# GEOMETRICAL DETERMINATION OF AMBIGUITIES IN BEARING ESTIMATION FOR SPARSE LINEAR ARRAYS

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## Abstract

The aim of this paper is to study the presence of manifold ambiguities [1], [2] in linear arrays. We propose a general framework for the analysis and so we obtain a generalisation of results given in recent publications [3], [4] for any rank ambiguities. We present a geometrical construction able to determine all the ambiguous directions which can appear for a given linear array. This is a geometrical approach closely connected to [4]. The method allows determination of any rank ambiguities and for each ambiguous direction set the rank of ambiguity is determined. The search is exhaustive. Application of the method requires no assumption for the linear array and is easy to implement. We apply the method to the search of ambiguities for sparse linear arrays, in particular minimum redundant and non redundant arrays [5], [6], [7]. We show how an ambiguous generator set can be associated to each intersensor distance if the intersensor distances (lags) are all multiples of the half wavelength.

## 1 INTRODUCTION

In the performance evaluation of sources localisation techniques, resolution is not the only criterion. Degradations may occur due to parasite peaks in the spectrum, which may be connected to high sidelobes in the beam pattern (sometimes referred as quasi-ambiguities) or to ambiguities themselves. In 1996, Abramovich *et. al.* proposed in [1] a study on ambiguities in DOA estimation. In particular they discussed the ideas of trivial ambiguities, manifold ambiguities and inherent ambiguities. Ambiguities in the array manifold, called manifold ambiguities [2], provide parasite peaks in the spectrum of the high resolution methods based on signal and noise subspace decomposition, like MUSIC. The aim of this paper is to study the presence of manifold ambiguities (not inherent ambiguities [1]) for a linear array of given geometry.

We propose a general framework for the analysis and thus we obtain a generalisation of results given in recent publications [3], [4] for rank one and two ambiguities. For rank $k \geq 3$ ambiguities the study is restricted to linear arrays, for which we derive original and synthetic results.

We present a geometrical construction able to determine all the ambiguous directions which can appear for a given linear array. This is a geometrical approach closely connected to [4]. The method allows determination of any rank ambiguities and for each ambiguous direction set the rank of ambiguity is determined. The search is exhaustive. Application of the method requires no assumption for the linear array and is easy to implement.

The proposed method is used to study the ambiguities in nonuniformly spaced linear arrays, with particular attention given to minimum redundancy and non redundancy arrays [5], [6], [7]. We show that an ambiguous generator set can be associated to each intersensor distance when the intersensor distances are all multiples of the half wavelength.

## 2 PROBLEM FORMULATION AND DEFINITIONS

Consider an array with $M$ sensors receiving $N$ narrowband signals impinging on the array from $N$ different locations $\theta_1,\dots,\theta_N$. Note $\mathbf{A}(\theta_1,\dots,\theta_N)=[\mathbf{a}(\theta_1),\dots,\mathbf{a}(\theta_N)]$, the matrix which columns are the sources steering vectors, also called the array manifold vectors.

The simultaneous localisation of $N$ sources is only possible if the array manifold vectors $\mathbf{a}(\theta_1),\dots,\mathbf{a}(\theta_N)$ are linearly independent.

An array is said rank $k$ ambiguous for a set of $k+1$ directions of arrival $\theta_1,\dots,\theta_{k+1}$ if matrix $A$ is singular but rank $k$. This can be written [2] :

$$\exists \alpha_1 \neq 0,\dots,\alpha_{k+1} \neq 0 \text{ so that } \alpha_1\mathbf{a}(\theta_1)+\dots+\alpha_{k+1}\mathbf{a}(\theta_{k+1})=0 \quad (\alpha_1,\dots,\alpha_{k+1}) \in C^{k+1} \qquad (1)$$

## 3 RANK ONE AMBIGUITIES (FOR GENERAL ARRAYS)

The wavefronts are supposed straight-line and on the same plane as the sensors. $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{k}}_2$ being the ambiguous wave vectors for the array under consideration, the phase delay of signal $n$ from sensor $m$ to sensor one is $\varphi_{mn} = \vec{\mathbf{k}}_n \cdot \vec{\mathbf{r}}_m$, where

$\vec{\mathbf{r}}_m$ denotes the position of the $m^{\text{th}}$ sensor in half wavelength. The ambiguity condition is equivalent to :

$$\alpha_1 e^{-j\varphi_{m1}} + \alpha_2 e^{-j\varphi_{m2}} = 0 \Leftrightarrow \varphi_{m1} = \varphi_{m2} + 2n_m\pi \Leftrightarrow$$
$$\exists p_m, \text{ integer } \left(\vec{\mathbf{k}}_1 - \vec{\mathbf{k}}_2\right)\vec{\mathbf{r}}_m = 2\,p_m\pi \qquad (2)$$

with $|\vec{\mathbf{k}}| = 2\pi/\lambda$ where $\lambda$ stands for the wavelength. The consequence is that, for arrays of arbitrary geometry, rank 1 ambiguities can arise if all of its sensors are located on a set of parallel lines separated by a distance $a > \lambda/2$. In the case of a linear array this result refunds the classical Shannon condition.

