Blind separation of polarised waves

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Abstract

In a great many situations polarised waves are received on vectorial sensors. These waves are composed of several polarised sources associated with different modes of propagation. The objective of our work is to separate the polarised sources without *a priori* knowledge of the polarisation of each source and to apply this technique to elastic waves observed in a seismic sounding.

1 Introduction

In radar observation, in seismic investigation [4] and other physical observations the recorded waves are vectorial. Therefore it is possible to take advantage of the polarisation in the separation of the sources which generate these waves.

We will first give an account of the different descriptions of the state of polarisation. Then we will propose an algorithm for the separation of polarised waves using fourth order cumulants. The separation algorithm will be illustrated using both synthetic data and real seismic data.

2 Polarisation

2.1 Polarised modes of propagation

When the frequency band is sufficiently narrow polarisation does not vary (significantly) within the bandwidth of the signal. If this is the case, a polarised wave received on 2 sensors can be written as

$$\underline{m}(t) = \begin{pmatrix} m_x(t) \\ m_y(t) \end{pmatrix} = \frac{1}{\sqrt{1+\rho^2}} \begin{pmatrix} 1 \\ \rho e^{j\theta} \end{pmatrix} z(t) = \underline{p} z(t).$$

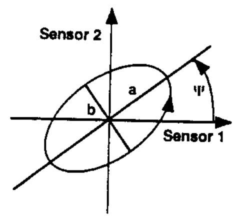

Figure 1: *Polarisation ellipse*

$z(t) = x(t) + jy(t)$ is the analytic signal describing the amplitude of the wave, ρ and θ caracterize the polarisation and $\underline{p}$ is a complex, normalised, vector describing the polarisation state.

2.2 Polarisation state

The polarisation state of a signal output received on the sensors is described by ρ and θ. For $\theta = 0$ we get a linear polarisation, for $\rho = 1$ and $\theta = \pi/2$ a circular polarisation, otherwhere the polarisation is elliptical. The two caracteristics of the ellipse of polarisation are the excentricity: $e = \frac{r_{max}}{r_{min}} = \frac{a}{b}$ and the angle ψ (figure 1).

A visual representation of the polarisation state is given by the Poincare sphere. In this representation a polarisation state is caracterized by a point on a sphere of radius unity. The azimut is represented by ψ and the declination by $\kappa = 2\arctan(b/a)$ with a positive value for a positive rotation and a negative value for a negative rotation. We represent the projection of the Poincare sphere on the Oxz plane (figure 2). Within the Oxz plane the coordinates of one state of polarisation are $x = \cos\kappa\cos\psi$ and $y = \sin\kappa$.

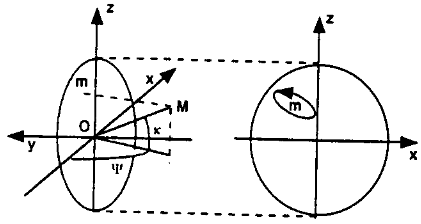

Figure 2: *A point* M *on the Poincare sphere, its projection* m *on the Oxz plane and the polarisation ellipse associated with the point* m

3 Polarised waves separation

3.1 The model

The combination of two polarised waves is received on two sensors. These two components have reached the sensors through different paths so we can suppose that their random amplitudes are statistically independent. The state of polarisation of each component and the mixing factors, α_1 and α_2 are unknown.The complex vectorial signal received on the sensors is given by:

$$\underline{m}(t) = \alpha_1 \underline{p}_1 z_1(t) + \alpha_2 \underline{p}_2 z_2(t)$$

which gives

$$\underline{m}(t) = \begin{pmatrix} \alpha_1 p_{11} & \alpha_2 p_{21} \\ \alpha_1 p_{12} & \alpha_2 p_{22} \end{pmatrix} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} = \mathbf{A}\ \underline{z}(t) \quad (1)$$

This matricial mixing corresponds exactly to the model used for sources separation [5]. In order to separate the two combined waves and to recover their state of polarisation we have to identify the matrix **A**. This identification cannot be acheived using only second order statistics but it must uses second and fourth order statistics [1, 5]. Source separation techniques allow the separation of two non-gaussian waves and the recover of their amplitude and of their state of polarisation.

