ITERATIVE BLIND IMAGE RESTORATION USING LOCAL CONSTRAINTS

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ABSTRACT

A new method of incorporating local image constraints into blind image restoration is proposed. The local mean and variance of the degraded image are used to obtain an initial estimate of the pixel intensity bounds. As the restoration proceeds, the bounds are updated from the current image estimate. The iterative-bound algorithm shows an improvement over the use of fixed bounds taken from the blurred image, in which case underestimation of the variance occurs at edges and textures. Simulations are presented for both the fixed- and iterative-bound implementations.

1 INTRODUCTION

During image formation and recording, blurring may occur due to relative motion between the object and camera, wrong focus, and atmospheric turbulence. Noise originating in the formation process, the transmission medium, or the recording process may further degrade the image.

Image degradation can be modelled as

$$y = Dx + n, \tag{1}$$

where x, y, and n represent, respectively, the lexicographically-ordered original and degraded images, and additive white Gaussian noise. The matrix D here represents a space-invariant linear distortion.

The goal of blind image restoration is to simultaneously estimate the blur and the original image, based on partial knowledge of their characteristics. The primary difficulty is insufficient information, as the problem admits a possibly infinite number of solutions in the absence of sufficient constraints on the blur and image. The question is how to develop a set of constraints which adequately characterise the unknown quantities. A number of approaches have been reported in the literature which provide solutions to the blind image restoration problem (for a recent review, see [1], [2]).

Recently, spatially-adaptive intensity bounds have been used to regularize the ill-conditioning of the restoration when the PSF was known explicitly [3]. The bounds were determined from the local mean, variance, and maximum of the current image estimate and used in conjunction with conventional regularisation operators. In [4], fixed intensity bounds, estimated from the degraded image, were applied to blind image restoration. However, estimation of the bounds from the degraded image produces over-smoothed texture and edge regions, due to underestimation of the variance in severely-blurred regions. By re-calculating the bounds from the current image estimate, we can obtain more accurate estimates of the variance in these regions, as shown in this paper.

During successive image updates, the intensity bounds associated with uniform regions (i.e., those with variances comparable to the noise level) do not need to be re-calculated. Since uniform regions typically comprise a large part of the image, this greatly reduces the number of computations. For the remaining pixels, the method of applying new bound estimates is extremely important. If the bounds in a region are "active", then re-estimation of the bounds from the smoothed region yields progressively smaller bounds. Therefore, the constraints should not be active when the bounds are updated. This can be accomplished by removing all local constraints at the beginning of each minimisation cycle. As the local variance converges between successive image estimates, the corresponding bounds are applied.

The organisation of this paper is as follows. In Section 2, a mathematical formulation for blind image restoration is given. The procedure for determining the pixel intensity bounds is then presented in Section 3. Section 4 describes the implementation of the algorithm, which is used to generate the experimental results in Section 5. The results are discussed, and areas for further research proposed, in Section 6.

2 BLIND IMAGE RESTORATION: PROBLEM FORMULATION

Blind deconvolution can be formulated as minimisation of the following cost function with respect to $\hat{x}$ and $\hat{d}$:

$$J(\hat{x}, \hat{d}) = ||y - \hat{D}\hat{x}||^2 + \alpha||C\hat{x}||^2, \tag{2}$$

subject to the constraints:

$$\begin{cases} \hat{d}(m,n) \geq 0, & m,n \in S_D \\ \hat{d}(m,n) = 0, & \text{otherwise,} \end{cases} \tag{3}$$

$$\sum_{i,j \in S_D} \hat{d}(m,n) = 1, \tag{4}$$

and

$$\begin{cases} \hat{x}(m,n) \geq 0, & m,n \in S_X \\ \hat{x}(m,n) = 0, & \text{otherwise.} \end{cases} \tag{5}$$

In equation (2), $\hat{x}$ is the image estimate, $\hat{d}$ is the estimate of the blur PSF which is used to form the blur matrix $\hat{D}$, C is a high-pass operator, and the regularisation parameter α controls the trade-off between fidelity to the data and smoothness of the solution. It is assumed that the PSF and image supports, S_D and S_X, are known. However, the assumption that S_D is known exactly can be relaxed, and the algorithm is then implemented by beginning with a conservatively large estimate of S_D, and then gradually pruning the region of support during successive updates of the PSF [5].

