BLIND SEPARATION OF CYCLOSTATIONARY SIGNALS

Adriana Dapena, Daniel Iglesia, Luis Castedo *
Departamento de Electrónica y Sistemas, Universidad de La Coruña
Campus de Elviña s/n, 15.071 La Coruña, SPAIN
Tel: +34 81 167100, e-mail: {adriana,daniel,luis}@des.fi.udc.es

ABSTRACT

In this paper we propose a new approach to blind source separation that exploits the cyclostationary nature of the sources typically found in communication applications. A new adaptive algorithm is proposed that simultaneously utilizes Higher Order Statistics (HOS) and cyclic moments. The approach enables exploitation of periodicities embedded in the sources such as the carrier frequency or the symbol rate. It is also presented an analysis to obtain the conditions under which the algorithm converges to the desired solutions.

1 INTRODUCTION

In this paper we address the problem of recovering non-Gaussian statistically independent signals from observations of linear mixtures of them. When the separation is carried out without resorting to an *a priori* knowledge of the sources or the mixing system, the problem is known as blind source separation [1]. A typical application is the extraction of signals arriving at an array of sensors in the base station of a cellular mobile communication system.

In communications applications the sources can be modeled as cyclostationary random processes since their statistical parameters fluctuate periodically with time. In this work we will interpret cyclostationarity as the property of generating spectral lines when a signal is passed through certain nonlinear transformations [2].

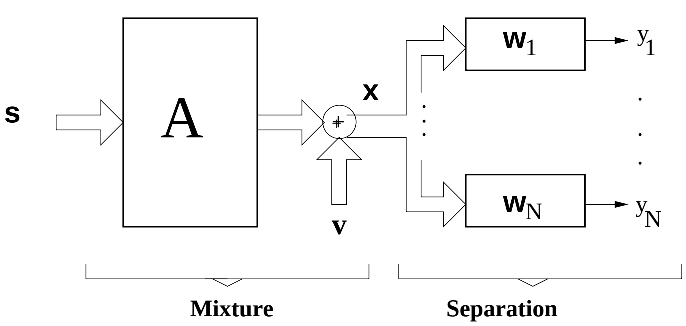

Figure 1: Mixture and separation model.

We have considered the signal model illustrated in figure 1 where $\mathbf{s} = [s_1, ..., s_N]^T$ is a vector of N statistically independent non-Gaussian sources and $\mathbf{x} = [x_1, ..., x_M]^T$ is a vector containing $M \geq N$ measured signals termed observations. The observations are related to the sources through the following linear model

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{v} \tag{1}$$

where $\mathbf{A}$ is the mixture matrix and $\mathbf{v}$ is the Gaussian noise vector. We assume that $\mathbf{A}$ is a full-rank matrix. The observations vector is processed through a bank of linear combiners to produce the output vector $\mathbf{y} = [y_1, ..., y_N]^T$ whose components are given by

$$y_i = \mathbf{w}_i^H \mathbf{x} \tag{2}$$

where $\mathbf{w}_i$ is the weight vector of the i-th linear combiner and the superindex ^{H} denotes conjugate transpose. The aim in blind source separation is to select the vectors $\mathbf{w}_i$ in order to recover the sources from the observations, assuming that both the mixture matrix $\mathbf{A}$ and the sources $\mathbf{s}$ are unknown.

Since the pioneering work of Herault and Jutten [3], many adaptive algorithms have been proposed to solve the blind source separation problem (see [4, 5, 6] and references therein). All of them have in common the use of Higher Order Statistics (HOS). In this paper we propose an algorithm that combines HOS and cyclic moments. This way we can incorporate into the algorithm information about periodicities embedded in the sources such as the carrier frequency or the symbol rate in digital modulated signals.

The remainder of the paper is organized as follows. Section 2 presents the proposed optimization problem and the resulting adaptive gradient algorithm that solves it. Section 3 analyzes the algorithm stability in the separating points. Section 4 shows simulation results and, finally, the conclusions are outlined in Section 5.