One particularity of this technique is that the polarised waves have complex values. Higher order statistics of signals with complex values are presented in [3]. It is shown there that we must simultaneously consider the signals and their complex conjugates. This begs the question of circularity. The definition of circularity is given in [5]. For circular signals it is sufficient to consider statistics that combine equals number of non-conjugated and conjugated terms. A true stationary frequency is circular and thus the narrow band signals that we consider here are circular or nearly circular.

3.2 Separation algorithm

The mixing matrix **A**, given in (1), is factorised into the product of 3 matrices:

$$\mathbf{A} = \mathbf{V}\ \Delta^{1/2} \mathbf{U}^{\dagger}.$$

V and **U** are unitary matrix and Δ is diagonal. The separation is acheived out in two steps using higher order statistics. The first step uses second order statistics and the second fourth order ones.

3.2.1 Second order step

With $E[\underline{z}(t)\underline{z}(t)^{\dagger}] = \mathbf{I}$ (identity matrix), the covariance matrix of the observed signal $\underline{m}(t)$ is:

$$\mathbf{C} = E[\underline{m}(t)\underline{m}(t)^{\dagger}] = \mathbf{V}\ \Delta\ \mathbf{V}^{\dagger}.$$

The matrix Δ is given by the eigenvalues of **C** and the matrix **V** by the eigenvectors of **C**.

Having estimated **V** and Δ we standardise the observation

$$\underline{l}(t) = \Delta^{-1/2}\ \mathbf{V}^{\dagger}\ \underline{m}(t) = \mathbf{U}^{\dagger}\ \underline{m}(t).$$

In order to achieve the separation we have to identify the matrix **U**. This is done using fourth order statistics.

3.2.2 Fourth order step

In order to identify the matrix **U** several methods have been proposed [5]. We use the approach based on the maximum likelihood [1, 2].

The unitary matrix **U** is written

$$\mathbf{U} = \begin{pmatrix} cos\theta & -sin\theta e^{-j\chi} \\ sin\theta & cos\theta e^{-j\chi} \end{pmatrix}.$$

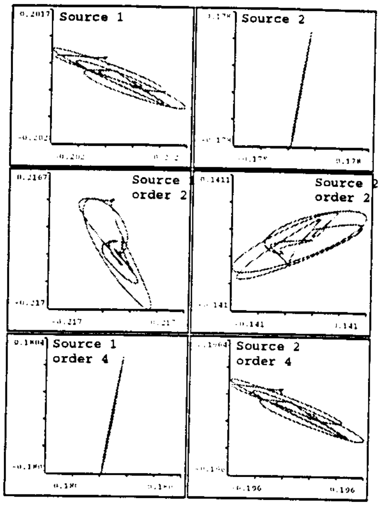

Figure 3: *Lissajou plots of the initial sources and of the sources obtained at order 2 and 4*

The maximum likelihood function of θ and χ is the sum of the squares of the fourth order cross-cumulants after the application of the matrix $\mathbf{U}$

$$M_l(\theta,\chi) = |C(1,1,1,2;\theta,\chi)|^2 + |C(2,2,2,1;\theta,\chi)|^2 + |C(1,1,2,2;\theta,\chi)|^2,$$

with $C(i,j,k,l) = Cumulant[l_i, l_j, l_k^*, l_l^*]$.

The values of θ and χ that maximize this function give the estimate of the matrix $\mathbf{U}$.

4 Examples

We present results obtained for both synthetic data and for real data for seismic soundings.

4.1 Synthetic data

We construct a two-dimensional signal which is a linear combination of two polarised waves. The amplitudes of the two waves are non-gaussian, white, digital signals containing 1024 samples. One of the waves is linearly polarised and the other one is elliptical. We show in figure 3 the Lissajou plot of the two sources and the sources separated at order 2 and at order 4.

