IMPROVING SIGNAL SUBSPACE ESTIMATION AND SOURCE NUMBER DETECTION IN THE CONTEXT OF SPATIALLY CORRELATED NOISES.

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ABSTRACT

This paper addresses the issue of Orthogonal Techniques for Blind Source Separation of periodic signals when the mixtures are corrupted with spatially correlated noises. The noise covariance matrix is assumed to be unknown. This problem is of major interest with experimental signals. We first remind that Principal Components Analysis (PCA) cannot provide a correct estimate of the signal subspace in this situation. We then decide to compute the spectral matrices using delayed blocks to eliminate the noise influence. We show that two of these delayed spectral matrices are enough to get the unnoisy spectral matrix. We also introduce a new source number detector which exploits the eigenvectors of a delayed matrix to estimate the signal subspace dimension. Simulation results show that the signal subspace estimation is improved and the source number detector is more efficient in this situation than the usual AIC and MDL criteria.

1 INTRODUCTION

Blind Source Separation consists in recovering the signals emitted by p sources from n ($n \geq p$) linear and stationary mixtures of these signals. The n sensors are receiving convolutive mixtures corrupted with additive noises. The observation vector $\underline{x}(t)$ is modelled as:

$$\underline{x}(t) = \sum_{i=1}^{p} \underline{h}_i(t) * s_i(t) + \underbrace{\sum_{j=1}^{n} \underline{g}_j(t) * n_j(t)}_{\underline{b}(t)} \qquad (1)$$

- $s_i(t)$ are periodic sources of different frequencies,
- the k-th component of $\underline{h}_i(t)$ is the impulse response characterizing the propagation from the i-th source to the k-th sensor,
- the elements of $\underline{b}(t)$ result from the filtering of n spectrally white gaussian noises $n_j(t)$.
- the sources are mutually independent and independent from the noises.

In the frequency domain, the convolutive mixture becomes an instantaneous mixture at each frequency bin

$$\underline{X}(f) = \underbrace{\underline{\underline{H}}(f).\underline{S}(f)}_{\underline{Y}(f)} + \underline{B}(f) \qquad (2)$$

$\underline{\underline{H}}(f)$ is the $n \times p$ matrix whose columns are the Fourier Transforms of vectors $\underline{h}_i(t)$. In order to simplify the notation, (f) will be omitted.
Orthogonal techniques use PCA as a first step, to whiten the observations. This relies on projecting the observations on an orthonormal base of the signal subspace E_s. Further separation is achieved using of 4-th order information [1] or joint diagonalisation [3] to find the exact base of the sources. Obviously, the efficiency of the whole process depends on the accuracy of the first step. The whitening matrix is built from the eigenvalues and eigenvectors of $\underline{\underline{\gamma}}_Y = E\{\underline{Y}.\underline{Y}^+\}$ which is the spectral matrix of the unnoisy mixtures. Unfortunately $\underline{\underline{\gamma}}_Y$ is estimated from $\underline{\underline{\gamma}}_X$ ($\underline{\underline{\gamma}}_X = \underline{\underline{\gamma}}_Y + \underline{\underline{\gamma}}_B$) where $\underline{\underline{\gamma}}_B$ is unknown. When the noise spectral matrix is not proportional to the identity matrix (i.e. when the noises are not spatially white), $\underline{\underline{\gamma}}_Y$ is ill estimated. Consequently the source number and the signal subspace are not correctly estimated. The PCA looses efficiency. This problem is reminded in the second part of this paper. In the third part we introduce a new estimator of $\underline{\underline{\gamma}}_Y$, using delays to eliminate the noise influence. This estimator involves the inverse of a matrix that is theorically of rank p but numerically of rank n. It is more robust to establish the source number directly from a delayed spectral matrix in order to use the pseudo-inverse algorithm to get $\underline{\underline{\gamma}}_Y$. A good estimation of $\underline{\underline{\gamma}}_Y$ will provide the orthonormal base that is necessary to whiten the observations. Consequently the fourth part is devoted to a new source number detector using the eigenvectors of a delayed spectral matrix. Simulation results for both the source number and the signal subspace estimation are given in the fifth part.