## 4 RANK $k$ AMBIGUITIES FOR LINEAR ARRAYS

Let us consider an array, assumed to present a rank $k$ ambiguity. By generalisation of previous results, see [8], [9], [10], we infer that the sensor array can be splitted in $k$ subarrays (which may be reduced to one sensor). In each subarray sensors are located on a grid of spacing denoted $a$. The $k$ grids are translated one from another. For the first grid $\vec{\mathbf{r}}_m = aN_m\vec{\mathbf{v}}$, where $\vec{\mathbf{v}}$ is the unitary vector of the linear array. Let us denote $\vec{\mathbf{k}} = (2\pi/\lambda)\vec{\mathbf{u}}$. The ambiguity condition can be written [9], [10]:

$$\vec{\mathbf{v}}\left(\vec{\mathbf{u}}_i - \vec{\mathbf{u}}_j\right) = n_{ij}(\lambda/a) \qquad (3)$$

where $n_{ij}$ is an integer. The corresponding set of ambiguous directions $\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2, \ldots, \vec{\mathbf{u}}_{k+1}$ may be obtained by the following geometrical construction.

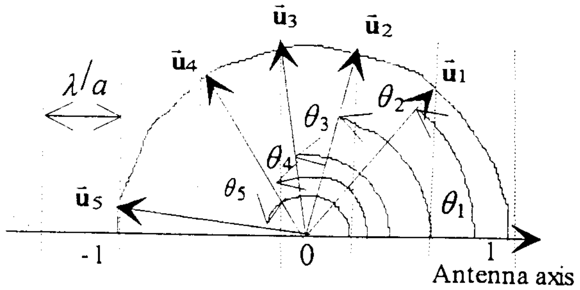


Figure 1 : Determination of the ambiguous directions of arrival $\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2, \ldots, \vec{\mathbf{u}}_5$ for a linear array.

Thus all the sets of vectors $\vec{\mathbf{u}}_1, \ldots, \vec{\mathbf{u}}_{k+1}$ which can be projected on the grid of step $\lambda/a$ are ambiguous. By arbitrary translation of this grid, an infinity of ambiguous direction sets can be obtained. One of these grids has a line passing by the point (1,0), this grid can be identified with the concept of "ambiguous generator set" introduced in [4].

It appears clearly on this figure that the condition for no rank $k$ ambiguities is :

$$k(\lambda/a) > 2 \qquad (4)$$

Based on the above considerations, we propose a geometrical method for the determination of all the ambiguous generator sets for a given linear array. For each ambiguous generator set the corresponding rank of ambiguity is also determined.

Let us consider a linear array of $M$ sensors. In order to determine all the ambiguous generator sets :

1- Compute all the intersensor distances. Note $r_{ij} = |\vec{\mathbf{r}}_j - \vec{\mathbf{r}}_i|$, $i = 1, \ldots, M$ and $j = 1, \ldots, M$, the intersensor distance in half wave-length.

2- All the intersensor distances $r_{ij}$ smaller than 1 cannot provide ambiguities because $(\lambda/r_{ij}) > 2$. This result follows from the construction depicted on figure 2. We will therefore work on the set $\mathfrak{R}$ of distinct intersensor distances $r_{ij}$, greater than one half wavelength.

3- Consider the first intersensor distance $r_{ij} \in \mathfrak{R}$. Compute the corresponding ambiguous generator set by using the geometrical construction with $a = r_{ij}$ (see figure 2).

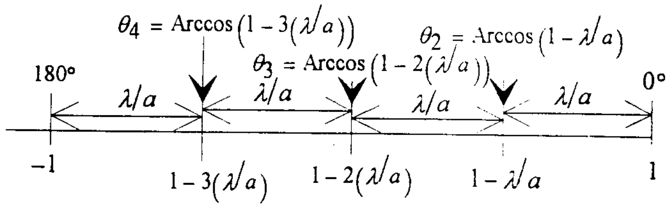


Figure 2 : Determination of the ambiguous generator set $\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\}$ for a linear array.

The ambiguous generator set is $\{0°, \theta_2, \ldots, \theta_l\}$, where $l$ is the number of ambiguous directions for this generator set.

