SELF CALIBRATION IN PRESENCE OF CORRELATED DISCRETE SOURCES

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ABSTRACT

In this paper, we are concerned by the self-calibration of a linear equispaced array impinged by correlated sources. We propose a new method based on the fact that sources have discrete distribution which is meaningful in a digital communication context. The assumption of discrete sources allows an estimation, in the LS sense, of the mixing matrix which is then used to compute the sensor gain, phase and location errors.

1 INTRODUCTION

The localization of radiating sources is of great interest in fields like sonar or radar, and has been extensively studied in the literature. However the various Direction of Arrival (DOA) estimation schemes, such as the so-called High-Resolution algorithms, are very sensitive to sensor gain, phase and location errors, especially when the DOA's are close [4] and for short observation durations (see fig.1). As a consequence Array Self-Calibration has become a critical issue.

Few works have been carried out for the identifiability conditions which were mainly treated in [10, 11, 3], but many algorithms have been proposed to recover the array calibration. In [6], the authors used the fact that in absence of gain decalibration the entries of the steering vectors are of unit modulus. This constraint yields an analytical estimation of the mixing matrix which is then used to compute the true sensor locations. The unit modulus constraint was already used in [13] in conjunction with the maximization of the orthogonal projection of the estimated steering vectors on the signal subspace. In [14], the calibration is performed using a joint estimation of DOAs and sensor locations by the means of a Fourier transform of the sensor outputs. Other papers, like [5] and [8] studied the single source case, which often occurs in sonar.

In this paper, we are concerned by the compensation for gain, phase and location uncertainties of pathological linear equispaced arrays [10], solely based on the fact that sources have the same discrete distribution.

This work has been supported in part by Eurecom's industrial partners: Ascom, Cegetel, Hitachi, IBM France, Motorola, Swisscom, and Thomson CSF, and in part by the CNRS telecommunications program.

2 PROBLEM STATEMENT

Assume that P source signals x_p impinge on an array of K sensors. Then, in a narrow band context and if the propagation medium is linear, the signals observed on the array can be modeled as:

$$\mathbf{y} = \sum_{p=1}^{P} x_p\, \mathbf{d}(p) + \mathbf{w} = D\,\mathbf{x} + \mathbf{w}, \qquad (1)$$

where $\mathbf{w}$ stands for additive noise, and D is a $K \times P$ matrix. Because of inherent indeterminacies [10], it is legitimate to assume that the first row of D has its entries equal to 1 (the first sensor is set as the reference sensor).

If the array is perfectly known, every DOA $d(p)$ coincides with some $\mathbf{d}(\theta(p))$, where $\mathbf{d}(\cdot)$ denotes the known array manifold [1],[12]. In presence of decalibration factors, the observation can be modeled as :

$$\mathbf{y} = A\,\mathbf{s} + \mathbf{w}, \qquad (2)$$

and A separates in $A = GD$, where G stands for the gain decalibration and D has unit modulus entries and contains the decalibrated DOAs.

Our problem can now be expressed as : given N independent observations $\mathbf{y}_n$ find G and the true sensor locations and phases assuming that :

- **[A1]** $\mathbf{x}$ and $\mathbf{w}$ are independent,
- **[A2]** the noise $\mathbf{w}$ is Gaussian,
- **[A3]** the sources may be partially correlated (multipath context) and have the same discrete distribution,
- **[A4]** there are more sensors than sources: $K \geq P$,
- **[A5]** there is no coupling between sensors so that the calibration matrix G is real diagonal ($G = diag(G_1, \cdots, G_K)$),
- **[A6]** by convention, the first row of D is formed of ones, without restricting the generality,

[A7] sensors and sources are coplanar and the sources are far enough from the observing array so that the signal wavefronts are planar over the array (far-field assumption).

Since the source signals x_p have a discrete distribution $\mathcal{C}$, but are known up to a scale factor F_p and a phase shift η_p, they can be modeled as : $x_p = F_p\, exp(i\eta_p)\, s_p$, where the support of s_p is $\mathcal{C}$. Thus (2) rewrites :

$$\mathbf{y} = GDF\,\mathbf{s} + \mathbf{w}, \tag{3}$$

where F is diagonal. This model contains an inherent indeterminacy that can be fixed by assuming that $G_1 = 1$

The calibration of the array will be performed in three steps. First, we estimate the impinging signals using their discrete distribution and we deduce the mixing matrix in the LS sense. Then we compute the estimation of the diagonal matrices F and G. Finally, we estimate the angles of arrival, the sensor location errors and the phase errors.

3 SOURCE EXTRACTION

The problem of interest in this section is : given $\mathbf{y}_n$ and assuming [**A4**], estimate the sources s_p so that they have a known discrete distribution.