2 MISMATCHING OF THE USUAL PCA IN SEVERE NOISE CONDITIONS

The vector of unnoisy mixtures can be written as:

$$\underline{Y} = \underline{\underline{H}}'.\underline{S}' \quad \text{with} \quad E\{\underline{S}'.\underline{S}'^{+}\} = \underline{\underline{I}}_p \qquad (3)$$

where $^+$ stands for the transconjugate. The Singular Value Decomposition of $\underline{\underline{H}}'$ is equal to:

$$\underline{\underline{H}}' = \underline{\underline{V}}.\underline{\underline{D}}^{1/2}.\underline{\underline{\Pi}} \qquad (4)$$

• $\underline{\underline{V}}$ and $\underline{\underline{\Pi}}$ are two unitarian matrices respectively $n \times n$ and $p \times p$.
• $\underline{\underline{D}}^{1/2}$ is a $n \times p$ diagonal matrix with elements $\sqrt{\lambda_i}, i = 1 \dots p$. When the first mixing matrix $\underline{\underline{H}}$ is unitarian, the λ_i are exactly the Power Spectral Densities (PSD) of the sources at the frequency f. The Eigenvalue Decomposition of $\underline{\underline{\gamma}}_Y$ can be written, using the singular elements of $\underline{\underline{H}}'$ as

$$\underline{\underline{\gamma}}_Y = \underline{\underline{V}}.\underline{\underline{D}}^{1/2}.\underline{\underline{\Pi}}.\underline{\underline{\Pi}}^{+}.\underline{\underline{D}}^{1/2+}.\underline{\underline{V}}^{+} = \underline{\underline{V}}.\underline{\underline{D}}.\underline{\underline{V}}^{+} \qquad (5)$$

$$\underline{\underline{D}} = \text{diag}(\lambda_1, \dots, \lambda_p, 0, \dots, 0) \qquad (6)$$

The eigenvalues λ_i are assumed to be ranged in decreasing order. The first p eigenvalues and their corresponding eigenvectors are representative of E_s.
PCA consists of projecting $\underline{X}$ on an orthonormal base of the signal subspace with a matrix $\underline{\underline{W}}$ verifying relation (7). The solution for $\underline{\underline{W}}$ is given in (8) where $\underline{\underline{D_s}}$ is the square submatrix containing the first p diagonal elements of $\underline{\underline{D}}$ and $\underline{\underline{V_s}}$ the rectangular submatrix containing the first p columns of $\underline{\underline{V}}$.

$$\underline{\underline{W}}.\underline{\underline{\gamma}}_Y.\underline{\underline{W}}^{+} = \underline{\underline{I}}_p \qquad (7)$$

$$\underline{\underline{W}} = \underline{\underline{D_s}}^{-1/2}.\underline{\underline{V_s}}^{+} \qquad (8)$$

Unfortunately, in a noisy context, one can only access to $\underline{\underline{\gamma}}_X$. When the noises are spatially white, the noise spectral matrix is on the form (9) and the Eigenvalue Decomposition of $\underline{\underline{\gamma}}_X$ provides the same eigenvectors as for $\underline{\underline{\gamma}}_Y$ (10).

$$\underline{\underline{\gamma}}_B = \sigma_b^2.\underline{\underline{I}}_n \qquad (9)$$

$$\underline{\underline{\gamma}}_X = \underline{\underline{V}}.\underbrace{(\underline{\underline{D}} + \sigma_b^2.\underline{\underline{I}}_n)}_{\underline{\underline{\Omega}}}.\underline{\underline{V}}^{+} \qquad (10)$$