4- The rank of ambiguity corresponding to this ambiguous generator set is given by the number of subarrays. Therefore, it is necessary to split the array into subarrays so that in each subarray sensors are located on grids of step $a$. The grids are all translated one from another. The construction must be done in order to get a minimum of subarrays. Note $q$ the number of subarrays.

5- If $q \geq l$, there is no ambiguous generator set for this value of $r_{ij}$.

If $q < l$, then the array presents a rank $q$ ambiguity, the ambiguous generator set is given by $\{0°, \theta_2, \ldots, \theta_l\}$.

6- Continue with the step 3- until all the intersensor distances of the set $\mathfrak{R}$ have been taken under consideration.

The method is very easy to implement and needs no assumption. Thus all ambiguous generator sets are determined for the considered linear array.

## 5. ILLUSTRATION OF THE METHOD BY AN EXAMPLE

The proposed method can be applied to linear arrays which intersensor distances are integers or reels. In [9] we have studied arrays which intersensors distances are reels. Here we consider the family of sparse linear arrays. All the intersensor distances are integers (in half wavelength).
Let us consider the non redundant four sensors array on figure 3.

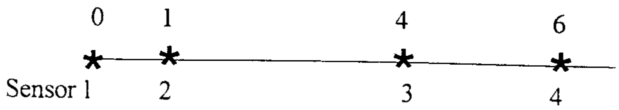


Figure 3 : Non redundant array of four sensors. The stars represent the sensors. The numbers represent the sensor positions.

First we search all the distinct intersensor distances : $r_{12} = 1$, $r_{34} = 2$, $r_{23} = 3$, $r_{13} = 4$, $r_{24} = 5$, $r_{14} = 6$. This array is called an optimum non redundant array because it presents a zero redundancy on spacings. Thus there is one, and only one, pair of elements separated by each multiple of the unit spacing out to a maximum spacing equal to the distance between the end elements. For arrays with more than four sensors, such a sensor configuration does not exist. Therefore minimum redundant and so called non redundant arrays have been obtained [5], [6], [7]. These arrays are known for their good resolution.

<u>$a$=1 :</u> $\lambda/a = 2$.

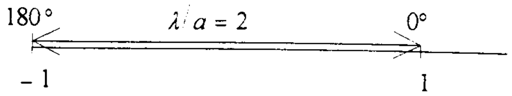


Figure 4 : Ambiguous generator set for $a$=1.

The geometrical construction gives an ambiguous generator set $\{0°, 180°\}$, thus $l$=2. All the sensors are located on a grid of step $a$=1, then $q$=1. The condition $q < l$ is verified, therefore the array presents a rank one ambiguity, the corresponding generator set being $\{0°, 180°\}$.

<u>$a$=2 :</u> $\lambda/a = 1$.

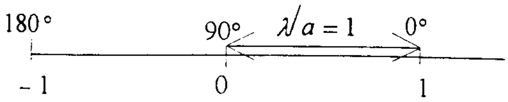


Figure 5 : Ambiguous generator set for $a$=2.

The geometrical construction gives an ambiguous generator set $\{0°, 90°, 180°\}$, thus $l$=3. Now the array is splitted in subarrays.

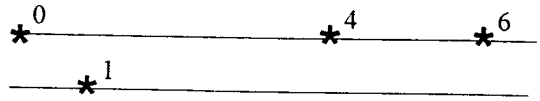


Figure 6 : Construction of the two subarrays.

The array can be splitted into two subarrays, then $q$=2. The condition $q < l$ is verified, therefore the array presents a rank two ambiguity, the corresponding generator set being $\{0°, 90°, 180°\}$.

<u>$a$=3 :</u> $\lambda/a = 2/3$. The geometrical construction gives an ambiguous generator set $\{0°, 70.5°, 109.5°, 180°\}$, thus $l$=4. The array can be splitted into two subarrays which are (0,6) and (1,4), then $q$=2. The condition $q < l$ is verified, therefore the array presents a rank two ambiguity, the corresponding generator set being $\{0°, 70.5°, 109.5°, 180°\}$.

<u>$a$=4 :</u> $\lambda/a = 1/2$. The geometrical construction gives an ambiguous generator set $\{0°, 60°, 90°, 120°, 180°\}$, thus $l$=5. The array can be splitted into three subarrays which are (0,4), (1) and (6), then $q$=3. The condition $q < l$ is verified, therefore the array presents a rank three ambiguity, the corresponding generator set being $\{0°, 60°, 90°, 120°, 180°\}$.