2 OPTIMIZATION CRITERION

A zero-mean complex cyclostationary signal $s(t)$ generates a spectral line at a frequency α when it is raised to a real number p if it has a non-zero *p-th order cyclic moment* defined as

$$m_{ps}^{\alpha} = \left\langle s^p(t) e^{-j2\pi\alpha t} \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s^p(t) e^{-j2\pi\alpha t} dt \tag{3}$$

The operator $< \cdot >$ denotes *time average*. Typically, one dimensional constellations contain second order cyclic moments at $\alpha = 2f_c \pm kf_s$, $k = 0, 1, 2, ...$ (f_c is the carrier frequency and f_s is the symbol rate) whereas two dimensional

* This work has been supported by CICYT, grant TIC96-0500-C10-02, and Xunta de Galicia, grant XUGA 10502A96.

constellations (QAM) have fourth order cyclic moments at $\alpha = 4f_c \pm kf_s$, $k = 0, 1, 2, \ldots$.

Let us assume now that each source $s_i(t)$ is a cyclostationary signal and contains a nonzero cyclic moment $m^{\alpha_i}_{p_i s_i}$. In case that several sources have the same cyclostationary properties let us denote $\mathcal{S}^{\alpha_l}_{p_l}$, $l = 1, \cdots, L$ the set of sources that contain a nonzero cyclic moment of order p_l at the frequency α_l and let us term $\mathcal{N}^{\alpha_l}_{p_l}$ $l = 1, \cdots, L$ the number of sources in $\mathcal{S}^{\alpha_l}_{p_l}$. For simplicity reasons, we will assume that the sets $\mathcal{S}^{\alpha_l}_{p_l}$ are disjoint, i.e., $\sum_{l=1}^{L} \mathcal{N}^{\alpha_l}_{p_l} = N$.

To separate the $\mathcal{N}^{\alpha_l}_{p_l}$ sources corresponding to the set $\mathcal{S}^{\alpha_l}_{p_l}$ we propose that the vectors $\mathbf{w}_i$, $i = 1, \cdots, \mathcal{N}^{\alpha_l}_{p_l}$ be adjusted in order to minimize the following cost function

$$J^{\alpha_l}_{p_l} = \sum_{i=1}^{\mathcal{N}^{\alpha_l}_{p_l}} J_i - \gamma \sum_{i=1}^{\mathcal{N}^{\alpha_l}_{p_l}} \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}^{\alpha_l}_{p_l}} J_{ij} \tag{4}$$

where γ is a positive real number. The cost function J_i is given by

$$J_i = \left\langle |e^{j2\pi\alpha_l t} - y_i^{p_l}(t)|^2 \right\rangle \tag{5}$$

and $J_{ij} = Cum(y_i, y_i^*, y_j, y_j^*) = \left\langle |y_i|^2 |y_j|^2 \right\rangle - \left\langle |y_i|^2 \right\rangle \left\langle |y_j|^2 \right\rangle - | \left\langle y_i y_j^* \right\rangle |^2$ is the fourth-order cross-cumulant between the outputs y_i and y_j.

The optimization problem (4) is motivated as a generalization of a statistical criterion for blind adaptive beamforming proposed in [7]. When the sources are statistically independent and have negative kurtosis, the only existing minima of J_i correspond to the extraction of a single source having a non-zero cyclic moment $m^{\alpha_l}_{p_l s_i}$ [7]. Therefore, if we adjust the separating vectors $\mathbf{w}_i$ according to the maximization of the first sum in (4), we ensure that each output extracts a single source from the set $\mathcal{S}^{\alpha_l}_{p_l}$. However, since all the sources in $\mathcal{S}^{\alpha_l}_{p_l}$ generate a spectral line in the same frequency α_l, it may occur that the same source is extracted in several outputs simultaneously. This undesirable situation is penalized by the set of cross-terms J_{ij}: when y_i and y_j extract the same source $Cum(y_i, y_i^*, y_j, y_j^*)$ will take a negative value whereas if both outputs extract different sources this term will be zero.