3 ITERATIVE IMPLEMENTATION OF LOCAL CONSTRAINTS

The above optimisation problem is often solved by using gradient-projection methods to minimise alternately with respect to the PSF and the image. At the beginning of each minimisation cycle l, the local variance $\sigma^2_{l,0}(m,n)$ is calculated. The intensity bounds for those pixels with $\sigma^2_{l,0}(m,n) < \sigma^2_n$, where σ^2_n is the noise variance estimated from a uniform region of the degraded image, are fixed during the minimisation, according to the definition given shortly. For the remaining pixels, only the positivity and support constraints are used initially to define the lower and upper intensity bounds $U_l(m,n)$ and $L_l(m,n)$. At each iteration k of the steepest descent algorithm, the change in local variance is calculated. If

$$|\sigma^2_{l,k}(m,n) - \sigma^2_{l,k-1}(m,n)| < \gamma\sigma^2_{l,k-1}(m,n), \tag{6}$$

where γ is a small constant, then the bounds for pixel (m,n) are defined as:

$$L_l(m,n) = \begin{cases} \max(\overline{x}_{l,k}(m,n) - \beta_l\sigma^2_{l,k}(m,n), 0), & m,n \in S_X \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

$$U_l(m,n) = \begin{cases} \overline{x}_{l,k}(m,n) + \beta_l\sigma^2_{l,k}(m,n), & m,n \in S_X \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$

where the local mean $\overline{x}_{l,k}(m,n)$ and variance $\sigma^2_{l,k}(m,n)$ are measured typically over a 3×3 or 5×5 window. These bounds are subsequently fixed during the minimisation cycle l.

The parameter β_l controls the tightness of the bounds. At the end of each minimisation cycle, β_l is adjusted according to the total change in variance of the image:

$$\beta_{l+1} = \beta_0 \frac{\sum \sigma^2_{0,0}(m,n)}{\sum \sigma^2_{l,k}(m,n)}, \tag{9}$$

where $\sigma^2_{0,0}(m,n)$ is the variance of the degraded image. (It should be noted that for the initial variance estimate only, the estimated noise variance was subtracted, and any resulting negative values were set to 0.)

The projection operator expressing local smoothness is then defined as:

$$P_{l,k}(\hat{x}_{l,k}(m,n)) = \begin{cases} L_l(m,n), & \hat{x}_{l,k}(m,n) < L_l(m,n) \\ U_l(m,n), & \hat{x}_{l,k}(m,n) > U_l(m,n) \\ \hat{x}(m,n), & \text{otherwise.} \end{cases} \tag{10}$$

4 BLIND IMAGE RESTORATION ALGORITHM

The ideas presented in Section 3 are incorporated into the following algorithm.

1. Determine the initial bounds from the degraded image using β_0. Initialise the iteration numbers $l, k = 0$ and set $\hat{x}_{0,0} = P_{0,0}y$ and $\hat{d}_0 = \delta(m,n)$.

2. Increment l. Minimise (2) with respect to the PSF to obtain $\hat{d}_l$.

3. Minimise (2) with respect to the image:

   - Reset the bounds as described in Section 3.
   - Set $\hat{x}_{l,k+1} = P_{l,k}(\hat{x}_{l,k} + \mu(\hat{D}^T_l y - (\hat{D}^T_l \hat{D}_l + \alpha C^T C)\hat{x}_{l,k}))$, where the step-size μ satisfies

     $$0 < \mu < \frac{2}{\lambda_{\max}(\hat{D}^T_l \hat{D}_l + \alpha C^T C)},$$

     with $\lambda_{\max}(A)$ denoting the maximum eigenvalue of A.
   - If $||\hat{x}_{l,k+1} - \hat{x}_{l,k}||^2/||\hat{x}_{l,k}||^2 > 10^{-6}$ and $k < k_{\max}$, then calculate the change in local variance. If any bounds satisfy the convergence criterion, then update the corresponding bounds. Increment k and repeat step 3.

4. Update and apply any unconverged bounds using the last image estimate.

5. If the change in PSF estimate was significant, i.e. $||\hat{d}_l - \hat{d}_{l-1}||^2/||\hat{d}_{l-1}||^2 > 10^{-3}$, then calculate β_{l+1} and go to step 2. Reset $k = 0$.