This problem has already been studied by the authors in [9] in the case of uncorrelated sources. There, it has been shown (see also appendix) that the MAP criterion, which is the most natural in this context, can be approximated by a polynomial criterion of the form :

$$\Phi_N(\mathbf{b}) = \frac{1}{N}\sum_{n=1}^{N}\prod_{j=1}^{D} |\mathbf{b}^\dagger \mathbf{y}(n) - C_j|^2. \tag{4}$$

where C_j represents the elements of the constellation $\mathcal{C}$ and D its cardinality. Furthermore, this latter criterion, first introduced in [7], is computationally less complex and MAP-equivalent in presence of PSK-sources (cf. appendix). An efficient minimisation algorithm, AMiSRoF dedicated to multivariate polynomial criteria, has been developed in [9] and allows a fast convergence.

When the sources are uncorrelated, criterion (4) allows to extract the sources one by one using a deflation approach. However, in the correlated case, a regression is prohibited and it is necessary to resort to other means to avoid obtaining several times the same source. Our method is simple and consists of including a penalty term in the polynomial criterion :

$$\Phi_N(\mathbf{b}) = \frac{1}{N}\sum_{n=1}^{N}\prod_{j=1}^{D} |\mathbf{b}^\dagger \mathbf{y}(n) - C_j|^2 + \frac{\alpha}{p}\sum_{i=1}^{p} \mathbf{b} Y s_p^\dagger \tag{5}$$

where p is the number of different extracted sources and s_p is the p^{th} estimated source.

In order to 'guide' AMiSRoF on a viable source estimation, we use a singular value decomposition of the sensor cross-correlation matrix. Indeed, for each source extraction, we initialize the algorithm with one of the unused singular vectors corresponding to the signal subspace.

4 CALIBRATION

Once the sources $s_p(n)$, $1 \le n \le N$, have been estimated, it is possible to invert the relation $Y = AS + W$ in the LS sense [6] and we can now compute the calibration.

4.1 Entries of A

¿From the assumptions [**A5**], [**A6**], since F is diagonal and $G_1 = 1$, one can easily deduce that A is equal to :

$$A = \begin{pmatrix} F_1 & \cdots & F_P \\ G_2 D_{21} F_1 & & G_2 D_{2P} F_P \\ \vdots & & \vdots \\ G_2 D_{K1} F_1 & & G_K D_{KP} F_P \end{pmatrix} \tag{6}$$

The specific structure of A will be now used to estimate G and F.

4.2 Estimation of F and G

As (6) shows, the first row of $\hat{A}$ gives a straight estimation of the entries of F : $\hat{F}_p = \hat{A}_{1p}$.

Thanks to $\hat{F}$, we can deduce $\widehat{GD} = \hat{A}\hat{F}^{-1}$ and since D has unit modulus entries, it obviously comes out that $Diag(\widehat{GD}\widehat{GD}^\dagger) = \hat{G}\hat{G}^\dagger$ which gives $\hat{G}_k$ (since the phase of G_k is included in the corresponding sensor phase error).

4.3 Estimation of sensor phase and location

Now, the only unknowns we are left with are the sensor phases and locations and the Angles of Arrival (AOA).

Since D has unit modulus entries, one can express its elements as : $D_{kp} = exp(i\Psi_{kp})$. With assumption [**A7**], the phase Ψ_{kp} rewrites :

$$\Psi_{kp} = \frac{\omega}{c}\left(x_k sin(\theta_p) + y_k cos(\theta_p)\right) + \Phi_k \tag{7}$$

where (x_k, y_k) is the true location of the k^{th} sensor and Φ_k its phase.

Now denote (x_k^o, y_k^o) the nominal k^{th} sensor location and θ_p^o an initialization of the p^{th} AOA; one can use, for example, the so-called BlindMaxCorr technique [2] or the method used in [14]. Then (7) becomes :

$$\begin{aligned}\Psi_{kp} &= \frac{\omega}{c}\Big((x_k^o + \delta x_k) sin(\theta_p^o + \delta\theta_p) \\ &\quad + (y_p^o + \delta y_p) cos(\theta_p^o + \delta\theta_p)\Big) + \Phi_k\end{aligned}$$

where $(\delta x_k, \delta y_k)$ and $\delta\theta_p$ are respectively the location errors and the AOA errors. If we suppose them sufficiently small, a first-order Taylor expansion leads to :

$$\begin{aligned}\frac{c}{\omega}\Psi_{kp} &= x_k^o sin\theta_p^o + y_k^o cos\theta_p^o \delta x_k sin\theta_p^o + \delta y_k cos\theta_p^o \\ &\quad + \delta\theta_p (x_k^o cos\theta_p^o + y_k^o sin\theta_p^o) + \Phi_k\end{aligned}$$

so we can estimate $(\delta x_k, \delta y_k)$ and Φ_k in the LS sense.