2.1 Adaptive Algorithm

An advantage of our method is that the optimum weight vectors $\mathbf{w}_i$ can be iteratively computed using a simple gradient algorithm

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) - \mu \left(\nabla_{\mathbf{w}_i} J_i(n) - \gamma \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}^{\alpha_l}_{p_l}} \nabla_{\mathbf{w}_i} J_{ij}(n) \right) \tag{6}$$

where μ is the algorithm step size and $\nabla_{\mathbf{w}_i}$ represents the complex gradient with respect to $\mathbf{w}_i$. Evaluating the gradients in (6) and replacing the statistical moments by estimates, we obtain the following stochastic gradient algorithm

$$\begin{aligned} \mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \mu \mathbf{x}(n) \Big(p_l e_i^*(n) y^{p_l - 1}(n) \\ +2\gamma \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}^{\alpha_l}_{p_l}} (y_i^*(n)|y_j(n)|^2 - y_i^*(n) E_j - y_j^*(n) E_{ij}) \Big) \end{aligned} \tag{7}$$

where $e_i(n) = e^{j2\pi\alpha_l n} - y_i^{p_l}(n)$. E_j and E_{ij} are the following estimators

$$\begin{aligned} E_j &= (1-\mu_1)E_j + \mu_1 |y_j(n)|^2 \\ E_{ij} &= (1-\mu_1)E_{ij} + \mu_1 y_i(n)^* y_j(n) \end{aligned} \tag{8}$$

where μ_1 is a real positive number less than 1.

3 STATIONARY POINTS ANALYSIS

In this section we analyze the stationary points of (4) where each output extracts a single and different source. For simplicity reasons, we will restrict ourselves to a situation where only the $\mathcal{N}^{\alpha_l}_{p_l}$ sources belonging to the set $\mathcal{S}^{\alpha_l}_{p_l}$ are present in the observations. In addition, we will also assume that $p_l = 2$ and that there is no noise. Without any loss of generality, sources can be assumed to have unit power since differences between powers can be included in matrix $\mathbf{A}$.

The first step is to express $J^{\alpha_l}_2$ in terms of $g_{ij} = \mathbf{w}_i^H \mathbf{a}_j$ where $\mathbf{a}_j$ is the j-th column of $\mathbf{A}$. Under the assumptions about the sources, the second order cyclic moment in (4) can be written as

$$\left\langle y_i^2(t) e^{-j2\pi\alpha_l t} \right\rangle = \sum_{j=1}^{\mathcal{N}^{\alpha_l}_2} g_{ij}^2 m^{\alpha_l}_{2s_j} = \mathbf{g}_i^T \mathbf{\Gamma} \mathbf{g}_i \tag{9}$$

where $\mathbf{\Gamma}$ is a diagonal matrix with elements $m^{\alpha_l}_{2s_i} = \left\langle s_i^2(t) e^{-j2\pi\alpha_l t} \right\rangle$ and $\mathbf{g}_i = [g_{i1} \ldots g_{i\mathcal{N}^{\alpha_l}_2}]^T$. In addition,

$$\begin{aligned} \left\langle |y_i(t)|^4 \right\rangle &= \sum_{j=1}^{\mathcal{N}^{\alpha_l}_2} |g_{ij}|^4 k_j + 2 \left(\sum_{j=1}^{\mathcal{N}^{\alpha_l}_2} |g_{ij}|^2 \right)^2 \\ &= \sum_{j=1}^{\mathcal{N}^{\alpha_l}_2} |g_{ij}|^4 k_j + 2(\mathbf{g}_i^H \mathbf{g}_i)^2 \\ Cum(y_i, y_i^*, y_j, y_j) &= \sum_{m=1}^{\mathcal{N}^{\alpha_l}_2} |g_{im}|^2 |g_{jm}|^2 k_m \end{aligned} \tag{10}$$

