HIGH RESOLUTION ARRAY PROCESSING FOR NON CIRCULAR SIGNALS

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ABSTRACT

We present in this article a direction of arrival estimation algorithm for non circular sources. We show how to take into account non circularity of signals in array processing and develop extensions of classical algorithms. The main improvement linked to the non circularity concerns the resolution, the variance of estimation and the number of resolvable sources. These characteristics are illustrated by simulations and theoretical analysis.

1 INTRODUCTION

Classical sensor array processing has been studied for many years and a lot of dedicated publication can be found. New methods based on specific signal properties have also been presented. The aim of these new methods is to introduce as much information as possible concerning the signal in order to improve the processing performances. This a priori information could be cyclostationarity or high order statistics for non gaussian signals.

More recently, several paper have studied the characteristics of non circular complex random signals [1] [5].

We present in this article a method allowing to introduce this characteristic in array processing technics. The algorithm used here is MUSIC but the approach can be extended to other algorithms.

2 NON CIRCULAR SIGNALS

The aim of this part is to present (briefly) the main characteristics of non circular signals that is used throughout this paper. We do not take into consideration statistics of order higher than two. Moreover, the notations used in this paper are : ^{T} for *transpose*, * for *conjuguate* et ^{H} for *transpose conjuguate*.

A vector of random variables Z is said non circular (at the order two) if $E\left\{ZZ^T\right\} \neq \mathbf{0}$; wich, in term of probability density, means that no angle $\phi \neq k\pi$ can be found such that Z and $Ze^{i\phi}$ would have the same probability density.

In array processing, circularity is a natural property in narrow band analysis because the signal phase is often uniformly distributed in $[0, 2\pi]$. Nevertheless, it is not universal and we can find a lot of signals wich are not circular (Amplitude Modulation, Binary Phase Shift Keying, ...)

For such signals, classical array processing (using only $E\left\{ZZ^H\right\}$) are sub optimal and significant gains are expected by conducting a complete analysis (using $E\left\{ZZ^H\right\}$ and $E\left\{ZZ^T\right\}$).

3 MUSIC FOR NON CIRCULAR SIGNALS

3.1 Notations

Let us consider an array of M sensors receiving the contribution of P sources.

The observation vector can be written as follow :

$$\mathbf{y}(t) = \mathbf{A}.\mathbf{x}(t) + \mathbf{b}(t) \tag{1}$$

with

- $\mathbf{y}(t) \in \mathcal{C}^{M\times 1}$: Observation at time t.
- $\mathbf{x}(t) \in \mathcal{C}^{P\times 1}$: Emitted signal of the P sources.
- $\mathbf{b}(t) \in \mathcal{C}^{M\times 1}$: Additive noise.
- $\mathbf{A} \in \mathcal{C}^{M\times P}$: Steering matrix.

$$\mathbf{A} = [a_1, a_2, \ldots, a_P] \tag{2}$$

$$a_k^T = \left[1, e^{-j\phi_1}, \ldots, e^{-j\phi_{M-1}}\right] \tag{3}$$

The P sources are assumed narrow-band, centered on ν_0, uncorrelated and $P < M$.

Moreover, let us consider the following assumptions :

- $E\{\mathbf{b}(t)\} = \mathbf{0}$ and $E\{\mathbf{x}(t)\} = \mathbf{0}$
- $E\{\mathbf{b}(t)\mathbf{b}(t)^H\} = \sigma^2\mathbf{Id}$ and $E\{\mathbf{b}(t)\mathbf{b}(t)^T\} = \mathbf{0}$
- $E\{\mathbf{x}(t)\mathbf{x}(t)^H\} = \Gamma_1$ and $E\{\mathbf{x}(t)\mathbf{x}(t)^T\} = \Gamma_2$

3.2 Principle of the algorithm

The classical correlation matrix defined by

$$\mathbf{R} = E\{\mathbf{y}(t)\mathbf{y}(t)^H\} = \mathbf{A}\Gamma_1\mathbf{A}^H + \sigma^2\mathbf{Id} \qquad (4)$$

summarizes completely the statistical characteristics of the received signal when the sources are gaussian and circular. However, if the emitted signals are not circular, we have to take into account not only $E\{\mathbf{y}(t)\mathbf{y}(t)^H\}$ but also $E\{\mathbf{y}(t)\mathbf{y}(t)^T\}$.