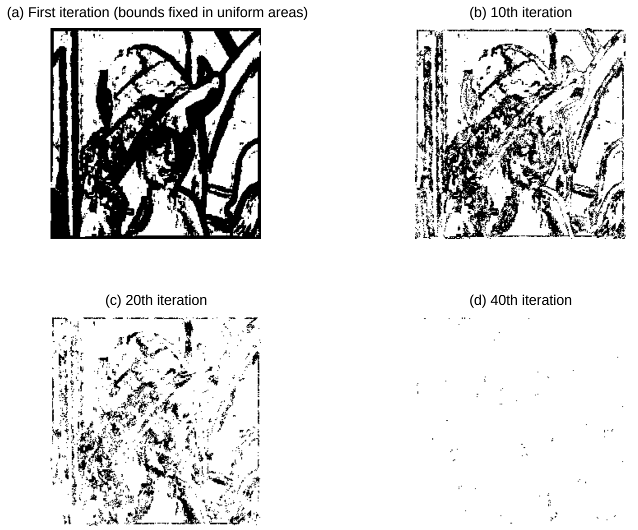

Figure 1: Convergence of intensity bounds during the first image minimisation

5 EXPERIMENTAL RESULTS

The fixed- and iterative-bound algorithms were tested on the 256×256 image of "Lena" for various noise levels and blurs. The original image was superimposed on a black background, in accordance with the assumption that the image has a known finite ROS, outside of which the intensity is negligible. The blur PSF, either 5×5 uniform or Gaussian ($\sigma^2 = 1$), was assumed to be separable and circularly symmetric, and sequential quadratic programming was used to perform the constrained nonlinear minimisation in step 2. The maximum number of function evaluations for each blur minimisation was set to $100 \times$ (no. of independent PSF parameters). For the image minimisation, the regularisation parameter α in equation (2) was set to 0, and the parameters $\mu = 1.99$ and $k_{\max} = 100$ were used in the steepest descent algorithm. The convergence parameter $\gamma = 0.01$ was chosen so that convergence of the last bounds corresponded closely to termination of the image minimisation. The parameter β was selected to give near-optimal Δ_{SNR} for the fixed-bound restoration, which then became β_0 for the iterative-bound algorithm.

As a measure of the quality of the restoration, the improvement in SNR (dB) was used:

$$\Delta_{SNR} = 10 \log_{10} \frac{||y - x||^2}{||\hat{x} - x||^2}.$$

The quality of the PSF estimate was measured by:

$$\epsilon_d = \frac{||d - \hat{d}||}{||d||}.$$

The results are shown in Table 1. For the iterative-bound algorithm, two values of β are given, corresponding to the initial and final values, respectively. In the last two columns, the number of minimisation cycles, l, and the total number of image updates, $\sum k$, are listed.

Table 1: Comparison of fixed- and iterative-bound restorations

Blur, BSNR (dB)	β_0	β_f	Δ_{SNR} (dB)	ϵ_d	l	$\sum k$
Uni., 30	15		2.65	0.70	4	32
	15	6.25	3.38	0.18	6	535
Uni., 20	7.5		2.19	0.50	4	35
	7.5	3.50	2.63	0.22	7	625
Gauss., 30	12		1.64	0.33	10	51
	12	6.85	1.50	0.10	13	827
Gauss., 20	7.5		0.95	0.31	10	51
	7.5	5.70	1.36	1.27	6	185

The progress of the iterative-bound algorithm during the first minimisation cycle is shown in Figure 1 for the uniform PSF (BSNR = 20 dB). The bounds which have not yet converged are indicated by the black pixels. It can be seen that the bounds surrounding the edges are the slowest to converge, although this is not as obvious during the first minimisation cycle, since the PSF estimate still strongly resembles a delta function. The application of the edge bounds at a later stage of the image minimisation enables better estimation of the variance in these regions. Furthermore, noise amplification due to the absence of local constraints for these pixels is partially compensated for by the noise-masking properties of the human visual system [3], [6].

It is clear from Table 1 that the use of iterative bounds generally yields a significantly better estimate of the PSF. By comparison of Figures 2–5 (BSNR = 30 dB), it can be seen that the iterative-bound implementation produces sharper image estimates, although the improvement is more noticeable for the uniform PSF, as expected. The texture region of the feathers illustrates the limitations of the variance-based bound definition, although it is here that much of the improvement is seen in the iterative implementation. While Δ_{SNR} of the iterative-bound algorithm is higher in all cases except for the Gaussian blur with 30 dB BSNR, it should be noted that the use of Δ_{SNR} as a means of comparison is slightly misleading, as the iterative-bound algorithm allows some noise amplification at the edges, which is then the primary error contribution. However, as indicated in the previous paragraph, noise at the edges is partially masked, and there may be an improvement in subjective image quality even if the change in Δ_{SNR} does not reflect this.

Figure 2: Uniform PSF, fixed bounds, $\Delta_{SNR} = 2.65$ dB

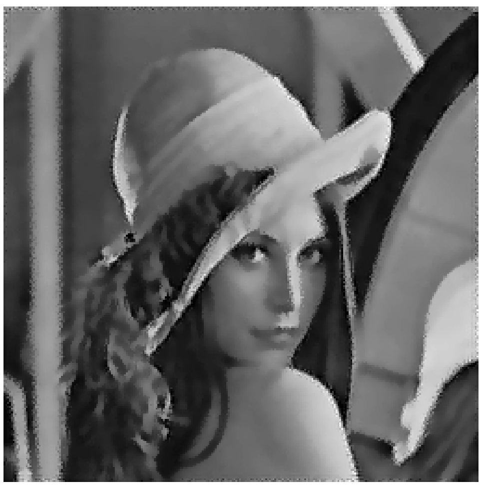

Figure 3: Uniform PSF, iterative bounds, $\Delta_{SNR} = 3.38$ dB

Figure 4: Gaussian PSF, fixed bounds, $\Delta_{SNR} = 1.64$ dB

Figure 5: Gaussian PSF, iterative bounds, $\Delta_{SNR} = 1.50$ dB

6 CONCLUSIONS

In this paper, a blind image restoration algorithm which makes use of iterative updates of local intensity bounds was proposed. Iterative and non-iterative implementations of the algorithm were compared, with the iterative method generally showing an improvement in the image and PSF estimates. Further research will examine other methods of applying iterative bounds, as well as alternative bound definitions which incorporate more accurate estimates of local activity.

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