where $k_i = \left\langle |s_i(t)|^4 \right\rangle - 2$ is the kurtosis of s_i. Substituting (9) and (10) into (4)

$$\begin{aligned} J^{\alpha_l}_2 &= \mathcal{N}^{\alpha_l}_2 + \sum_{i=1}^{\mathcal{N}^{\alpha_l}_2} \left(2(\mathbf{g}_i^H \mathbf{g}_i)^2 - \mathbf{g}_i^T \mathbf{\Gamma} \mathbf{g}_i - \mathbf{g}_i^H \mathbf{\Gamma}^* \mathbf{g}_i^* \right) \\ &+ \sum_{m=1}^{\mathcal{N}^{\alpha_l}_2} \sum_{i=1}^{\mathcal{N}^{\alpha_l}_2} (|g_{im}|^4 - \gamma \sum_{\substack{j=1 \\ i \neq j}}^{\mathcal{N}^{\alpha_l}_2} |g_{im}|^2 |g_{jm}|^2) k_m \end{aligned} \tag{11}$$

Considering that $\mathbf{A}$ is a full-rank matrix, the stationary points of $J^{\alpha_l}_2$ are the points where the following gradient vanishes

$$\nabla_{\mathbf{g}_i} J^{\alpha_l}_2 = 4(\mathbf{g}_i^H \mathbf{g}_i)\mathbf{g}_i - 2\mathbf{\Gamma}^* \mathbf{g}_i^* + 2diag(\mathbf{g}_i \mathbf{d}_i^T) \tag{12}$$

where $diag(.)$ denotes elements into the diagonal and $\mathbf{d}_i$ is a vector with components

$$d_{im} = (|g_{im}|^2 - \gamma \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}^{\alpha_l}_2} |g_{jm}|^2) k_m, \; m = 1, \ldots, \mathcal{N}^{\alpha_l}_2 \tag{13}$$

When the separation is achieved, the vectors $\mathbf{g}_i$ contain a single and different non-zero element. Assuming $g_{ii} \neq 0$, $i = 1, ..., \mathcal{N}_2^{\alpha_l}$ and $g_{ij} = 0$ $i \neq j$, the gradient (12) vanishes when

$$2|g_{ii}|^2 g_{ii}(k_i + 2) = 2(m_{2s_i}^{\alpha})^* g_{ii}^*, \quad i = 1, ..., \mathcal{N}_2^{\alpha_l} \Rightarrow$$

$$\Rightarrow \begin{cases} |g_{ii}|^2 = \frac{|m_{2s_i}^{\alpha}|}{k_i+2}, \\ arg(g_{ii}) = -\frac{arg(m_{2s_i}^{\alpha})}{2}, \quad i = 1, ..., \mathcal{N}_2^{\alpha_l} \end{cases} \tag{14}$$

where $arg(\cdot)$ denotes the phase of a complex number.

The next step is to examine the positive definiteness of the Hessian matrix at this point to determine under which conditions (14) is a minimum. This analysis is carried out in appendix A and it is shown that (14) is a minimum if all the sources have negative kurtosis and

$$\gamma^2 > \frac{(k_i + 2)^2}{k_i^2} \tag{15}$$

4 SIMULATION RESULTS

Computer simulations were carried out to illustrate the performance of the proposed algorithm. We considered a 10-elements uniform linear array whose spacing is half wavelength. The input signals are sampled at a rate five times faster that the symbol rate (i.e., $f_s = 0.2$).