In these conditions it is possible to use algorithms such as AIC and MDL for estimating the source number p. Then σ_b^2 can be estimated from the $n-p$ last eigenvalues of $\underline{\underline{\Omega}}$ and substracted to the p first eigenvalues to get an estimate of $\underline{\underline{D_s}}$.
With experimental signals $\underline{\underline{\gamma}}_B$ is hardly ever on the form of (9) and the factorization (10) is not possible anymore. Consequently, in the Eigenvalue Decomposition of $\underline{\underline{\gamma}}_X$, the first p column vectors don't span anymore the signal subspace but a p dimensional subspace in the n dimensional space of observations. The eigenvalues are ill estimated too and the AIC and MDL methods fail. Consequently the whitening matrix $\underline{\underline{W}}$ is ill estimated because of errors on the signal subspace dimension and the eigenvectors in $\underline{\underline{V_s}}$. In the case of periodic sources, we propose in the next section, a new estimator of $\underline{\underline{\gamma}}_Y$, robust to spatially correlated noise.

3 A DIRECT ESTIMATOR OF THE UNNOISY SPECTRAL MATRIX

The spectral matrix $\underline{\underline{\gamma}}_X$ is estimated from the N-point Discrete Fourier Transform of $\underline{x}$ on M sliding blocks. In the case of periodic signals it is interesting to exploit the fact that the autocorrelation lengths of the sources are larger than the correlation lengths of all the noises. Let τ_b be the greater correlation or cross-correlation length of the n noises. Let $\underline{X}$ be the DFT of $\underline{x}$ on a temporal block and $\underline{X}^{\tau}$ the DFT on a block delayed of τ samples. If $\tau \geq \tau_b$, at each frequency bin, the covariance matrix $\underline{\underline{\gamma}}_X^{\tau} = E\{\underline{X}.\underline{X}^{\tau +}\}$ contains only information about the sources. Suppose that the sources have harmonic frequencies f_i close to the analysis frequency. Using (3) $\underline{\underline{\gamma}}_X^{\tau}$ can be written

$$\underline{\underline{\gamma}}_X^{\tau} = \underline{\underline{H}}'.\underbrace{E\{\underline{S}'.\underline{S}'^{\tau +}\}}_{\underline{\underline{\theta}}}.\underline{\underline{H}}'^{+} \qquad (11)$$

$$\underline{\underline{\theta}} = \begin{pmatrix} e^{-j2\pi f_1 \tau} & & 0 \\ & \ddots & \\ 0 & & e^{-j2\pi f_p \tau} \end{pmatrix} \qquad (12)$$

From (4) and (11) we get:

$$\underline{\underline{\gamma}}_X^{\tau} = \underline{\underline{V}}.\underline{\underline{D}}^{1/2}.\underline{\underline{\Pi}}.\underline{\underline{\theta}}.\underline{\underline{\Pi}}^{+}.\underline{\underline{D}}^{1/2+}.\underline{\underline{V}}^{+} \qquad (13)$$

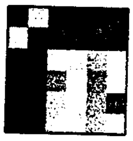
It is theorically possible to find back the Eigenvalue Decomposition of $\underline{\underline{\gamma}}_Y$ (5) with the use of a second spectral matrix obtained with delay -2τ. The final relation is:

$$\underline{\underline{\gamma}}_Y = \underline{\underline{\gamma}}_X^{\tau}.(\underline{\underline{\gamma}}_X^{\tau +})^{-1}.\underline{\underline{\gamma}}_X^{-2\tau} \qquad (14)$$

We must pay attention to the fact that this expression involves the inverse of a matrix of size n but rank p. As a practical consideration, stability is improved using the pseudo-inverse algorithm, so that only the non zero eigenvalues of $\underline{\underline{\gamma}}_X^{\tau +}$ are inversed. Unfortunately the $n-p$ last eigenvalues resulting from numerical computations are hardly ever equal to zero. Choosing the number of eigenvalues to inverse is the same as choosing the number of sources. Since the eigenvalues of the delayed spectral matrices are not easy to handle, we are going to implement, in the next section, a source number detector using the eigenvectors.

