REDUCED-COMPLEXITY DECISION-FEEDBACK EQUALIZER FOR NONLINEAR CHANNELS

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ABSTRACT

This paper deals with the compensation for nonlinear distortions introduced by power-efficient amplifiers on linear modulations by means of equalization. In our approach, we employ a Decision-Feedback Equalizer (DFE) based on the Generalized Cerebellar Model Arithmetic Computer. The new scheme is compared with the conventional Linear DFE, the Volterra and the Multi-Layer Perceptron-based DFE in terms of their convergence rates, Bit-error rates and Signal-to-Noise Ratio degradation.

1 INTRODUCTION

Practical digital communication systems employing multilevel Pulse Amplitude Modulation (PAM), one kind of bandwidth-efficient transmission method for digital signals, require a compromise between power efficiency and linearity of the transmitter amplifiers. If the amplifiers are working near the saturation point (maximum output power), a better use of the available power is achieved, but the PAM signal is severely distorted due to envelope fluctuation. It is possible to reduce the nonlinear distortion by making the amplifier operate in a quasi-linear region far from the saturation point (the diminution of output power is called output back-off). However, this strategy reduces the transmitted signal power and, therefore, the noise margin.

The need of some compensation technique has been recognized long time ago. Nonlinear compensators consist of controlling either the signal before it is sent (predistortion) or the noisy received signal (equalization). In this paper we consider the compensation problem from the receiver perspective.

The optimal solution is the Maximum Likelihood Sequence Detector (MLSD) using the Viterbi Algorithm [1], however its large complexity makes this method useless for practical channels. Suboptimal equalizer structures based on Multi-Layer Perceptron (MLP) [2], the Radial Basis Function (RBF) [3] and the Volterra Filters [4, 5] have been shown to offer a significant performance improvement over the traditional linear equalizers on account of their ability to approximate the optimal symbol-decision equalizer implicitly. However, the MLP presents problems of slow convergence and unpredictable solutions during training; while the RBF network's ability to realize the optimal equalization solution is crucially dependent upon whether its centers can be positioned correctly at the desired channel states; whereas the polynomial filters suffers the drawback of slow convergence and exponentially increasing filter dimensions.

In this paper, a new approach for the Decision-Feedback Equalizer using the Generalized Cerebellar Model Arithmetic Computer (GCMAC) [6, 7] is presented. The GCMAC network possesses nonlinear decision making capabilities and yet has a linear-in-the-parameters structure. The former property is essential for realizing the optimal equalization solution and the latter characteristic is beneficial in practical implementation. In this paper, the GCMAC-based DFE is compared with the conventional Linear DFE, the Volterra and the MLP-based DFE in terms of their convergence rates, Bit-Error rates (BER) and Signal-to-Noise Ratio (SNR) degradation. The simulation results show that the GCMAC improves the performance of previous networks, specially when strong nonlinearities are present.

The paper is organized as follows. The effects of nonlinear amplification of PAM signals are analyzed in Section 2. The structure of the GCMAC network is discussed in Section 3. Two GCMAC-based DFE schemes are presented in Section 4. Simulation results and performance comparisons are explained in Section 5. Finally, conclusions are given in Section 6.

2 PROBLEM STATEMENT

Figure 1 depicts the block diagram of a baseband equivalent PAM system including a memoryless nonlinear High-Power Amplifier (HPA). Driving the communication system with the stream of M-ary complex PAM symbols, the the baseband complex signal at the output of receiver filter can be written as :

$$R[k] = F(\ldots, A[k-1], A[k], A[k+1], \ldots) + Z[k] \quad (1)$$

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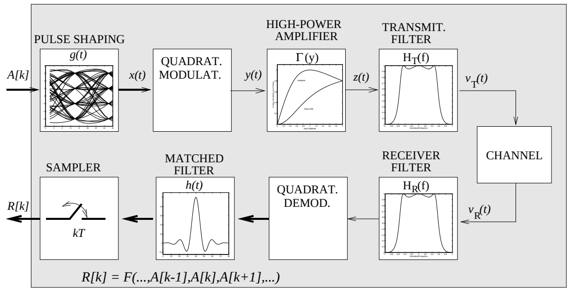
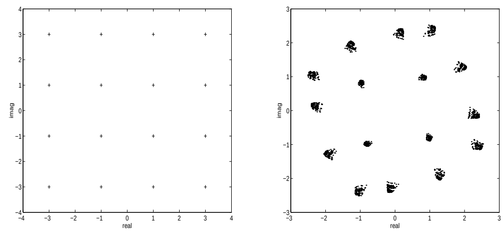


Figure 1: Block diagram of the PAM communication system.



