Nth-Band Filter Design

Tapio Saramäki and Markku Renfors
Department of Information Technology Tampere University of Technology
P.O.Box 553, FIN-33101, Tampere, FINLAND
E-mails: ts@cs.tut.fi, mr@cs.tut.fi

ABSTRACT

Digital Nth-band linear-phase nonrecursive and Nth-band recursive filters are special digital filter classes playing an important role in various applications. Both these filter classes are named according to their frequency-domain characteristics. This paper reviews the properties of these filters as well as their usefulness in several digital signal processing applications. Also their optimization for various applications is considered.

1 Introduction

Digital Nth-band FIR and IIR filters [1]–[11] (see also references in [11]) have somewhat different frequency domain and time domain properties, but they posses also many common characteristics. In the lowpass case, these filters have a (3 dB or 6 dB) bandwidth of π/N and the transition band is approximatively symmetric around this frequency. Both FIR and IIR Nth-band filters are quite efficient to implement. Especially, downsampling and upsampling operations can be combined very efficiently with these filters. This property makes Nth-band filters very interesting for all multirate signal processing applications. Also bandpass and highpass version can be obtained, e.g., by complex or cosine-modulation. In this way, also efficient Hilbert transfomers can be derived from lowpass Nth-band filters [12]. Also critically-sampled perfect reconstruction analysis-synthesis filter banks and transmultiplexers have a close relationship to Nth-band filters [13]–[18]. However, in this paper we concentrate on the lowpass case.

In the time domain, a characteristic property of Nth-band filters is that its impulse response has zero crossings at a regular distance, at all multiples of N samples away from the central sample. This is the so-called zero intersymbol interference property of the Nyquist pulse-shaping filters, a concept which is one of the corner-stones of bandwidth-efficient digital transmission systems [19]. Traditionally, raised cosine filters or a pair of square-root raised-cosine filters have has been utilized in digital transmission systems. A digital implementation of a raised cosine filter is actually a special type of Nth-band filter. In practise, raised-cosine filters suffer from the nonideal stopband response due to truncation of the ideal infinite-length impulse response, and better solutions can be found by filter optimization techniques [1], [3], [6], [9].

This paper reviews some basic properties of Nth-band FIR and IIR filters and illustrates their usefulness in various digital signal processing applications in terms of examples.

2 Nth-Band Linear-Phase FIR Filters

This section considers some basic properties of Nth-band linear-phase FIR filters. Their transfer function is of the form

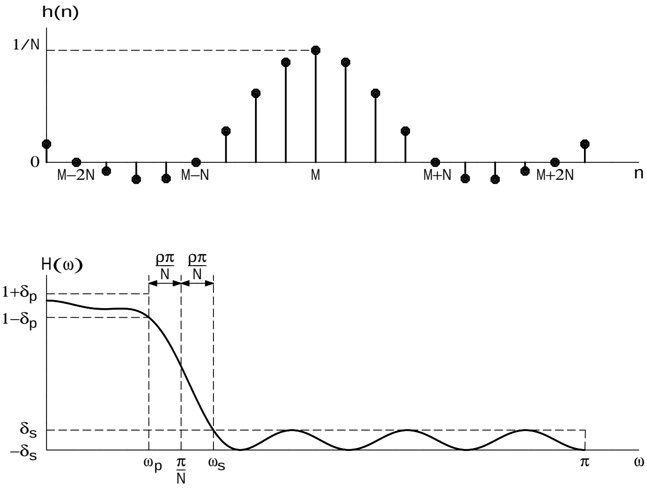

Figure 1: *Typical impulse response and zero-phase frequency response for an FIR Nth-band FIR filter.*

$$H(z) = \sum_{n=0}^{2M} h(n) z^{-n}, \tag{1}$$

where the impulse-response coefficients are symmetric, that is, $h(2M-n) = h(n)$ for $n = 0, 1, \cdots, 2M$. This filter is said to be an Nth-band filter if (see Fig. 1)

$$h(M) = 1/N, \quad h(M \pm rN) = 0 \quad \text{for} \quad r = 1, 2, \cdots, \lfloor M/N \rfloor, \tag{2}$$

where $\lfloor x \rfloor$ stands for integer part of x.