This can be realised by using the vector $\mathbf{y_{nc}}(t)$:

$$\mathbf{y_{nc}}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \mathbf{y}(t)^* \end{bmatrix} \qquad (5)$$

With this new observation vector, it is possible to define a correlation matrix summarizing all the second order statistical characteristics of the received signal :

$$\mathbf{R_{nc}} = E\{\mathbf{y_{nc}}(t)\mathbf{y_{nc}}(t)^H\} \qquad (6)$$

It can be show [3] that $\mathbf{R_{nc}}$, a $2M \times 2M$ matrix, is written as :

$$\mathbf{R_{nc}} = \mathbf{A}_{nc}\mathbf{\Gamma_{nc}}\mathbf{A}_{nc}^H + \sigma^2\mathbf{Id} \qquad (7)$$

where $\mathbf{A}_{nc}\mathbf{\Gamma_{nc}}\mathbf{A}_{nc}^H$ is a $2P$ rank matrix.

Consequently, we can distinguish a signal subspace spanned by the first $2P$ eigenvectors of $\mathbf{R_{nc}}$ and a noise subspace spanned by the $2M-2P$ last eigenvectors of $\mathbf{R_{nc}}$. If we have a steering vector model, the principle of the algorithm can be easily deduced from the classical MUSIC algorithm.

4 AMPLITUDE MODULATED SIGNALS

Let us consider the case of amplitude modulated signals. The signal $s_p(t)$ emitted by the p source is written as :

$$s_p(t) = e_p(t)e^{-2\imath\pi\nu_0 t} \qquad (8)$$

After demodulation and low-pass filtering, the signal can be written as :

$$x_p(t) = e_p(t)e^{\imath\Psi_p} \qquad (9)$$

Evidently, these signals are not circular and moreover :

$$E\{\mathbf{x}(t)\mathbf{x}(t)^H\} = \Gamma_1 \qquad (10)$$

and

$$E\{\mathbf{x}(t)\mathbf{x}(t)^T\} = \Gamma_1\mathbf{\Phi} \qquad (11)$$

with

$$\mathbf{\Phi} = \begin{bmatrix} e^{\imath 2\Psi_1} & 0 & \cdots & 0 \\ 0 & e^{\imath 2\Psi_2} & 0 & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & e^{\imath 2\Psi_P} \end{bmatrix} \qquad (12)$$

For this kind of signals, expression (7) becomes after simplification :

- $\mathbf{\Gamma_{nc}} = \Gamma_1$
- $\mathbf{A_{nc}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^*.\mathbf{\Phi}^* \end{bmatrix}$

These situation is particularly interesting since the dimension of the signal subspace is P even though the dimension of the observation is $2M$. It is then possible to locate up to $2(M-1)$ sources (to be compared at $M-1$ sources with standard MUSIC).

The algorithm implementation is very similar to the standard one. After the eigen elements decomposition of $\mathbf{R_{nc}}$, we define the signal and noise subspaces with respectively the first P eigenvectors of $\mathbf{R_{nc}}$ and the last $2M-P$ eigenvectors of $\mathbf{R_{nc}}$:

$$\mathbf{R_{nc}} = \mathbf{U_{s\_nc}}\Lambda_{nc}\mathbf{U_{s\_nc}}^H + \sigma^2\mathbf{U_{b\_nc}}\mathbf{U_{b\_nc}}^H \qquad (13)$$

The steering vector can be deduced by the expression of $\mathbf{A_{nc}}$:

$$\mathbf{a_{nc}}(\theta,\Psi) = \begin{bmatrix} \mathbf{a}(\theta) \\ \mathbf{a}(\theta)^*.e^{2i\Psi} \end{bmatrix} \qquad (14)$$

where $\mathbf{a}(\theta)$ is the classical steering vector representing the phase shifts between sensors for a source with parameter θ. This steering vector depends on the direction of arrival but also on the demodulation phase Ψ_p for the source p. Since the eigenvectors associated with the noise are orthogonal to $\mathbf{A_{nc}}$, we obtain, when θ and Ψ correspond to the true parameters of a sources :

$$\mathbf{U_{b\_nc}}^H\mathbf{a_{nc}}(\theta,\Psi) = 0 \qquad (15)$$

This lead to maximise the expression :

$$f(\theta,\Psi) = \frac{1}{\mathbf{a_{nc}}(\theta,\Psi)^H\mathbf{U_{b\_nc}}\mathbf{U_{b\_nc}}^H\mathbf{a_{nc}}(\theta,\Psi)} \qquad (16)$$

The expression (16) is a two dimension functionnal. Meanwhile, for a given value of θ, it is possible to compute analyticaly (see [3] for demonstration) the value of Ψ maximising (16).

