

ROBUSTNESS OF LEAST-SQUARES AND SUBSPACE METHODS FOR BLIND CHANNEL IDENTIFICATION/EQUALIZATION WITH RESPECT TO CHANNEL UNDERMODELING

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ABSTRACT

The least-squares and the subspace methods are well known approaches for blind channel identification/equalization. When the order of the channel is known, the algorithms are able to identify the channel, under the so-called *length* and *zero* conditions. Furthermore, in the noiseless case, the channel can be perfectly equalized. Less is known about the performance of these algorithms in the cases in which the channel order is underestimated. We partition the true impulse response into the significant part and the tails. We show that the m -th order least-squares or subspace methods estimate an impulse response which is “close” to the m -th order significant part of the true impulse response. The closeness depends on the diversity of the m -th order significant part and the size of the “unmodeled” part.

1 INTRODUCTION

The recent development of second order statistics (SOS) based blind channel identification methods under a single-input/multiple-output (SIMO) channel setting [1], derived either from fractionally sampling (FS) of the receiver or from the use of an array of sensors at the receiver, has created intensive research interest. Many novel schemes have been developed, which can claim exact channel identification/equalization, in the noiseless case, under the so-called *zero forcing* conditions. The most well known approaches are the least-squares (LS) [2], the subspace (SS) [3] and the linear prediction (LP) [4] methods.

While all the aforementioned methods claim exact channel identification/equalization, under the zero forcing conditions, in the noiseless case, their behavior may change dramatically [5]–[10] under more realistic conditions, including:

- the presence of non-negligible additive channel noise;

- the presence of long tails of “small” leading and/or trailing impulse response terms.

Motivated by [8], we partition the true impulse response into the *significant part* and the *tails*. By *significant part* is meant the part which is usually found at the middle of the impulse response and contains all the large terms (it may contain some small intermediate terms as well); the small leading and trailing terms compose the *tails*. We show that the m -th order LS or SS methods estimate a channel, which is “close” to the m -th order significant part of the true channel; the closeness depends on the diversity of the m -th order significant part and the size of the “unmodeled” part. Furthermore, we show that if we try to model not only the significant part of the channel but also (a part of) the tails, then the quality of the estimate may degrade dramatically. Thus we should avoid modeling tails.

Therefore, we call a case *overmodeled* (resp. *undermodeled*), if the assumed channel length is bigger (resp. smaller) than the *effective channel length*, which is the length of the significant part of the true channel.

2 LS/SS METHODS: EXACT-ORDER CASE

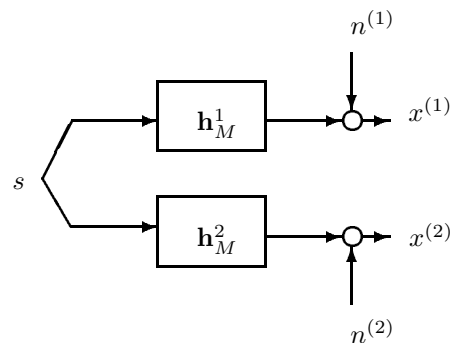


Fig. 1. Single-input/two-output channel setting.

In this section, we describe the LS and SS methods for blind system identification for the single-input/two-output channel setting presented in Fig. 1; this setting

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can be obtained by channel oversampling by a factor of 2, which is quite common in telecommunications. If the true channel order is M , the output of the j -th channel, $x_i^{(j)}$, for $j = 1, 2$, is given by

$$x_i^{(j)} = s_i \otimes h_i^{(j)} + n_i^{(j)} = \sum_{k=0}^M h_k^{(j)} s_{i-k} + n_i^{(j)},$$

where \otimes denotes the convolution operator, $\{s_i\}$ the input sequence, which is assumed zero-mean unit-variance i.i.d., $\{h_i^{(j)}\}$ the impulse response of the j -th channel and $\{n_i^{(j)}\}$ the additive white channel noise. We denote the impulse response of the j -th channel, $j = 1, 2$, as $\mathbf{h}_M^j \triangleq [h_0^{(j)} \dots h_M^{(j)}]^T$, and the entire channel parameter vector as $\mathbf{h}_M \triangleq \begin{bmatrix} \mathbf{h}_M^1 \\ \mathbf{h}_M^2 \end{bmatrix}$.

By stacking the $(L+1)$ most recent samples of each channel, we construct the data vector

$$\mathbf{x}_L(i) \triangleq [x_i^{(1)} \dots x_{i-L}^{(1)} x_i^{(2)} \dots x_{i-L}^{(2)}]^T,$$

which can be expressed as

$$\mathbf{x}_L(i) = \mathcal{H}_L(\mathbf{h}_M) \mathbf{s}_{L+M}(i) + \mathbf{n}_L(i),$$

using input and noise vectors

$$\begin{aligned} \mathbf{s}_{L+M}(i) &\triangleq [s_i \dots s_{i-L-M}]^T, \\ \mathbf{n}_L(i) &\triangleq [n_i^{(1)} \dots n_{i-L}^{(1)} n_i^{(2)} \dots n_{i-L}^{(2)}]^T. \end{aligned}$$

The convolution matrix $\mathcal{H}_L(\mathbf{h}_M)$ is defined as

$$\mathcal{H}_L(\mathbf{h}_M) \triangleq \begin{bmatrix} \mathcal{F}_L(\mathbf{h}_M^1) \\ \mathcal{F}_L(\mathbf{h}_M^2) \end{bmatrix},$$

where $\mathcal{F}_L(\mathbf{h}_M^i)$ is the $(L+1) \times (M+L+1)$ matrix:

$$\mathcal{F}_L(\mathbf{h}_M^i) \triangleq \begin{bmatrix} h_0^{(i)} & \dots & \dots & h_M^{(i)} \\ & \ddots & & \ddots \\ & & h_0^{(i)} & \dots & \dots & h_M^{(i)} \end{bmatrix}.$$

In order to review the LS and the SS methods for the identification of \mathbf{h}_M , we consider the case $L = M$. Furthermore, we assume that the subchannels do not share common zeros, guaranteeing their identifiability.