The frequency response of the above filter is expressible as

$$H(e^{j\omega}) = e^{-jM\omega} H(\omega), \tag{3a}$$

where the zero-phase frequency response $H(\omega)$ is given by

$$H(\omega) = 1/N + 2\sum_{n=1}^{M} h(M-n)\cos(n\omega). \tag{3b}$$

It can be shown [2] that the time-domain conditions of Eq. (2) are equivalent to the following frequency-domain condition:

$$\sum_{r=0}^{N-1} H(\omega + 2\pi r/N) = 1. \tag{4}$$

Based on this condition, the passband (stopband) in the lowpass case is restricted to be smaller (larger) than π/N.

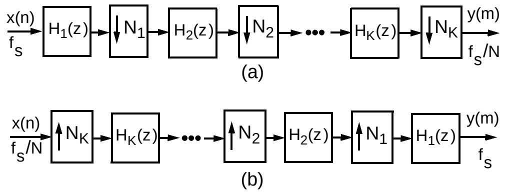

Figure 2: *Implementations of a multistage FIR Nth-band filter for sampling rate conversion. (a) Decimator. (b) Interpolator.*

Usually, the passband and stopband edge angles, denoted by ω_p and ω_s, are specified as (see Fig. 1)

$$\omega_p = (1-\rho)\pi/N, \quad \omega_s = (1+\rho)\pi/N, \tag{5}$$

where $\rho > 0$ is called the rolloff of the filter. For designing Nth-band FIR filters for other cases, see [10].

It follows also that if the maximum deviation of $H(\omega)$ from zero in the stopband region $[\omega_s,\ \pi]$ is δ_s, then δ_p, the maximum deviation of $H(\omega)$ from unity in the passband region $[0,\ \omega_p]$ is satisfies $\delta_p \le (L-1)\delta_s$. This implies that for a small value of δ_s, δ_p is automatically small. Therefore, when designing FIR Nth-band filters the synthesis can concentrate on shaping the stopband response.

3 Various Classes of Nth-Band FIR filters

This section considers different classes of Nth-band FIR filters.

3.1 Multistage Filters

If N is factorizable into the product $N = N_1 \cdot N_2 \cdots N_K$, then the overall Nth-band filter can be contsructed in terms of K N_kth-band FIR filters with transfer functions of the form

$$H_k(z) = \sum_{n=0}^{2M_k} h_k(n) z^{-n}, \quad \text{for} \quad k = 1,2,\cdots,K, \tag{6}$$

where each impulse response $h_k(n)$ is symmetric and satisfies the conditions of Eq. (2) with $N = N_k$ and $M = M_k$. The desired overall filter is then expressible as [6]

$$H(z) = \prod_{k=1}^{K} H_k(z^{L_k}), \tag{7a}$$

where

$$L_K = 1, \quad L_k = \prod_{l=k+1}^{K} N_k \quad k = 1,2,\cdots K-1. \tag{7b}$$

In the above equation, instead of a unit delay z^{-1}, there is a block delay z^{-L_k} for all the terms except for $H_K(z)$. The order of this filter is $2M = 2(L_1M_1 + L_2M_2 + \cdots + L_KM_K)$.

The main advantage of the above decomposition is that the number of multipliers is significantly reduced compared to the direct-form implementation. Furthermore, if the overall filter is used for decimation or interpolation by a factor of N, then it can be implemented as shown in Fig. 2. Note that in these implementations unit delays are used.

The zero-phase frequency response of the above multistage filter is given by

$$H(\omega) = \prod_{k=1}^{K} H_k(L_k\omega). \tag{8}$$

3.2 Separable Filters

In pulse shaping in telecommunication applications, it is desired that the overall Nth-band (Nyquist) filter is factorizable as [9], [19]

$$H(z) = T(z)R(z) \tag{9a}$$

where

$$T(z) = \prod_{k=1}^{K} T_k(z^{L_k}), \quad R(z) = \prod_{k=1}^{K} R_k(z^{L_k}). \tag{9b}$$

Here, the half-Nyquist filters $T_k(z)$ and $R_k(z)$ are obtained by factorizing $H_k(z)$ as

$$H_k(z) = T_k(z)R_k(z) \tag{10}$$

where $T_k(z)$ and $R_k(z)$ have the same magnitude responses and their impulse responses are time-reversed versions of each other, that is, $R_k(z) = z^{-M_k}T_k(z^{-1})$, where M_k is half the order of $H_k(z)$.