(a) (b)

Figure 2: (a) Original symbol constellation. (b) Received symbol constellation after nonlinear distortion (amplifier delivering maximum output power).

where $R[k]$ is the received symbol, $\{A[k]\}$ are the original PAM symbols, $F(\bullet)$ is the nonlinear mapping which describes the behavior of the channel[1] and $Z[k]$ is the observation noise. Figure 2 represents a typical received symbol constellation after driving the previous scheme with a 16-QAM signal.

The general Decision-Feedback Equalizer transforms a finite sequence of P correlative received and detected PAM symbols and produces the estimated symbol $\hat{A}[k]$:

$$\begin{aligned} \hat{A}[k] &= H(\hat{A}[k-N_b], \ldots, \hat{A}[k-1], \\ &\qquad R[k], R[k+1], \ldots, R[k+M_f]) \\ &= H(\boldsymbol{x}_P[k]) \; . \end{aligned} \tag{2}$$

where N_b is the order of the feedback part, M_f is the order of the feedforward part and $P = N_b + M_f + 1$ is the order of the equalizer (it is assumed that the channel does not introduces additional delay).

3 NETWORK STRUCTURE

An important feature of the function $H(\bullet)$ is the hybrid nature of its input space. Unlike the feedforward part, driven by symbols with continuous-amplitudes, the feedback part, excited by past M-ary PAM symbol decisions, has a discrete-amplitude nature. This particular feature suggests the use of a network which allows both classes of input variables, such as the GCMAC network.

[1] We consider the channel including all the elements and devices between the modulator and the detector.

The GCMAC network approximates the desired nonlinear function (i.e. the optimal decision function) using a set of overlapped Local Basis Functions (LBFs). The LBFs are placed at fixed intervals using a lattice which discretizes the input space into cells. The support of LBFs are P-dimensional rectangles whose size is specified by the vector $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_P]^T$, where $1 \leq \rho_i < L_i$, and L_i is the number of cells along the i-th input axis. To provide the network with generalization abilities, $\rho_{max} = \max(\boldsymbol{\rho})$ LBFs cover every cell of input space. As the generalization is influenced by the geometry of the local domains, $\boldsymbol{\rho}$ is called generalization vector[2].

The output of GCMAC is a linear combination of LBFs:

$$y = H(\boldsymbol{x}) = \sum_{j=1}^{N} \boldsymbol{w}_j \Phi_j(\boldsymbol{x}) \; . \tag{3}$$

where $\{\Phi_1(\boldsymbol{x}), \ldots, \Phi_N(\boldsymbol{x})\}$ is the set of basis functions, and $\boldsymbol{w}$ is the vector of weights. Hence, the approximation used by the GCMAC network is linear in the unknown coefficients $\boldsymbol{w}$ and, therefore, simple instantaneous learning laws can be used, for which convergence can be established subject to well-understood restrictions [8].

4 EQUALIZER ARCHITECTURES

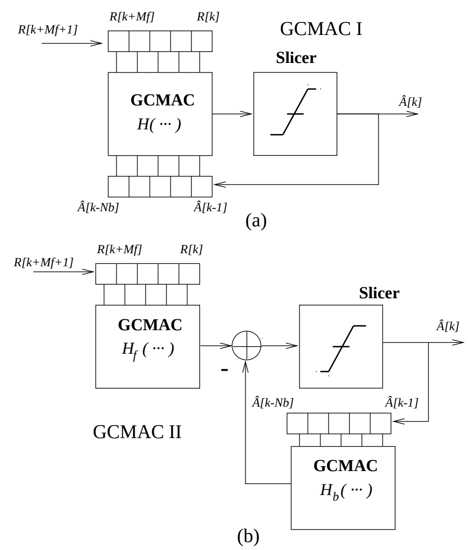


Figure 3: Equalizer architectures. (a) General DFE: GCMAC I (b) Conventional DFE: GCMAC II.