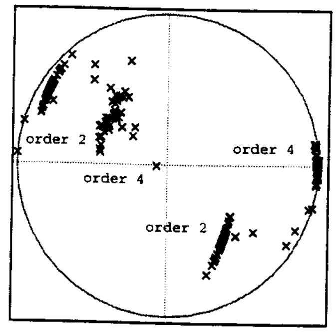

Figure 4: *Points representative of the polarisation at orders 2 and 4*

In order to give an experimental account of the precision we plot (figure 4) the points representative of the polarisation state on the projection of the Poincare sphere obtained in 50 independent tests of the sources.

4.2 Seismic data

The technique is applied to a seismic data set collected by M. Dietrich of the LGIT.

4.2.1 Physics of seismic observation

In a seismic investigation the signal is conveyed by elastic waves propagating within the ground [4]. Different modes of propagation exist : P-waves with a linear polarisation parallel to the direction of propagation, S-waves with a polarisation in the plane perpendicular to the direction of propagation, Rayleigh waves that are elliptically polarised waves guided along the surface of the ground...Therefore in this context the use of polarisation is useful in separating the modes of propagation.

4.2.2 Seismic data separation

The outputs from 10 equispaced sensors, with a 10m separation, give the vertical and one horizontal component of the ground displacement in the wave cre-

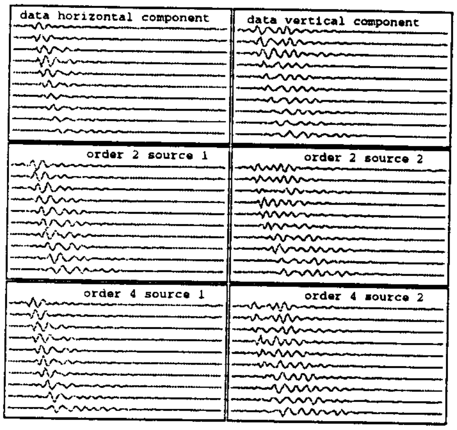

Figure 5: *Waveforms of the sensors outputs and of the sources at order 2 and 4*

ated by an explosive source situated at 400m from the first sensor. The covariance matrix and the fourth order cumulants are estimated on the 10 sensors output.

Figure 5 shows the waveforms of the observed waves, of the sources separated at the second order and at the fourth order. Figure 6 shows the polarisation of the recorded waves and the polarisations of the sources separated at order 2 and 4.

The second order step leads to a physically non-coherent separation. In the fourth order step, one of the separated sources has an elliptical polarisation. This is coherent with the propagation of Rayleigh waves. The other one is linearly polarised and is nearly vertical.

Following the separation we can measure the velocity of these two waves. The values obtained (337 m/s for the source 1 and 480 m/s for the source 2) confirm that the elliptically polarised wave is a Rayleigh wave and indicate that the linear one is an acoustic wave which propagated in the air.

5 Conclusion

We have shown using synthetic data and real data that the application of fourth order algorithms for source separation is able to recover the elementary components of combined polarised waves. This result has potentially a wide field of application in the areas concerned with combinations of polarised waves. For example, remote sensing with radar, communications with polarised waves and the list go on.

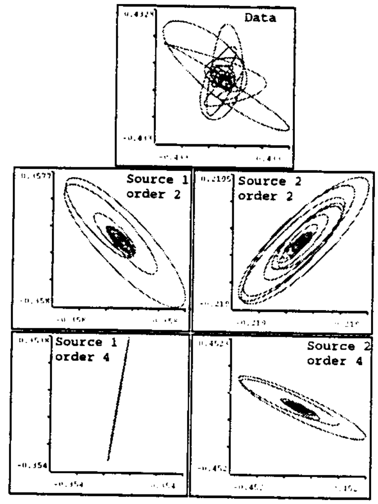

Figure 6: *Lissajou plots of the initial sources and of the sources obtained at order 2 and 4*

References

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