In the first experiment we considered the separation of three BPSK signals with normalized carrier frequency $f_c = 0.1$, input Signal to Noise Rate (SNR) of 0, 5 and 10 dB and arrival angles of 50^o,-50^o and 0^o, respectively. These signals generate several spectral lines when they are raised to $p_l = 2$, being the most powerful at $\alpha_l = 2f_c = 0.2$. Figure 2 shows the $SINR$ time evolution when $\alpha_l = 0.2$ and $\gamma = 1.5$. Figure 3 plots the results obtained when the Constant Modulus (CM) function $J_i = \left\langle |1 - |y_i(t)|^2|^2 \right\rangle$ is used [6]. We can see that both algorithms converge to the optimum $SINR$ but the proposed method is faster because the Gaussian noise does not have non-zero cyclic moments.

In a second experiment, we considered the separation of the four signals in table 1. The MSK modulation also generates spectral lines when it is raised to $p_l = 2$ but these appear in $\alpha_l = 2f_c \pm f_s/2$. To perform the separation, we divided the sources into two disjoint sets $\mathcal{S}_2^{0.4} = \{s_1, s_2\}$ and $\mathcal{S}_2^{0.9} = \{s_3, s_4\}$. In this case we can select $\mathbf{w}_1$ and $\mathbf{w}_2$ to minimize

$$J_2^{0.4} = \sum_{i=1}^{2} \left\langle |e^{2\pi 0.4t} - y_i^2(t)|^2 \right\rangle - 2\gamma Cum(y_1, y_1^*, y_2, y_2^*) \tag{16}$$

and $\mathbf{w}_3$ and $\mathbf{w}_4$ to minimize

$$J_2^{0.9} = \sum_{i=3}^{4} \left\langle |e^{2\pi 0.9t} - y_i^2(t)|^2 \right\rangle - 2\gamma Cum(y_3, y_3^*, y_4, y_4^*) \tag{17}$$

Note that we only make use of two cross-terms whereas if we use the CM cost function we would need six cross-terms. Figure 4 plots the $SINR$ time evolution for this second experiment. It is clearly seen that it has converged to the optimum values in less that 500 symbols.

5 CONCLUSIONS

Most existing approaches to blind source separation are based on the use of Higher Order Statistics (HOS). In this paper we investigate a new approach that combines HOS and cyclic moments. This way we can exploit the cyclostationary nature of digital modulated signals. In particular, we can incorporate into the algorithm information about periodicities embedded in the sources such as the carrier frequency or the symbol rate. This information enables us to obtain simpler adaptive algorithms. We also present a stationary point analysis and obtain convergence conditions for the algorithms. Finally, computer simulations show that the rate of convergence in the presence of noise is faster than existing approaches.

Signal	Modulation	Direction	SNR	f_c	α_l
s_1	BPSK	-40^o	6 dB	0.2	0.4
s_2	BPSK	40^o	4 dB	0.2	0.4
s_3	BPSK	-20^o	2 dB	0.45	0.9
s_4	MSK	20^o	0 dB	0.5	0.9

Table 1: Experiment 1: Signals parameters

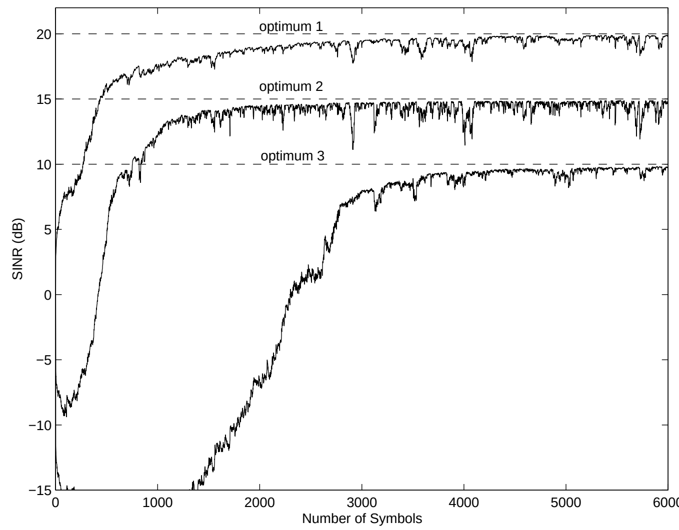

Figure 2: Experiment 1 with proposed algorithm

A HESSIAN MATRIX ANALYSIS

The Hessian matrix can be decomposed as follows

$$\mathbf{H_g} J_2^{\alpha_l} = \begin{bmatrix} \mathbf{E_g} J_2^{\alpha_l} & \mathbf{S_g} J_2^{\alpha_l} \\ \mathbf{S_g^*} J_2^{\alpha_l} & \mathbf{E_g} J_2^{\alpha_l} \end{bmatrix}$$