4 A NEW SOURCE NUMBER DETECTOR

Denote $\underline{\underline{V_\tau}}.\underline{\underline{D_\tau^{1/2}}}.\underline{\underline{U_\tau^+}}$ the SVD of $\underline{\underline{\gamma_X^\tau}}$. Using relation (13) it is possible to show that the first p column vectors of $\underline{\underline{V_\tau}}$ and $\underline{\underline{U_\tau}}$ result from unitary transforms of $\underline{\underline{V_s}}$. They are orthonormal bases of E_s. The $n-p$ last vectors span $E_\perp$ that is orthogonal to E_s. As a conclusion the matrices $\underline{\underline{V_\tau}}$ and $\underline{\underline{U_\tau}}$ can be written as $[\underline{\underline{V'}}.\underline{\underline{V_\perp}}]$ and $[\underline{\underline{U'}}.\underline{\underline{U_\perp}}]$. The matrix $\underline{\underline{A}} = \underline{\underline{V_\tau^+}}.\underline{\underline{U_\tau}}$ involves the cross products $\underline{\underline{V'^+}}.\underline{\underline{U_\perp}}$ and $\underline{\underline{V_\perp^+}}.\underline{\underline{U'^+}}$ that are theoretically zero. This particular structure of $\underline{\underline{A}}$ reveals the number of sources. The picture below shows a matrix $\underline{\underline{A}}$ obtained with the noisy mixtures of two sources observed on six sensors. The noises are spatially correlated and the Signal to Noise Ratio is around -5 dB on every sensor. Since this criterion is quite visual, we are going to process $\underline{\underline{A}}$ as an image. We first expand $\underline{\underline{A}}$ to a $3n \times 3n$ matrix, in order to process filters of size 3×3. Then, a Laplacian filter is applied to enhance the contrasts. The positive pixels are set to one and the others to zero. As a result, the left and upper contours of the submatrix $\underline{\underline{V_\perp^+}}.\underline{\underline{U_\perp}}$ are only constituted of ones. This quite simple test on the matrix gives the number of sources.



5 DISTANCE TO THE SIGNAL SUBSPACE - SIMULATIONS RESULTS

As we said in section 2, the efficiency of PCA relies on both the estimation of the source number and the estimation of an orthonormal base of E_s. We now need a criterion to measure jointly the accuracy of the estimated eigenvalues and the closeness to the signal subspace. Denote $\|.\|$ the matrix 2-norm, and $\widehat{\underline{\underline{\gamma_Y}}}$ the estimate of $\underline{\underline{\gamma_Y}}$. The distance $\|\underline{\underline{\gamma_Y}} - \widehat{\underline{\underline{\gamma_Y}}}\|$ is not appropriate since $\widehat{\underline{\underline{\gamma_Y}}}$ can be close to $\underline{\underline{\gamma_Y}}$ without the good eigenvalues and eigenvectors. Consequently the criterion must rely on the whitening matrix $\underline{\underline{W}}$. The usual rejection rates referred in [2] cannot be used here since the rotation matrix $\underline{\underline{\Pi}}$ is undetermined after the PCA. We must pay attention to the fact that the estimated whitening matrix $\widehat{\underline{\underline{W}}}$ is not uniquely determined. Denote $\widehat{\underline{\underline{W}}} = \tilde{\underline{\underline{D_s}}}^{-1/2}.\tilde{\underline{\underline{V_s}}}^+$. If the column vectors in $\tilde{\underline{\underline{V_s}}}$ form an orthonormal base of E_s then $\tilde{\underline{\underline{V_s}}}^+.\underline{\underline{V_s}}$ is close to a unitarian matrix $\underline{\underline{P}}$ that is in fact diagonal when the mixing matrix $\underline{\underline{H}}$ is unitarian. In this particular situation, if the estimated eigenvalues in $\tilde{\underline{\underline{D_s}}}$ are closed to the real ones, then the product (15) is close to $\underline{\underline{P}}$.