In this case, it is required that the zero-phase frequency responses $H_k(\omega)$ for $k = 1,2,\cdots,K$ are non-negative on $[0,\ \pi]$ in order to make $H_k(z)$ factorizable in the desired manner.

In communication theory, $T(z)$ and $R(z)$ are referred to as a matched filter pair and they are used as transmitter and reciever filters, respectively. $T(z)$ and $R(z)$ and can be effectively implemented in a manner similar to Figs. 2(b) and 2(a), respectively.

4 Optimization of Nth-Band FIR Filters and Design Examples

This section illustrates the filter optimization in terms of examples.

4.1 Example 1: Design of Nonseparable Filters

It is desired to design an Nth-band FIR filter to meet in the minimax sense the criteria: $N = 8$, $\rho = 0.2$, and the minimum stopband attenuation is at least 40 dB. Given K, the number of stages, the problem is to find N_k's and the minimum overall orders $2M_k$ for $k = 1,2,\cdots,K$ to meet the given criteria and then to optimize the filter parameters to minimize

$$E_\infty = \max_{\omega\in[(1+\rho)\pi/N,\ \pi]} |W(\omega)H(\omega)|, \tag{11}$$

where $H(\omega)$ is given by Eq.(8) and $W(\omega)$ is a positive weight function on $[(1+\rho)\pi/N,\ \pi]$.

For $K = 1$, these criteria are met by $2M = 74$. When exploiting the coefficient symmetry and the facts that $h(37) = 2^{-3}$ and $h(37 \pm 8r) = 0$ for $r = 1,2,3,4$, only 32 multipliers are required to implement this filter. The implementation of the central coefficient $h(37) = 2^{-3}$ is trivial. For $K = 3$, the given criteria are met by $N_1 = N_2 = N_3 = 2$ and $2M_1 = 18$, $2M_2 = 2M_3 = 6$. In this case, only $4+2+2 = 8$ multipliers are required. The price paid for the reduction in the number of multipliers from 32 to 8 compared to the direct design is a slight increase in the overall filter order (from 74 to 90). Figures 3(a) and 3(b) show the responses for $H_1(z^4)$, $H_2(z^2)$, and $H_3(z)$ as well as that of the overall filter. The subfilters have been iteratively designed using the technique proposed in [6]. As seen from these two figures, $H_1(z^4)$ provides for the overall filter an equiripple stopband behavior on $[(1+\rho)/8,\ \pi/4]$, whereas $H_2(z^2)$ and $H_3(z)$ and attenuate in the minimax sense the extra passbands and transition bands of $H_4(z^4)$ located around $\omega = \pi/2$ and $\omega = \pi$, respectively.

The impulse response of the overall filter is depicted in Fig. 3(c), whereas the overall filter optimized in the least-mean-square sense with the same subfilter orders is shown