where

$$\mathbf{E_g} J_2^{\alpha_l} = \begin{bmatrix} \nabla_{\mathbf{g}_1}^H \nabla_{\mathbf{g}_1} & \cdots & \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}}^H \nabla_{\mathbf{g}_1} \\ \vdots & \ddots & \vdots \\ \nabla_{\mathbf{g}_1}^H \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}} & \cdots & \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}}^H \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}} \end{bmatrix}$$

$$\mathbf{S_g} J_2^{\alpha_l} = \begin{bmatrix} \nabla_{\mathbf{g}_1}^T \nabla_{\mathbf{g}_1} & \cdots & \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}}^T \nabla_{\mathbf{g}_1} \\ \vdots & \ddots & \vdots \\ \nabla_{\mathbf{g}_1}^T \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}} & \cdots & \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}}^T \nabla_{\mathbf{g}_{\mathcal{N}_2^{\alpha_l}}} \end{bmatrix}$$

From (12) it is obtained

$$\begin{aligned} \nabla_{\mathbf{g}_i}^H \nabla_{\mathbf{g}_i} &= 4(\mathbf{g}_i^H \mathbf{g}_i)\mathbf{I}_{\mathcal{N}_2^{\alpha_l}} + 4\mathbf{g}_i \mathbf{g}^H \\ &+ 2diag(|g_{i1}|^2 k_1 + d_{i1}, \cdots, |g_{i\mathcal{N}_2^{\alpha_l}}|^2 k_{\mathcal{N}_2^{\alpha_l}} + d_{1\mathcal{N}_2^{\alpha_l}}) \end{aligned}$$