The general GCMAC-based DFE transforms a correlative sequence of both received and past decision feedback samples (see Eq. (2), and Figure 3.a). Although, the arithmetic cost per sample is independent of the order[3] P, the total number of LBFs grows exponentially

[2] When $\rho_i = 1, \forall i$, the GCMAC becomes a LUT (no generalization); the larger ρ_i is, the more generalization is obtained.

[3] The number of weights used to update or construct the output, ρ_{max}, is independent of the dimension of input space.

with the order. As a result, the rate of convergence decreases for channels with large memory. In order to alleviate the memory requirements and accelerate the convergence, we have used the structure depicted in Figure 3.b. Using this scheme, the feedforward part elliminates the precursor (anticausal) intersymbol interference (ISI) and the feedback part cancels the remaining nonlinear postcursor ISI. Obviously, the difference in relative computational complexities of both structures becomes more significant for large feedforward and feedback equalizer orders M_f and N_b.

5 PERFORMANCE ANALYSIS

We have simulated a typical 16-QAM system with root-raised cosine pulse shaping filter ($\alpha = 0.5$) and a High Power Amplifier delivering maximum output power. The channel is assumed to have a flat frequency response with additive, white, circularly symmetric, Gaussian noise. For simplicity, we have also assumed that slicer produces right decisions, avoiding the propagation of errors in the feedback branch. For a fair comparison, all of the equalizers analyzed were chosen to be of order $P = 3$ ($M_f = N_b = 1$).

The networks used used in the comparison are a fifth-order Volterra Filter, a MLP with two-hidden layers (10 nodes in the first hidden layer and 6 in the second one), and two GCMAC networks whose configuration is explained as follows. The inputs coming into the feedforward part are quantized using 32 nonuniform-spaced levels; the corresponding generalization vector is $\boldsymbol{\rho}_f = [16, 16, 16, 16]^T$ (it should be noticed that the dimension of input space is doubled to process complex inputs). Since, the previous decisions are already discrete in amplitude, no quantization is needed, and the selected generalization vector is $\boldsymbol{\rho}_b = [3, 3]^T$. For simplicity, simulations were carried out with constant LBFs.

5.1 *Convergence characteristics*

The Volterra and the GCMAC equalizers have been trained using the LMS algorithm. The MLP was trained using the Back-Propagation (BP) algorithm modified with a momentum term that increases the convergence rate and produces smooth weight changes. The Mean Square Error (MSE) curves achieved by the analyzed equalizers are represented in Figure 4. It is observed that the Volterra equalizer presents the fastest convergence (curve 1). The learning curve of the MLP (curve 2) reveals the irregular behavior of the BP algorithm; in spite of this, the MLP outperforms the Volterra, although it requires a larger training time. The full GCMAC-based DFE (GCMAC I) produces the least final MSE outperforming both the MLP and the Volterra equalizers in 6 dB and 9 dB, respectively. Finally, the structures shown in Figure 3 are compared. The simplified GCMAC-based DFE (GCMAC II) presents a relative high initial rate of convergence compared with the general structure (curve 4 in Figure 4); however, its final

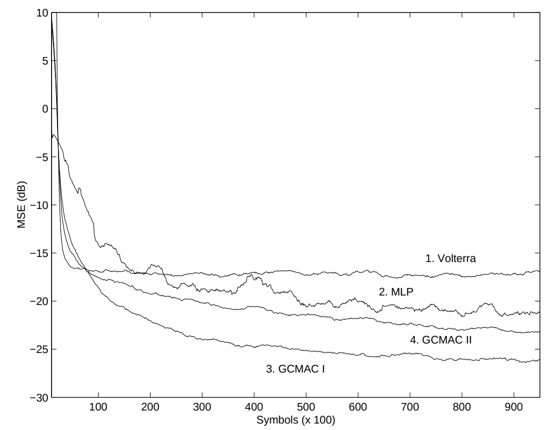
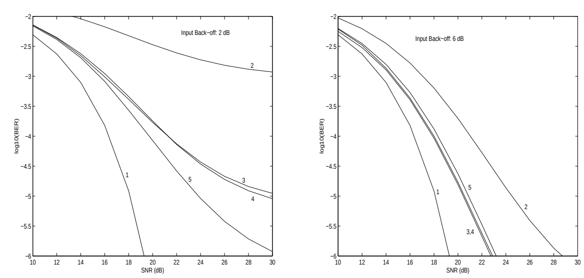


Figure 4: Convergence curves when the HPA is saturated. Curve 1: Fifth-order Volterra equalizer; curve 2: Multi-Layer Perceptron (6-10-6-2); curve 3: GCMAC I DFE ($\boldsymbol{\rho}_f = [16, 16, 16, 16]^T$, $\boldsymbol{\rho}_b = [3, 3]^T$); curve 4. GCMAC II DFE. Traces are ensembled average of 15 convergence curves.