The maximisation of (16) is then reduced to the maximisation of a one dimensional functionnal :

$$f(\theta) = \frac{1}{P_1 - ||P_{12}||} \qquad (17)$$

with

- $\mathbf{U_{b\_nc}} = \begin{bmatrix} \mathbf{U_1} \\ \mathbf{U_2} \end{bmatrix}$
- $P_1 = \mathbf{a}(\theta)^H\mathbf{U_1}\mathbf{U_1}^H\mathbf{a}(\theta)$
- $P_{12} = \mathbf{a}(\theta)^T\mathbf{U_2}\mathbf{U_1}^H\mathbf{a}(\theta)$
- $\mathbf{a}(\theta)$: classical steering vector.

5 MORE SOURCES THAN SENSORS

As $\mathbf{y_{nc}} = \begin{bmatrix} 0 & \mathbf{Id} \\ \mathbf{Id} & 0 \end{bmatrix} \mathbf{y_{nc}}^*$ it can be shown that the eigenvectors $\mathbf{U_1}$ and $\mathbf{U_2}$ are in relationship :

$$\mathbf{U_1} = \mathbf{U_2}^H \mathbf{D} \tag{18}$$

where $\mathbf{D}$ is a unitary diagonal matrix.

When the dimension of the noise subspace is equal to one ($2M-1$ signals) then $P_1 = \|P_{12}\|$ whatever the value of the angle θ. The maximum number of localizable signals is equal to $2(M-1)$.

We can find an intuitive solution of this problem. We can imagine that we make a 'virtual' antenna which holds the informations about $\mathbf{y}(t)^*$ (figure 1). Yet, the reference sensor used for the first antenna is also used for the virtual one. So, the number of 'sensors' becomes $2M-1$ for this kind of treatment, and we can locate $2M-2$ sources.

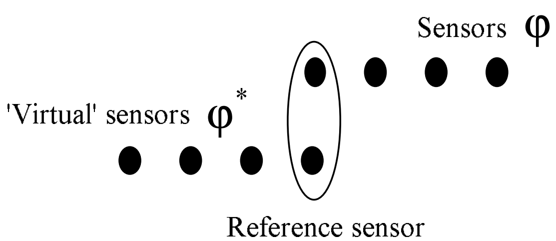

Figure 1: Localization of 2M-2 sources with M sensors

6 THEORETICAL VARIANCE

The MUSIC functionnal to be minimized reads :

$$f(\theta, \Psi) = trace(\hat{\mathbf{\Pi}}_\mathbf{n} \mathbf{a_{nc}}(\theta, \Psi) \mathbf{a_{nc}}(\theta, \Psi)^H) \tag{19}$$

which can be written as :

$$f(\theta, \Psi) = trace(\hat{\mathbf{\Pi}}_\mathbf{n} \mathbf{S}) \tag{20}$$

where $\hat{\mathbf{\Pi}}_\mathbf{n}$ is the estimated projector on the noise subspace.

The computation of the variance is based on a first order expansion of the first order derivatives of $f(\theta, \Psi)$:

$$\mathbf{0} = \dot{\mathbf{f}} + \mathbf{H} \begin{bmatrix} \mathbf{\Delta}\theta \\ \mathbf{\Delta}\psi \end{bmatrix} \tag{21}$$

with

- $\dot{\mathbf{f}} = \begin{bmatrix} \frac{\partial f}{\partial \theta}(\theta_0, \Psi_0) \\ \frac{\partial f}{\partial \Psi}(\theta_0, \Psi_0) \end{bmatrix}$
- $\mathbf{H} = \begin{bmatrix} \frac{\partial^2 f}{\partial^2 \theta}(\theta_0, \Psi_0) & \frac{\partial^2 f}{\partial \Psi \partial \theta}(\theta_0, \Psi_0) \\ \frac{\partial^2 f}{\partial \theta \partial \Psi}(\theta_0, \Psi_0) & \frac{\partial^2 f}{\partial^2 \Psi}(\theta_0, \Psi_0) \end{bmatrix}$

and a first order expansion of $\hat{\mathbf{\Pi}}_\mathbf{n} = \mathbf{\Pi_n} + \delta \mathbf{\Pi_n}$.