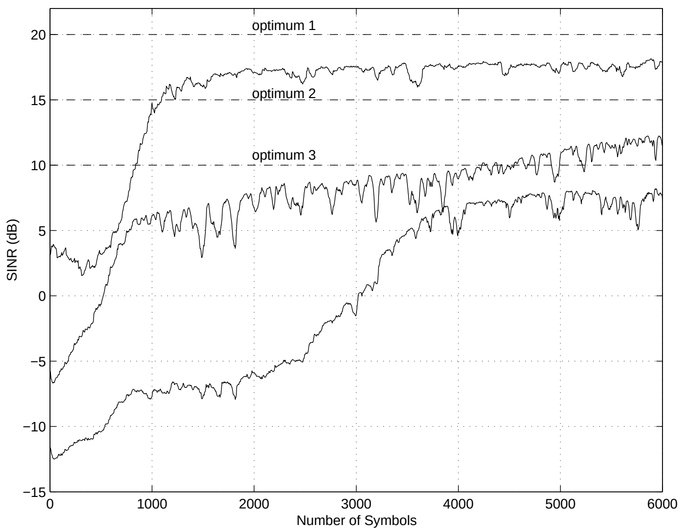

Figure 3: Experiment 1 with Constant Modulus algorithm

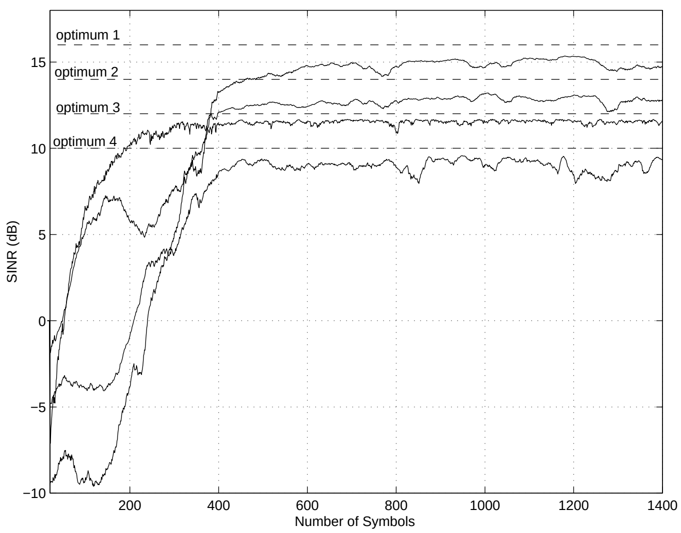

Figure 4: Experiment 2 with proposed algorithm

$$\begin{aligned}
\nabla^H_{\mathbf{g}_j}\nabla_{\mathbf{g}_i} &= -2\gamma diag(g_{i1}g^*_{j1}k_1,\cdots,g_{i\mathcal{N}_2^{\alpha_l}}g^*_{j\mathcal{N}_2^{\alpha_l}}k_{\mathcal{N}_2^{\alpha_l}})\ i\neq j\\
\nabla^T_{\mathbf{g}_i}\nabla_{\mathbf{g}_i} &= 4\mathbf{g}_i\mathbf{g}_i^T-2\mathbf{\Gamma}^*+2diag(g^2_{i1}k_1,\cdots,g^2_{i\mathcal{N}_2^{\alpha_l}}k_{\mathcal{N}_2^{\alpha_l}})\\
\nabla^T_{\mathbf{g}_j}\nabla_{\mathbf{g}_i} &= -2\gamma diag(g_{i1}g_{j1}k_1,\cdots,g_{i\mathcal{N}_2^{\alpha_l}}g_{j\mathcal{N}_2^{\alpha_l}}k_{\mathcal{N}_2^{\alpha_l}})\ i\neq j
\end{aligned}$$

where $\mathbf{I}_{\mathcal{N}_2^{\alpha_l}}$ is the $\mathcal{N}_2^{\alpha_l}\times\mathcal{N}_2^{\alpha_l}$ identity matrix. Substituting (14) into the Hessian matrix terms

$$\begin{aligned}
\nabla^H_{\mathbf{g}_i}\nabla_{\mathbf{g}_i} &= 2diag(2|g_{ii}|^2-\gamma|g_{11}|^2k_1,\cdots,\\
&\quad\overbrace{2|g_{ii}|^2(k_i+2)}^{i},\cdots,2|g_{ii}|^2-\gamma|g_{\mathcal{N}_2^{\alpha_l}\mathcal{N}_2^{\alpha_l}}|^2k_{\mathcal{N}_2^{\alpha_l}})\\
\nabla^T_{\mathbf{g}_i}\nabla_{\mathbf{g}_i} &= -2diag((m^{\alpha}_{21})^*,\cdots,\\
&\quad\overbrace{g^2_{ii}(k_i+2)-(m^{\alpha_l}_{2s_i})^*}^{i},\cdots(m^{\alpha_l}_{2\mathcal{N}_2^{\alpha_l}})^*
\end{aligned}$$

Using a permutation matrix, it is possible to transform $\mathbf{H}_{\mathbf{g}}J_2^{\alpha_l}$ in a block matrix with the same positive definiteness nature. The new matrix only contains non-zero blocks into its diagonal