Note that the computation of Ψ_{kp} needs a phase unwrapping and that some indeterminacies are inherent in the problem. As shown in [3], a manner to fix them and restore identifiability in the case of far-field linear equispaced array is to assume that a second sensor is completly known (true location and phase) and that there are at least 3 source signals.

5 COMPUTER RESULTS

Simulations have been carried out using a uniformly spaced linear array with $K = 4$ sensors. The first sensor was taken as the reference and we assumed that the last one was completly known. The element spacing was $\lambda/2$, where λ is the wavelength of the propagating waveforms. We used $P = 3$ BPSK sources whose AOAs were $\theta_1 = -15°$, $\theta_2 = 3°$ and $\theta_3 = 20°$ and they were partially correlated (arround 30%). We tested our algorithm with datalength $N = 1000$, over 500 trials and for various SNRs. The calibration errors were simulated as follows :

$$
\begin{aligned}
G_k &= 1 + \sigma_G |w_G(k)|, k \neq 1 \\
\Phi_k &= \sigma_\Phi w_\Phi(k) \\
x_k &= x_k^o + \sigma_L w_x(k), k \neq 1, K \\
y_k &= y_k^o + \sigma_L w_y(k), k \neq 1, K
\end{aligned}
$$

where w_G, w_Φ, w_x and w_y are independent uniform processes in $]-1,\ 1[$, and $\sigma_G = 1$, $\sigma_\Phi = 0.05$ and $\sigma_L = 4\%$ of the sensor spacing.

Figure 1 shows the behaviour of raw MUSIC in presence of decalibration and with a $20dB$ SNR. It shows that the algorithm is either unable to detect all the sources or unable to estimate accuratly the AOAs.

The estimation results are sumarized in table 1 in terms of the Median of the Quadratic Error on the DOAs (MQED), on the sensors gain (MQEG), on the sensors phase (MQEP), on the sensor location (MQEx,MQEy). Under each median we also indicate the standard deviation.

If we consider the median of the quadratic errors, the behaviour of the algorithm appears to be quite good. However an overview of the standard deviations shows that the proposed algorithm may be unreliable. Nevertheless, even though it reveals the existence of ill-convergences due to the correlation between source signals, they can be detected by looking at the intercorrelation between the last estimated signal and the previous ones. For the moment, this test has not been implemented yet. After calibrating the array, the MUSIC response recovers a performance close to nominal, according to figure 2.

6 CONCLUSIONS

We proposed a new self-calibration method which does not need cooperating sources. It is solely based on the fact that the impinging source signals have a known discrete distribution. This feature is used in the estimation of the source signals which allows a Least-Square estimation of the mixing matrix A. The decalibration is then computed using the particular structure of A. The results are quite good as far as the median of the quadratic errors is concerned but the algorithm suffers from a few ill-convergences that can be easily detected by a correlation test.

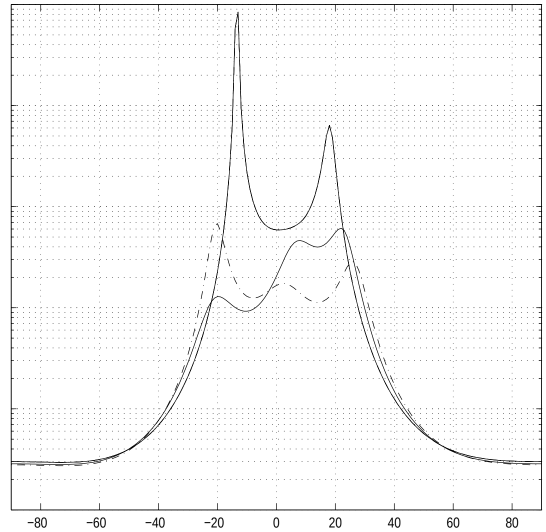

Figure 1: 3 typical responses of MUSIC in presence of decalibration.