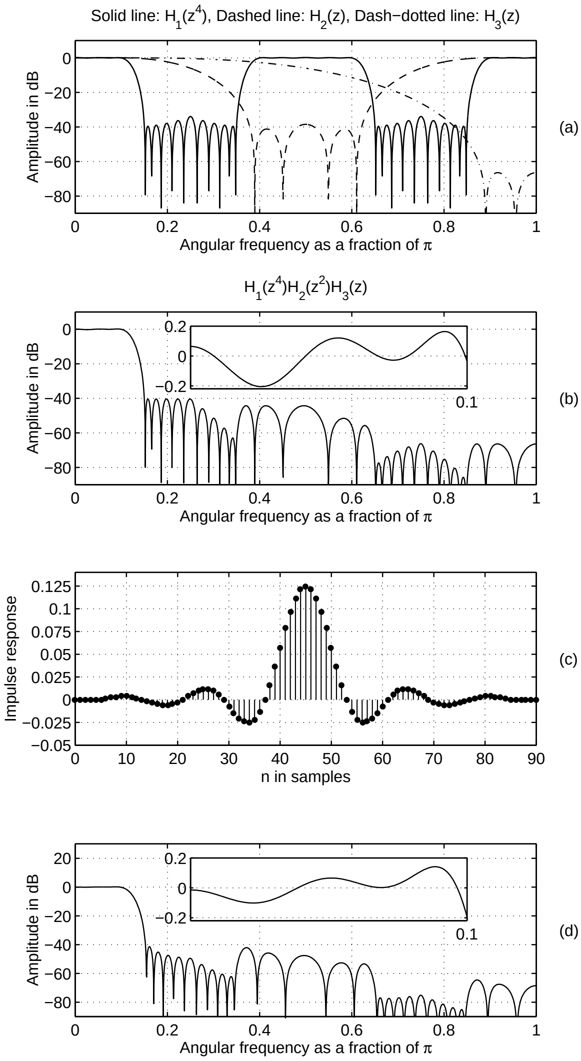

Figure 3: *Responses for three-stage filters of Example 1. (a) Amplitude responses for the subfilters. (b) and (c) Amplitude and impulse responses for the overall minimax Nth-band FIR filter. (d) Amplitude response for the least squared filter design.*

in Fig. 3(d). In this case, the impulse-response coefficients are determined to minimize

$$E_2 = \int_{(1+\rho)\pi/N}^{\pi} [W(\omega)H(\omega)]^2 d\omega. \tag{12}$$

The frequency-response-shaping responsibilities are shared like for the corresponding minimax filter design. It should be point out that linear programming [7], [8] can be also used for designing subfilters in the minimax sense, whereas the synthesis method proposed in [5] can be used for designing these filters in the least-mean square sense.

4.2 Example 2: Design of Separable Filters

It is desired to design a separable Nth-band FIR filter to meet in the minimax sense the criteria: $N = 8$, $\rho = 0.2$, and the minimum stopband attenuation is at least 40 dB for $R(z)$ and $T(z)$. For the overall separable filter, the minimum attenuation is thus 80 dB. In this case, the problem is to find the filter parameters to mininize E_∞ as given by Eq. (11) subject to the condition that $H(\omega)$ is nonnegative.

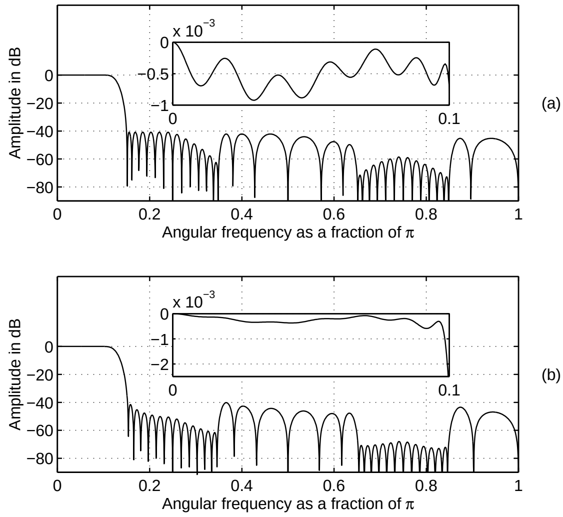

Figure 4: *Responses for three-stage filters of Example 2. (a) Common amplitude response for $T(z)$ and $R(z)$ designed in the minimax sense. (b) Common amplitude response for $T(z)$ and $R(z)$ designed in the least-mean-square sense.*

The subfilters can be effectively optimized using the synthesis scheme proposed in [20]. To meet these criteria with $K = 1$, $2M = 202$ is required For $K = 3$, the criteria are met by $N_1 = N_2 = N_3 = 2$, $2M_1 = 50$, $2M_2 = 18$, and $2M_3 = 10$. When the overall filter is split into the minimim-phase part $T(z)$ and the maximum phase part $R(z)$, both $T(z)$ and $R(z)$ require 102 multipliers in the $K = 1$ case and 42 multipliers in the $K = 3$ case. The price paid for this reduction is a slight increase in the overall filter order (from 202 to 246). Figure 4(a) shows in the three-stage case the common amplitude response $T(z)$ and $R(z)$, whereas Fig. 4(b) shows the corresponding response for a filter designed in the least-mean-square sense. In this case, the filter parameters are desired to be determined to minimize

$$\widehat{E}_2 = \int_{(1+\rho)\pi/N}^{\pi} [W(\omega)H(\omega)] d\omega \tag{13}$$

subject to the condition that $H(\omega)$ is nonnegative on $[0,\ \pi]$. For this purpose, the authors have generated a MATLAB routine. Note that in this case, $H(\omega) = |H_T(e^{j\omega})|^2 = |H_R(e^{j\omega})|^2$.