<u>$a$=5 :</u> $\lambda/a = 2/5$. The geometrical construction gives an ambiguous generator set $\{0°, 53°, 78°, 101°, 127°, 180°\}$, thus $l$=6. The array can be splitted into three subarrays which are (0), (1,6) and (4), then $q$=3. The condition $q < l$ is verified, therefore the array presents a rank three ambiguity, the corresponding generator set being $\{0°, 53°, 78°, 101°, 127°, 180°\}$.

<u>$a$=6 :</u> $\lambda/a = 1/3$. The geometrical construction gives an ambiguous generator set $\{0°, 41°, 70°, 90°, 109°, 132°, 180°\}$, thus $l$=7. The array can be splitted into three subarrays which are (0,6), (1) and (4), then $q$=3. The condition $q < l$ is verified, therefore the array presents a rank three ambiguity, the corresponding generator set being $\{0°, 41°, 70°, 90°, 109°, 132°, 180°\}$.

In order to see the importance of the parasite peaks, we have simulated the determination of directions of arrival for this array with the well known high resolution method MUSIC.
We illustrate the rank three ambiguity. Three sources are located at 0°, 60° and 90°.

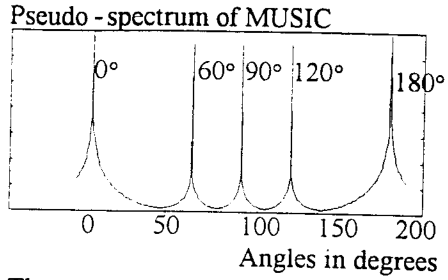


Figure 7 : Three sources are located at 0°, 60° and 90°.

Two parasite peaks appear exactly where they were predicted. The parasite peaks are here as high as the real peaks (see figure 7).
If we simulate three sources located at 0°, 41° and 90°, the predicted parasite peaks all appear but the peaks are less high (see figure 8).

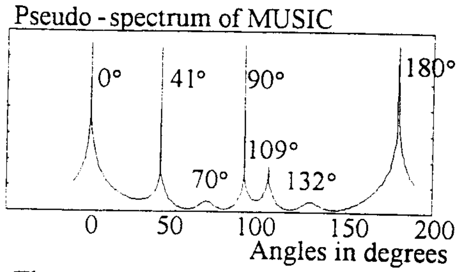


Figure 8 : Three sources are located at 0°, 41° and 90°.

The construction of minimum redundant arrays is justified by their great resolution. The above study shows that the presence of ambiguities in such arrays increases with the aperture for a given number of sensors.
In order to compare the performances we will study the uniform linear array. By applying the proposed method, it appears that there are three different intersensor distances 1, 2 and 3. For each there is one ambiguous generator set. Only three generator sets are present : rank one, $\{0°,180°\}$,rank two, $\{0°,90°,180°\}$ and rank three, $\{0°,70.5°,109.5°,180°\}$.
In the next section, a study of the presence of generator ambiguous sets is done for minimum redundant arrays and non redundant arrays. Array design by using D-optimality is also considered.

## 6 GENERAL STUDY OF AMBIGUITIES IN NON UNIFORMLY SPACED LINEAR ARRAYS

Let us now apply the proposed method to the research of ambiguous generator sets for sparse linear arrays. In many papers, [5], [6], [7] non uniformly spaced linear arrays are designed, in particular minimum redundant, non redundant and D-optimal arrays. We have study ambiguities for these arrays until 11 sensors.

In our investigations, we have done simulations on four families of linear arrays. The minimum redundant arrays proposed by Moffet [7], which can be splitted in restricted and unrestricted minimum redundant arrays. A restricted array is characterized by a minimum of redundancy under the constraint that no intersensor spacing miss. An unrestricted array is characterized by a minimum of redundancy under the constraint that the greatest number of intersensor spacings are contiguous.

The non redundant arrays are developed by Vertatschitsch [5]. No intersensor spacing is redundant but some spacings are missing. The last family is the D-optimal array [6]. It use a statistical approach.

We have computed all the ambiguous generator sets for all the arrays of these four families until 11 sensors. From these systematic simulations follow that there are the same number of ambiguous generator sets as different intersensor spacings. Thus for a given number of sensors, the non redundant arrays present the more ambiguous generator sets

Therefore application of the proposed method brings some enlighten in the study of the performances of sparse arrays. In linear arrays the research of minimum redundancy in the intersensor distances increases the risk of apparition of manifold ambiguities.

## 7. CONCLUSION

We propose a general framework to study ambiguities for general arrays. For linear arrays, a geometrical construction is presented and is able to predict all the ambiguous directions for the considered array. The presented method opens a new way to study non uniformly spaced linear arrays.

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