$$\widehat{\underline{\underline{W}}}.\underline{\underline{W}}^\# = \tilde{\underline{\underline{D_s}}}^{-1/2}.\tilde{\underline{\underline{V_s}}}^+.\underline{\underline{V_s}}.\underline{\underline{D_s}}^{1/2} \qquad (15)$$

# denotes the pseudo-inverse. This considerations leads to the following criterion of distance :

$$d(\widehat{\underline{\underline{W}}}, \underline{\underline{W}}) = \left| \ \|\widehat{\underline{\underline{W}}}.\underline{\underline{W}}^\#\| - 1 \ \right| \qquad (16)$$

We show simulation results on figure 1, 2 and 3. Two sources are mixed and observed on 6 sensors. Each source is composed of 2 pure frequencies (0.14,0.36 and 0.15,0.37). The mixture is obtained from AR1 filters. The spatially correlated and spectrally colorated noises result from the filtering of white noises with AR1 filters too. The noise power spectral densities are different on every sensor and the corresponding Signal to Noise Ratios are about -5 dB. Computations are processed on 600 sliding blocks of 64 samples with $\tau = 90$ samples. Denote $\underline{\underline{\gamma_1}}$ and $\underline{\underline{\gamma_2}}$, the estimation of $\underline{\underline{\gamma_Y}}$ respectively with the usual PCA method, and the new method involving the delay τ. The number of sources p is supposed to be known here, to compare only the distance to the signal subspace without influence of the detector. $d1$ and $d2$ are the corresponding distances to the signal subspace. In figure 1 one can see that the eigenvalues of $\underline{\underline{\gamma_1}}$ in dotted line are very far from the eigenvalues of $\underline{\underline{\gamma_Y}}$ in solid line whereas in figure 2, the eigenvalues of $\underline{\underline{\gamma_2}}$ are very close to the eigenvalues of $\underline{\underline{\gamma_Y}}$. The distance $d2$ to the signal subspace is much lower than the distance $d1$ as shown in figure 3.

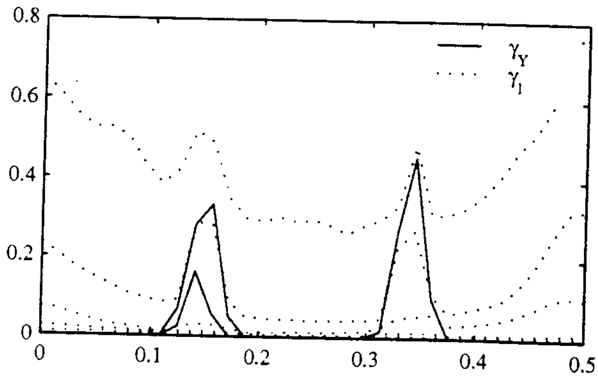


Figure 1: *Eigen values of* $\underline{\underline{\gamma_Y}}$ *and* $\underline{\underline{\gamma_1}}$

Figure 4 and 5 are devoted to the source number detection. We compare the new detector based on the eigenvectors of $\underline{\underline{\gamma_X^\tau}}$ to the usual AIC and MDL criteria based on the eigenvalues of $\underline{\underline{\gamma_X}}$. These two figures

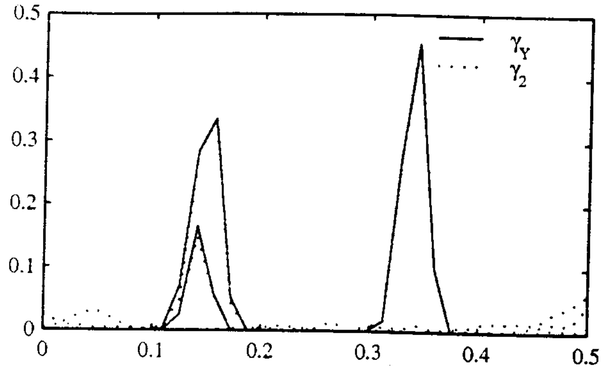


Figure 2: Eigen values of $\underline{\underline{\gamma}}_Y$ and $\underline{\underline{\gamma_2}}$