5 Nth-Band IIR filters

This section considers some basic properties of N-band IIR filters. These filters are a special class of filters having the following polyphase decomposition

$$H(z) = \frac{1}{N} \sum_{n=0}^{N-1} z^{-n} A_n(z^N). \tag{14}$$

For these filters, the $A_n(z)$'s are stable allpass filters of the form

$$A_n(z) = z^{-k_n} \frac{\sum_{l=0}^{K_n} a^{(n)}(l) z^{-(K_n - l)}}{\sum_{l=0}^{K_n} a^{(n)}(l) z^{-l}}. \tag{15}$$

The order of $A_n(z)$ is $k_n + K_n$ and it contains K_n adjustable parameters so that the overall number of parameters is

$$K = \sum_{n=0}^{N-1} K_n \tag{16}$$

The order of the nth branch filter $z^{-n}A_n(z^N)$ is $n+N(k_n+K_n)$. In order to achieve a good frequency response, it is required that the branches can be ordered such that $n+N(k_n+K_n)$ increases by one between two consecutive branches [11].

There exist two classes of Nth-band IIR filters, namely nonlinear phase filters and approximately linear phase filters. For the first filter class, $k_n \equiv 0$ for $n = 0, 1, \cdots, N-1$. These filters have a very attractive property that all the allpass filters are cascades of first-order section, making the implementation very efficient. For the second class, there exists one branch having $k_n \neq 0$ and $K_n = 0$, whereas for the other braches $k_n = 0$ and $K_n \neq 0$.

It has been shown in [11] that for the Nth-band IIR filters it is valid that

$$\sum_{r=0}^{N-1} |H(e^{j(\omega+2\pi r/N)})|^2 = 1. \tag{17}$$

Based on this fact, the passband and stopband edges for these filters are given by Eq. (5). However, for these IIR filters, the stopband region is a multiband region between ω_s and π. It constains don't-care bands of width $2\rho\pi/N$ at odd multiples of π/N. This is illustrated in Fig. 5 showing the amplitude response for a seventh-band IIR filter with $\rho = 0.2$ and $K = 18$. In this case $A_k(z)$ for $k = 0, 1, 2, 3$ are cascades of three first-order allpass filters, whereas $A_k(z)$ for $k = 4, 5, 6$ are cascades of two first-order allpass filters.

Secondly, if the maximum deviation of the squared-magnitude response from zero on the multiband stopband region is $\widehat{\delta}_s$, then the squared-magnitude response oscillates in the passband region between 1 and $1-\widehat{\delta}_p$, where $\widehat{\delta}_p \leq (N-1)\widehat{\delta}_s$. As seen from Fig. 5, this makes the passband ripple extremely small for a small value of $\widehat{\delta}_s$.

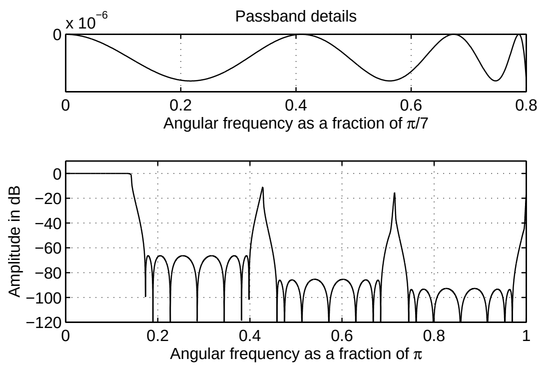

Figure 5: *Amplitude response for a seventh-band IIR filter.*

6 Applications of Recursive Nth-Band Filters

This section concentrates on the usefulness of recursive Nth-band filters in various digital signal processing applications.