It can be shown [4] that :

$$-\delta \mathbf{\Pi_n} = \delta \mathbf{\Pi_S} = \mathbf{M \Delta R \Pi_n} + \mathbf{\Pi_n \Delta R M} \tag{22}$$

with

$$\mathbf{M} = \sum_{i=1}^{P} \frac{1}{\lambda_i - \lambda} \mathbf{u_i u_i}^H \tag{23}$$

- $\mathbf{u_i}$ signal subspace eigenvectors
- λ noise power
- $\mathbf{\Delta R}$ errors in the covariance matrix estimation.

The error covariance $Cov \begin{bmatrix} \mathbf{\Delta}\theta \\ \mathbf{\Delta}\psi \end{bmatrix} = \mathbf{H}^{-1} E[\dot{f} \dot{f}^T] \mathbf{H}^{-1}$ is computed from the following quantities :

- $\frac{\partial f}{\partial \theta}(\theta_0, \Psi_0) = -trace(\delta \mathbf{\Pi_S} \frac{\partial \mathbf{S}}{\partial \theta})$
- $\frac{\partial f}{\partial \Psi}(\theta_0, \Psi_0) = -trace(\delta \mathbf{\Pi_S} \frac{\partial \mathbf{S}}{\partial \Psi})$
- $\frac{\partial^2 f}{\partial^2 \theta}(\theta_0, \Psi_0) = 2trace(\mathbf{\Pi_n} \frac{\partial \mathbf{a_{nc}}}{\partial \theta}^H)$
- $\frac{\partial^2 f}{\partial \Psi \partial \theta}(\theta_0, \Psi_0) = 2Real(\frac{\partial \mathbf{a_{nc}}}{\partial \theta}^H \mathbf{\Pi_n} \frac{\partial \mathbf{a_{nc}}}{\partial \Psi})$

In order to compute $E[\dot{f}\dot{f}^T]$ one needs to evaluate terms like

$$E[trace(\delta \mathbf{\Pi_S A}) trace(\delta \mathbf{\Pi_S B})] = E[trace(\mathbf{\Delta R A_1}) trace(\mathbf{\Delta R B_1})] \tag{24}$$

with $\mathbf{A_1} = \mathbf{MA\Pi_n} + \mathbf{\Pi_n AM}$.

They may be expressed as [2] :

$$E[trace(\mathbf{\Delta R A_1}) trace(\mathbf{\Delta R B_1})] = \frac{1}{N}[trace(\mathbf{RB_1RA_1}) + trace(\tilde{\mathbf{R}} \mathbf{B_1^T} \tilde{\mathbf{R}}^* \mathbf{A_1})] \tag{25}$$

with $\mathbf{R} = E[\mathbf{y_{nc} y_{nc}}^H]$, and $\tilde{\mathbf{R}} = E[\mathbf{y_{nc} y_{nc}}^T]$. Variance of Music and Non Circular Music are compared in the next section.

7 ILLUSTRATIONS

The ability of the algorithm to resolve more sources than sensors has been verified with many simulations. We have performed theoritical and experimental studies of both MUSIC and Non Circular MUSIC estimation variance and resolution. They showed that Non Circular MUSIC outperform MUSIC.

In the simulations, we choose an array of 4 sensors ($\theta_{3db} = 45$ degrees). Six sources are simulated and their directions of arrival (DOA) are equal to $-60, -30, -10, 0, 20, 50$ degrees. The results of the DOA estimation with Non Circular MUSIC are presented on figure 2 for 100 simulations. With 4 sensors, we can locate 6 sources.

