V.

A similar behavior can be analyzed just considering the Godard parameters also adaptive, playing the role of an adaptive gain tracking the values of the firs samples in the virtual channels (assuming α=0.5).

$$J_5 = \min_{\mathbf{A}} \left(\sum_{j=1}^{q} \sum_{i \geq j}^{q} E\left\{ \left| i_i\, i_j^H - \gamma_i \gamma_j^H \right|^2 \right\} \right) \tag{16}$$

Let us recall that in previous criteria (J_1 to J_4), once the innovation is white, the transmitted symbol must be estimated (usually by a subspace technique from the innovation covariance matrix). In the present criterium, let us point out that the evolution of constants γ_i, allows us the proper linear combination of the innovation components for direct estimation of the transmitted sequence [7].

$$\hat{s}[n] = \sum_{i=1}^{q} i_i \gamma_i^H \Big/ \sum_{i=1}^{q} \gamma_i \gamma_i^H \tag{17}$$

4. SIMULATION RESULTS

To show the ability of our proposals to properly equalize a multichannel system, we have consider a binary transmission over an scenario of a bivariate system whose impulse response is given in table 1; the Gaussian noise power is set to -20 dB, and we have considered a linear prediction order of two. Also, we have introduced an automatic gain control in order to equalize the evolution of all the equalizers with the same step size. This last criterium J_5 (whose simmulation are included) provides the best observed performance, but however, the behavior of criteria J_1 to J_4 are at this moment under study for several scenarios and constellations in order to provide a wide set of comparative performances.

$\mathbf{h_1}$	1.1650	0.0751	-0.6965
$\mathbf{h_2}$	0.0352	-0.0697	0.1696

Table 1

5. SUMMARY AND CONCLUSIONS

We have shown that tap-anchoring allows the exploitation of some of the special features related with SIMO system: the minimum phase character of the multivariate transfer function and the different instantiations of the same input signal by multiple FIR channels. A family of linearly constrained cost functions is proposed in order to deconvolve the transmitted sequence as a trade off between convergence speed and computational complexity.

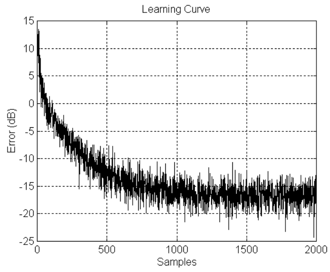

Fig.2 Learning Curve

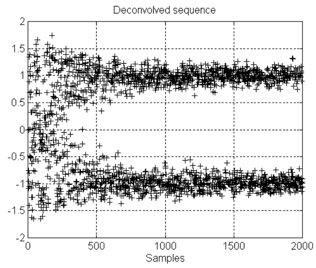

Fig.3 Deconvolved sequence

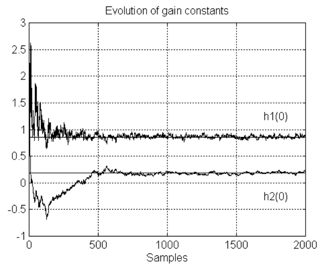

Fig.4 Evolution of gain constants γ_1, γ_2

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