6.1 Design of Decimators and Interpolators

These filters have turned out to be best ones in constructing nonlinear-phase decimators and interpolators. To illustrate this, we consider the following specifications: the decimation ratio is $N = 20$, the passband edge is $0.9\pi/N$, the stopband edge is π/N, the stopband ripple for the amplitude response is 0.005 (46-dB attenuation), and in the passband the amplitude response oscillates between unity and $1-0.1$. As shown in [11], an effective overall solution is obtained by using three filter stages. The first stage with transfer function $H_1(z)$ provides decimation by 5, the second stage with transfer function $H_2(z)$ decimates by 4, and the last

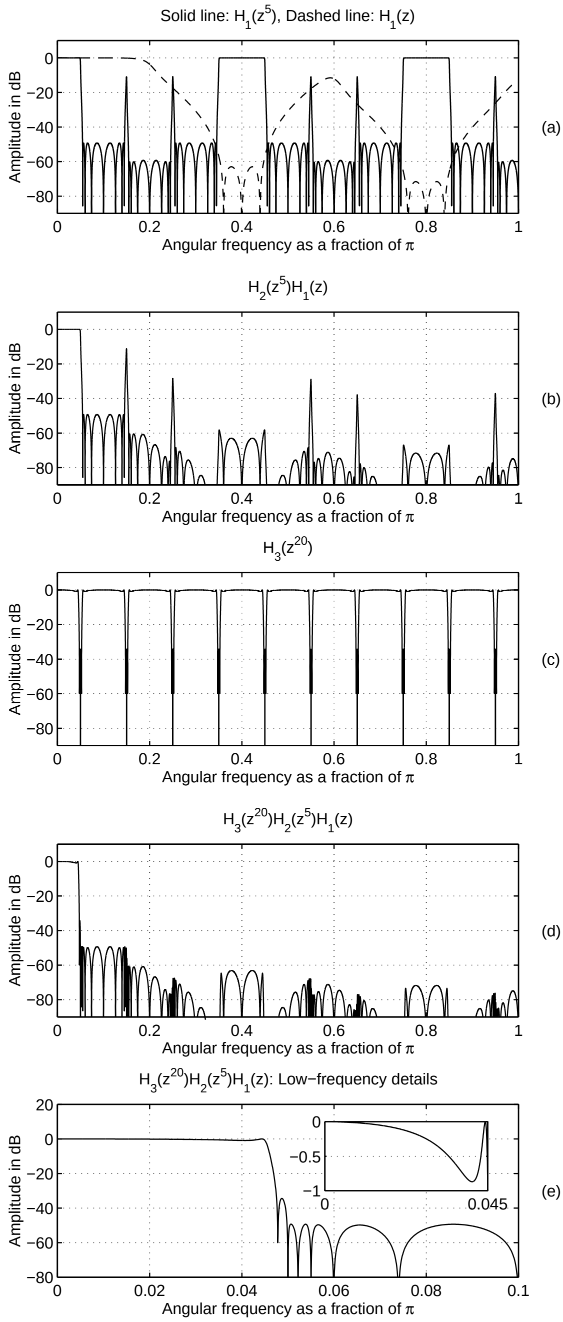

Figure 6: *Design of a 20-to-1 decimator using two Nth-band recursive filters and an corrector filter at the output sampling rate.*

filter stage with transfer function $H_3(z)$ works at the output sampling rate. For analysis purposes, this corresponds to the case where the single-stage overall transfer function is $H(z) = H_3(z^{20})H_2(z^5)H_1(z)$ before decimation by $N = 20$. Figure 6 illustrates how to construct the desired overall decimator. $H_2(z)$ has been designed in such a way that $H_2(z^5)$ provides for the overall filter the desired stopband attenuation in bands $[2\pi r/20 - 0.9\pi/20,\ 2\pi r/20 + 0.9\pi/20]$ for $r = 1, 2, 3, 5, 6, 7, 9, 10$. The role of $H_1(z)$ is to provide the desired attenuation in the bands for $r = 4$ and $r = 8$. For $H_2(z^5)H_1(z)$ the stopband starts at $\omega = 1.1\pi/20$, instead of $\omega = \pi/20$. Futhermore, there are peaks in the stopband region. The desired overall response is obtained by using $H_3(z)$ at the filter output in such a way that $H_2(z^{20})$ provides the desired attenuation in the band $[\pi/20,\ 1.1\pi/20]$ and attenuates the undesired peaks in the stopband region.