On figure 3 and 4, 2 sources are simulated. The DOA are equal to 0 and -3 degrees. The figure 3 shows the

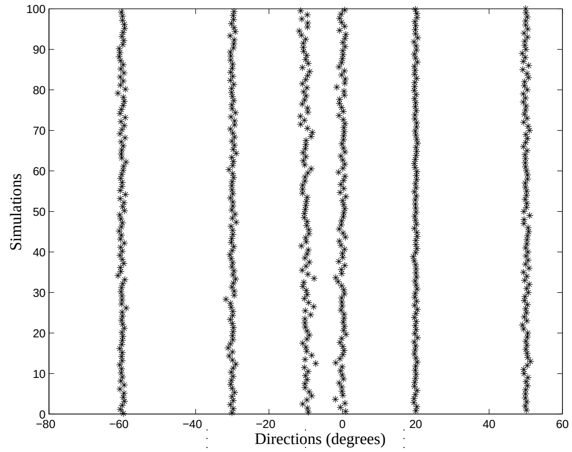

Figure 2: Localization of 6 sources with 4 sensors

results of the DOA estimated by the classical MUSIC algorithm for 100 simulations. The figure 4 shows the results for the same simulations with the modified algorithm and we can see the improvement in term of variance of estimation.

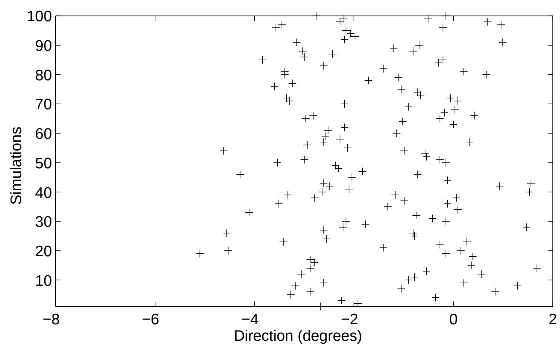

Figure 3: Localization with MUSIC

On figure 5, 2 sources are simulated. The DOA are equal to 0 and the second DOA varies from -65 to -5 degrees. This simulation shows the theoretical (lines) and experimental ($*$) standard deviation for the 2 sources. We can see that the standard deviation is divided by 10 when the 2 sources are close (5 degrees).

8 CONCLUSION

We have presented a new approach allowing to introduce the non circularity in the estimation of the angular localization of sources. The developed method, without drastically increasing the complexity of the MUSIC algorithm ,allows us to take into account the specificity of non circular signals. The performances of the estimator are significantly improved. The main improvements concern the number of resolvable sources , the variance of estimation and the resolution.

The algorithm used here is an extension of the MUSIC algorithm but other array processing, such Capon or

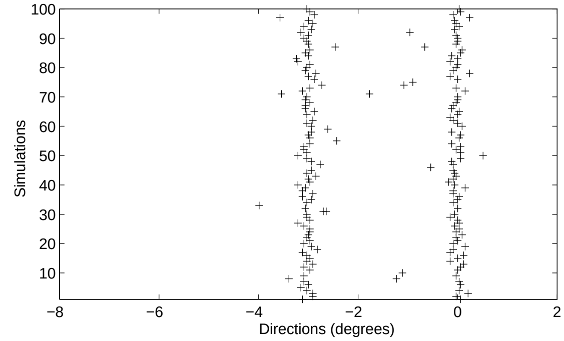

Figure 4: Localization with Non Circular MUSIC

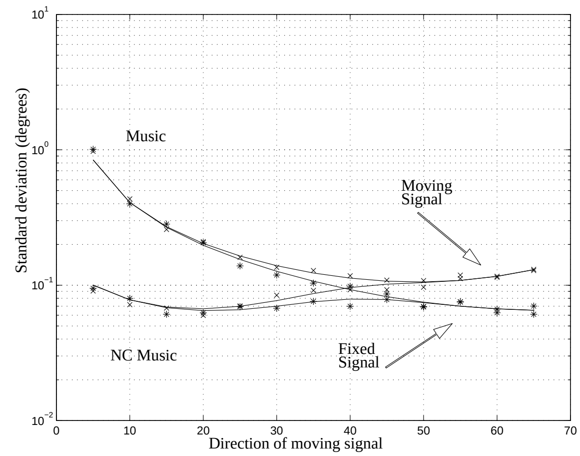

Figure 5: Standard deviation

root-MUSIC algorithm, could be considered from the non circular correlation matrix.

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