$$\begin{aligned}
\mathbf{\Delta}_{ii} &= \begin{bmatrix}\frac{\partial^2J_2^{\alpha_l}}{\partial g_{ii}\partial g^*_{ii}} & \frac{\partial^2J_2^{\alpha_l}}{\partial g^*_{ii}\partial g^*_{ii}}\\ \frac{\partial^2J_2^{\alpha_l}}{\partial g_{ii}\partial g_{ii}} & \frac{\partial^2J_2^{\alpha_l}}{\partial g^*_{ii}\partial g_{ii}}\end{bmatrix}\\
&= 2\begin{bmatrix}2|g_{ii}|^2(k_i+2) & g^2_{ii}(k_i+2)-(m^{\alpha_l}_{2s_i})^*\\ 2(g^2_{ii})^*(k_i+2)-m^{\alpha_l}_{2s_i} & 2|g_{ii}|^2(k_i+2)\end{bmatrix}\\
\mathbf{\Delta}_{ij} &= \begin{bmatrix}\frac{\partial^2J_2^{\alpha_l}}{\partial g_{ij}\partial g^*_{ij}} & \frac{\partial^2J_2^{\alpha_l}}{\partial g^*_{ij}\partial g^*_{ij}}\\ \frac{\partial^2J_2^{\alpha_l}}{\partial g_{ij}\partial g_{ij}} & \frac{\partial^2J_2^{\alpha_l}}{\partial g^*_{ij}\partial g_{ij}}\end{bmatrix}\\
&= 2\begin{bmatrix}2|g_{ii}|^2-\gamma|g_{jj}|^2k_j & -(m^{\alpha_l}_{2s_j})^*\\ -m^{\alpha_l}_{2s_j} & 2|g_{ii}|^2-\gamma|g_{jj}|^2k_j\end{bmatrix}
\end{aligned}$$

From (14) we obtain that the diagonal terms of the Hessian and the determinant $det(\mathbf{\Delta}_{ii})=16|g_{ii}|^4(k_i+2)^2$, are always positive. On the other hand, the matrix $\mathbf{\Delta}_{ij}$ reduces to

$$\mathbf{\Delta}_{ij}=2\begin{bmatrix}2\frac{|m^{\alpha}_{2s_i}|}{k_i+2}-\gamma\frac{|m^{\alpha}_{2s_j}|k_j}{k_j+2} & -(m^{\alpha}_{2s_j})^*\\ -m^{\alpha}_{2s_j} & 2\frac{|m^{\alpha}_{2s_i}|}{k_i+2}-\gamma\frac{|m^{\alpha}_{2s_j}|k_j}{k_j+2}\end{bmatrix}$$

It is apparent that the diagonal terms are positive if the sources have negative kurtosis and $\gamma>0$. In addition, it is easy to show that the determinant is positive when

$$\gamma^2>\frac{(k_i+2)^2}{k_i^2} \tag{18}$$

Therefore, the Hessian matrix at the desired stationary points is positive definite if the sources have negative kurtosis and γ satisfies the above condition.

References

[1] X. Cao and R. Wen-Liu, "General Approach to Blind Source Separation", *IEEE Transactions on Signal Processing*, vol.44, no. 4, pp. 562-571, March 1996.

[2] *Cyclostationarity in Communications and Signals Processing*, W.A. Gardner Ed., IEEE Press, New York, 1994.

[3] C. Jutten and J. Herault, "Blind Separation of Sources, Part I", *Signal Processing*, vol. 24, no. 1, pp. 1-10, July 1991.

[4] E. Moreau and O. Macchi, "High-Order Contrasts for Self-Adaptive Source Separation", *International Journal of Adaptive Control and Signal Processing*, 1996, pp. 19–46.

[5] J-F. Cardoso and B. Laheld, "Equivariant Adaptive Source Separation", *IEEE Transactions on Signal Processing*, 1996, vol, 44, no. 12, pp. 3017–3030.

[6] A. Dapena and L. Castedo, "Stochastic Gradient Adaptive Algorithms for Blind Source Separation", submitted to Signal Processing.

[7] L. Castedo and A. R. Figueiras-Vidal, "An Adaptive Beamforming Technique Based on Cyclostationary Signal Properties", *IEEE Trans. on Signal Processing*, vol. 43, no. 7, pp. 1637-1650, July 1995.