When practically implementing $H_1(z)$ and $H_2(z)$, a commutative structure is very useful [11]. In this case, there is no need to implement the extra delays in the branch filters and $A_n(z)$'s, instead of $A_n(z^N)$'s, are implemented. In our example case, the first four allpass filters of $H_1(z)$ are first-order sections, whereas the last one is just a direct line. For $H_2(z)$, the first allpass section is a cascade of three first-order allpass sections, whereas the last three filters are cascades of two first-order sections. $H_3(z)$ is simply a parallel connection of first-order and second-order allpass filters.

6.2 Design of Nyquist Filters

As we have defined them, IIR Nth-band filters have a 3-dB attenuation at π/N, whereas FIR Nth-band filters have a 6-dB attenuation at this frequency. Comparing Eqs. (4) and (17), we can see that the squared magnitude response of an IIR Nth-band filter satisfies the same property as the frequency response of the zero-phase FIR Nth-band filter. This means that the cascade of an Nth-band IIR filter, $H(z)$, and the anti-causal IIR filter $H(z^{-1})$ satisfies the frequency-domain condition of an FIR Nth-band filter and, consequently, satisfies the zero-crossing (zero ISI) property of Nyquist filters. $H(z)$ and $H(z^{-1})$ are a matched filter pair, i.e., the impulse responses are mirror images of each other. The anti-causal filter $H(z^{-1})$ can in some cases be implemented by filtering the data in the reverse direction with $H(z)$. For other cases, there are approximative methods to implement it [9].

In the case of $N = 2$, halfband IIR filters can be designed as a special case of elliptic filters (or other classical lowpass IIR filter types). This means that it is possible to satisfy the zero intersymbol interference property using an elliptic halfband filter together with the corresponding anticausal mirror image impulse response for pulse shaping. For power-of-two oversampling ratios, it is possible to design half-Nyquist filters using a cascade of halfband filters. For other values of N, Nth-band IIR filters cannot be used as Nyquist filters as such, since the don't-care bands between the stopbands cannot be tolerated. However, for even values of N, solutions where the first interpolator stage (last decimator stage) is a halfband IIR filter are feasible, since a continuous stopband from ω_s to π can be achieved in this case.

We can also notice that in the case of approximatively linear-phase IIR Nyquist filters, both the transmitter and receiver half-Nyquist filters, as well as the cascade satisfy the time-domain zero-crossing property. This is because in this case one of the polyphase branches of each interpolator/decimator stage is a pure delay. It can also be shown that in the case of $N = 2$, the approximatively linear-phase IIR half-Nyquist filters are the only filter class having this property [21].

Recent experience has revealed one drawback in using IIR Nth-band filters for transmitter pulse shaping: the peak envelope value of the modulated signal is higher than when using square-root raised-cosine filters. Separable FIR Nyquist filters with nonlinearphase filter for $T(z)$ suffer from the same problem, which is rather critical in applications where the transmitter power consumption should be minimized. On the other hand, for other applications IIR Nyquist filters offer a solution with low implementation complexity.

As an example we consider the design of a nonseparable Nyquist filter with $N = 8$ and $\rho = 0.8$. Such a filter can be implemented as $H(z) = H_1(z^4)H_2(z^2)H_3(z)$, where the $H_k(z)$ are of the form $H_k(z) = [z^{N_k} + A_k(z^2)]/2$. A 40-dB attenuation is achieved by $N_1 = 7$, $N_2 = 3$, $N_3 = 1$, and $A_k(z)$'s of orders 4, 2, and 1 for $k = 1, 2, 3$, respectively.

References

[1] P. R. Chevillat and G. Ungerboeck, "Optimum FIR transmitter and receiver filters for data transmission over bandlimited channels," *IEEE Trans. Commun.*, vol. COM-30, pp. 1909–1915, Aug. 1982.

[2] F. Mintzer, "On half-band, third-band, and Nth-band FIR filters and their design," *IEEE Trans. Acousts, Speech, Signal Process.,* vol. ASSP-30, pp. 734–738, Oct. 1982.