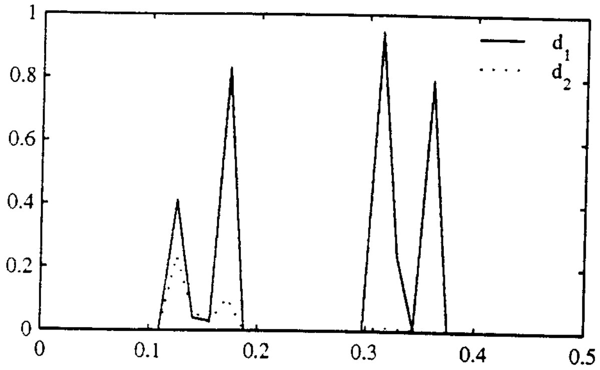


Figure 3: Distance to the signal subspace

give the probability of detection versus the Signal to Noise Ratio in dB. The simulation are run with two pure frequencies (0.14,0.15) mixed with AR1 filters and observed on 6 sensors. The probabilities of detection are estimated from 100 sets of data. For every set the spectral matrices are obtained from 600 FFT blocks of 64 samples. Figure 4 involves spatially and spectrally white noises, which corresponds to the optimal conditions for the AIC and MDL methods. The new method has similar performances as AIC. Figure 5 involves spatially correlated and spectrally colorated noises resulting from the filtering of white noises with AR1 filters. In this situation the usual AIC and MDL methods are not trustable at all even for high SNR whereas the new method is working still the same.

CONCLUSIONS

This paper involves simulation results with spatially correlated and spectrally colorated noise. In this context the PCA fails because the unnoisy spectral matrix is ill-estimated. We then propose a new estimator of this matrix computed from two interspectral matrices using

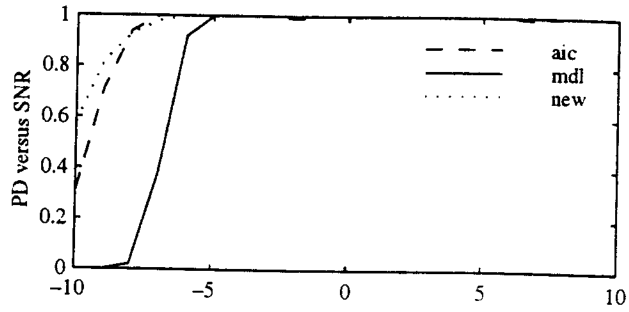


Figure 4: PD in spatially white noise

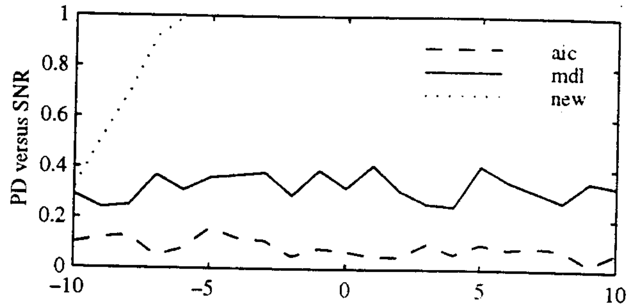


Figure 5: PD in spatially correlated noise

two different delays. We choose a distance criteria to the signal subspace and show the efficiency of the method with a simulation in severe conditions (Signal to Noise Ratio around -5dB). A quite simple source number detector using the eigenvectors of a delayed matrix is introduced. Simulations show that this detector is more robust than the usual AIC and MDL criterion in any situation. Knowing the source number makes the estimation of the unnoisy spectral matrix more robust. Consequently, using jointly the new detector and the new estimator of the unnoisy spectral matrix will provide a quite robust tool for second order whitening.

References

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