MSE is slightly worse, specially when strong nonlinearities are present.

5.2 *BER characteristics*



(a) (b)

Figure 5: BER comparison. Curve 1: ideal linear channel; curve 2: Linear DFE; curve 3: Volterra DFE; curve 4: MLP DFE; curve 5: GCMAC I DFE. (a) Input Back-off 2 dB. (b) Input Back-off 6 dB.

Additional simulations were made to determine the BER versus the Signal-to-Noise-Ratio. The results, averaged over 15 trials of length 10^6 bits, are depicted in Figure 5. It can be seen that the Linear DFE achieves poor performance when the received samples have suffered high nonlinear distortion (Figure 5.a). In this context, the GCMAC gives a more effective compensation for the nonlinear distortion than the other structures. However, for larger input back-offs, that is, lower nonlinear distortion, the MLP and the Volterra equalizers provide better results.

5.3 *Total Degradation characteristics*

Other way to quantify the validity of the analyzed equalizers is to compute the equivalent SNR degradation caused by the residual nonlinear distortion at a specified BER. The Total Degradation, expressed in dB, is defined as the difference between the required SNR by the equalized system to reach the specified BER at a given input back-off, and the required SNR to obtain the same BER on the Gaussian flat channel. The total degradation results in a convex function of the input back-off, taking the minimum value at the optimum input back-off (BO_{in}^{opt}). We have obtained this function after using the quasi-analytical procedure described in [9]. Results for a target BER of 10^{-4} are shown in Figure 6 and Table 1.

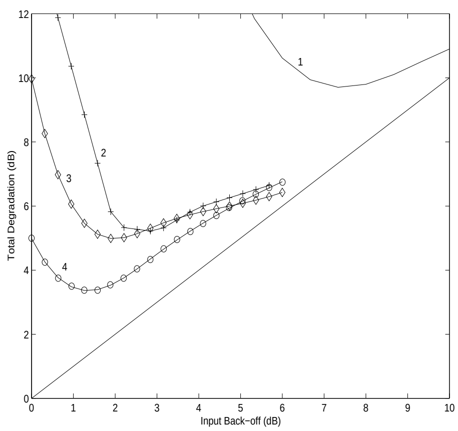


Figure 6: Total degradation for the analyzed equalizers. Curve 1: Linear DFE; curve 2: Fifth-order Volterra equalizer; curve 3: Multi-Layer Perceptron equalizer; curve 4: GCMAC I equalizer.

	[Gain]	[BO_{in}^{opt}]	Param.	Iter.
1. L-DFE	0	7.35	3	200
2. Volterra	4.44	2.79	57	$6\ 10^3$
3. MLP	4.67	1.98	150	$> 10^5$
4. GCMAC I	6.28	1.28	4924	$7\ 10^4$
5. GCMAC II	5.43	1.82	1243	$4\ 10^4$

Table 1: Gain, optimum input back-off, number of parameters and speed of convergence for the simulated equalizers.

Again, it is confirmed that the GCMAC-based equalizers perform clearly better than the other equalizers when strong nonlinearities are present in the received sequence. The gain[4] achieved by the GCMAC I equalizer is 6 dB with respect to the linear DFE. Furthermore, the optimum input back-off is only 1.28 dB, which means in practice a better use of the available power.

[4]The gain is defined as the difference between the values of the Total Degradation evaluated at the optimum input back-off.

6 CONCLUSIONS

In this paper we have proposed new structures to equalize nonlinear channels. We have focused the compensation for nonlinear distortion caused by power efficient amplifiers on Pulse Amplitude Modulation systems. By means of a GCMAC-based equalizer, it is possible to obtain effective compensation even for strong nonlinearities. The proposed equalizers provide better performance in steady state MSE, BER and SNR degradation over other nonlinear structures, namely the Volterra and the MLP-based equalizers.

7 References

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