[3] A.C. Salazar and V.B. Lawrence, "Design and implementation of transmitter and receiver filters with periodic coefficient nulls for digital systems," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, pp. 306–310, 1982.

[4] J. K. Liang, R. J. P. DeFigueiredo, and F. C. Lu, "Design of optimal Nyquist, partial response, Nth band, and nonuniform tap spacing FIR digital filters using linear programming techniques," *IEEE Trans. Circuits Syst.,* vol. CAS-32, pp. 386–392, Apr. 1985.

[5] P. P. Vaidyanathan and T. Q. Nguyen, "Eigenfilters: A new approach to the least-squares FIR filter design and applications including Nyquist filters," *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 11–23, Jan. 1987.

[6] T. Saramäki and Y. Neuvo, "A class of FIR Nyquist (Nth-Band) filters with zero intersymbol interference," *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 1182–1190, Oct. 1987.

[7] H. Samueli, "On the design of optimal equiripple FIR digital filters for data transmission applications," *IEEE Trans. Circuits Syst.*, vol. CAS-35, pp. 1542–1546, Dec. 1988.

[8] H. Samueli, "On the design of FIR digital data transmission filters with arbitrary magnitude specifications," *IEEE Trans. Circuits Syst.*, vol. CAS-38, pp. 1563–1567, Dec. 1991.

[9] M. Renfors and and T. Saramäki, "Pulse-shaping filters for digital transmission systems,", in *Proc. 1992 IEEE Global Telecommun. Conf.* (Orlando, FL), pp. 467–471, Dec. 1992.

[10] J. M. Nohrden and T. Q. Nguyen, "Constraints on the cutoff frequencies of Mth-band linear-phase FIR filters," *IEEE Trans. Signal Procc.*, vol. 43, pp. 2401–2405, oct. 1995.

[11] M. Renfors and T. Saramäki, "Recursive Nth-band digital filters— Part I: Design and properties, Part II: Design of multistage decimators and interpolators," *IEEE Trans. Circuits Syst.* vol. CAS-34, pp. 24–51, Jan. 1987.

[12] P. A. Regalia, "Special filter design", Chapter 13 in *Handbook for Digital Signal Processing*, edited by S. K. Mitra and J. F. Kaiser, John Wiley and Sons, New York, 1993, pp. 907–980.

[13] P. P. Vaidyanathan, T. Q. Nguyen, Z. Doganata, and T. Saramäki, "Improved Technique for design of perfect reconstruction FIR QMF banks with lossless polyphase matrices," *IEEE Transactions on Acoust., Speech, Signal Process.*, vol. ASSP-27, pp. 1042–1056, July 1989.

[14] P. P. Vaidyanathan, *Multirate Systems and Filter Banks.* Prentice Hall, Englewood Cliffs, New Jersey, 1993.

[15] H. S. Malvar, *Signal Processing with Lapped Transforms.* Artech House, Boston, 1991.

[16] M. Vetterli and J. Kovacevic, *Wavelets and Subband Coding.* Prentice Hall, 1995.

[17] H. Babic, S. K. Mitra, C. Creusere, and A. Das, "Perfect reconstruction recursive QMF banks for image subband coding," in *Proc. Asilomar Conf. Circuits, Signals, and Systems* (Pacific Grove, CA), pp. 746–750, Nov. 1991.

[18] T. A. Ramstad, S. O. Aase, and J. H. Husøy, *Subband Compression of Images – Principles and Examples.* North Holland: ELSEVIER Science Publishers BV, 1995.

[19] J. G. Proakis, *Digital Communications.* 3rd Edition, New York: McGraw-Hill, 1995.

[20] T. Saramäki, "Finite impulse reponse filter design", Chapter 4 in *Handbook for Digital Signal Processing*, edited by S. K. Mitra and J. F. Kaiser, John Wiley and Sons, New York, 1993, pp. 155–277.

[21] M. Renfors, K. Väisänen, "Efficient IIR filters for pulse-shaping and jitter-free frequency error detection and timing recovery, " in *Proc. IEEE Global Telecomm. Conf.* (GLOBE-COM'95, Singapore), pp. 1440-1444, Nov. 1995.