SNR(dB)	MQED	MGEG	MQEP	MQEx	MQEy
30	$1.3e^{-6}$	$4e^{-7}$	$6e^{-9}$	$7e^{-7}$	$4e^{-4}$
	$4e^{-2}$	$7e^{-3}$	$2e^{-3}$	$5e^{-2}$	29.2
25	$4e^{-6}$	$1.4e^{-6}$	$1.9e^{-8}$	$2.1e^{-6}$	$1.2e^{-3}$
	$7.8e^{-2}$	$2e^{-2}$	$2e^{-3}$	$7.4e^{-2}$	43.6
20	$1.3e^{-5}$	$4.5e^{-6}$	$6e^{-8}$	$6.7e^{-6}$	$4e^{-3}$
	0.1	$2.5e^{-2}$	$2e^{-3}$	0.4	226
15	$4.2e^{-5}$	$1.5e^{-5}$	$2e^{-7}$	$2.2e^{-5}$	$1.3e^{-2}$
	0.1	$1.5e^{-2}$	$2e^{-3}$	0.57	342
10	$1.6e^{-4}$	$8.2e^{-5}$	$9e^{-7}$	$1e^{-4}$	$6.1e^{-2}$
	0.1	$1.6e^{-2}$	$2e^{-3}$	0.7	394

Table 1: Decalibration estimation results for various SNR

APPENDIX

Given a number z in the complex plane, consider the the two optimization criteria $\Psi(z) = \min_j |z - C_j|^2$ and $\Phi(z) = \prod_j |z - C_j|^2$, and denote by ϵ the distance between z and the closest element of the constellation, C_p. Then $\Phi(z)$ can be written as:

$$
\begin{aligned}
\Phi(z) &= |z - C_p|^2 \prod_{j \neq p} |z - C_j|^2 \\
&= \Psi(z) \prod_{j \neq p} |C_p - C_j + \epsilon|^2
\end{aligned}
$$

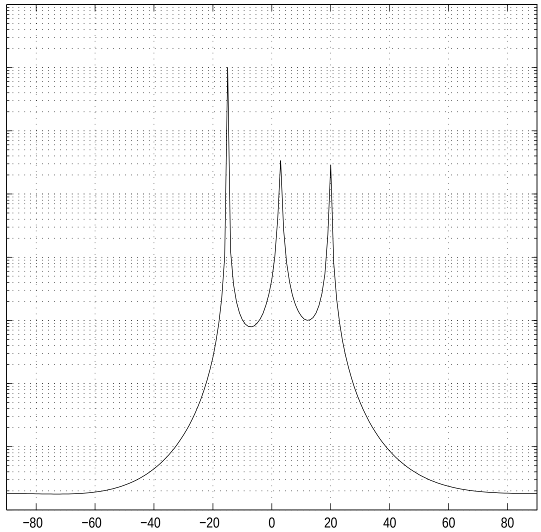

Figure 2: MUSIC response after compensating for errors with the help of the calibration matrix.

$$\begin{aligned}
&= \Psi(z)\prod_{j\neq p}|C_p - C_j|^2 \prod_{j\neq p}|1+\frac{\epsilon}{C_p - C_j}|^2 \\
&= \Psi(z)K(p)\prod_{j\neq p}|1+\frac{\epsilon}{C_p - C_j}|^2 \\
&= \Psi(z)K(p)\prod_{j\neq p}\left(1+2Re\left[\frac{\epsilon}{C_p - C_j}\right]+\frac{|\epsilon|^2}{|C_p - C_j|^2}\right) \\
&= \Psi(z)K(p)\left(1+f_p(\epsilon)\right)
\end{aligned}$$

A constellation $\mathcal{C}$ is said to have a constant power if $\exists\gamma/\forall c\in\mathcal{C},\ c^\gamma = 1$. This property characterizes usual PSK modulations. For these constellations, $K(p)$ and $f_p(\epsilon)$ do not depend on p, and we have:

$$\Psi(z) = \frac{1}{K(1+f(\epsilon))}\Phi(z).$$

If ϵ is small compared to the distance between constellation symbols (low noise assumption), then $|f(\epsilon)| \ll 1$, and a Taylor expansion is possible and yields:

$$\Psi(z) = \frac{1}{K}\,\Phi(z)(1+g(\epsilon)) + o(\epsilon).$$

Now let $z_n = \mathbf{b}^*\mathbf{y}(n)$. Then the MAP solution can be rewritten as $\text{Arg Min}_{\hat{\mathbf{b}}}\sum_n \Psi(z_n)$, with the notation of this appendix, and eventually takes the form:

$$\frac{1}{K}\text{Arg}\,\underset{\hat{\mathbf{b}}}{\text{Min}}\sum_n \Phi(z_n)(1+g(\epsilon_n)) \qquad (8)$$

which can be approximated by

$$\frac{1}{K}\text{Arg}\,\underset{\hat{\mathbf{b}}}{\text{Min}}\left[(1+\eta)\sum_n \Phi(z_n)\right],$$

where $\eta = \left[\frac{1}{N}\sum_n \Phi(z_n)g(\epsilon_n)\right]\left[\frac{1}{N}\sum_n \Phi(z_n)\right]^{-1}$ keeps small for bounded N or uniformly small ϵ_n. This eventually shows the MAP equivalence for PSK constellations and for sufficiently low noise levels.

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