It is shown in [11] that, for the two-channel case, the LS and SS estimators coincide. They both start from the data autocorrelation matrix

$$\mathbf{R}_M \triangleq E[\mathbf{x}_M(i) \mathbf{x}_M^T(i)]$$

and they identify, in the noiseless or the spatially and temporally white noise case, the unknown impulse response \mathbf{h}_M , by following the sequence of steps:

$$\mathbf{R}_M \rightarrow \mathbf{n}_M \rightarrow \mathbf{h}_M = \mathbf{T}_M \mathbf{n}_M$$

where \mathbf{n}_M is the minimal eigenvector of \mathbf{R}_M and

$$\mathbf{T}_M \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I}_{M+1} \\ -\mathbf{I}_{M+1} & \mathbf{0} \end{bmatrix},$$

with \mathbf{I}_{M+1} being the $(M+1)$ -dimensional identity matrix.

Having identified the channel \mathbf{h}_M , we can equalize it perfectly, in the noiseless case, by using the zero-forcing equalizers of order $(M-1)$, for delays $i = 0, \dots, 2M-1$:

$$\mathbf{g}_{M-1,i}^{\text{ZF}} \triangleq \mathcal{H}_{M-1}^{-T}(\mathbf{h}_M) \mathbf{e}_i, \quad (1)$$

where \mathbf{e}_i denotes the vector with 1 at the $(i+1)$ -st position and zeros elsewhere.

3 UNDERMODELING/OVERMODELING

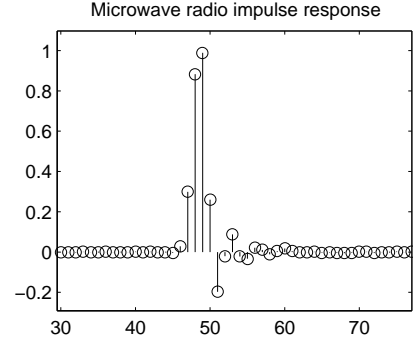


Fig. 2. Portion of the real part of microwave radio channel.

In Fig. 2, we plot a portion of the real part of the oversampled, by a factor of 2, radio microwave channel *chan1.mat* (<http://spib.rice.edu/spib/microwave.html>). The partitioning into the significant part and the tails is clear. In symbols, this partition can be expressed, for $0 \leq m_1 < m_2 \leq M$, as [8], [9]:

$$\mathbf{h}_M = \mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z, \quad (2)$$

with

$$\mathbf{h}_{m_1, m_2}^z \triangleq \begin{bmatrix} \mathbf{h}_{m_1, m_2}^{z1} \\ \mathbf{h}_{m_1, m_2}^{z2} \end{bmatrix}, \quad \mathbf{d}_{m_1, m_2}^z \triangleq \begin{bmatrix} \mathbf{d}_{m_1, m_2}^{z1} \\ \mathbf{d}_{m_1, m_2}^{z2} \end{bmatrix},$$

where, for $j = 1, 2$,

$$\mathbf{h}_{m_1, m_2}^{zj} \triangleq [\underbrace{0 \dots 0}_{m_1} \underbrace{h_{m_1}^{(j)} \dots h_{m_2}^{(j)}}_{m_2 - m_1 + 1} \underbrace{0 \dots 0}_{M - m_2}]^T,$$

$$\mathbf{d}_{m_1, m_2}^{zj} \triangleq [\underbrace{h_0^{(j)} \dots h_{m_1-1}^{(j)}}_{m_1} \underbrace{0 \dots 0}_{m_2 - m_1 + 1} \underbrace{h_{m_2+1}^{(j)} \dots h_M^{(j)}}_{M - m_2}]^T.$$

With \mathbf{h}_{m_1, m_2} we denote the non zero-padded vectors:

$$\mathbf{h}_{m_1, m_2} \triangleq \begin{bmatrix} \mathbf{h}_{m_1, m_2}^1 \\ \mathbf{h}_{m_1, m_2}^2 \end{bmatrix}, \quad \mathbf{h}_{m_1, m_2}^j \triangleq [h_{m_1}^{(j)} \dots h_{m_2}^{(j)}]^T.$$

In the sequel, we study the m -th order LS and SS methods, in the noiseless case, and we explore the relationship

between the “identified” m -th order impulse response and the true M -th order impulse response \mathbf{h}_M .

If $m = m_2 - m_1$ and the true impulse response is \mathbf{h}_{m_1, m_2} , then the autocorrelation matrix of \mathbf{x}_m , \mathbf{R}_m , provides sufficient information for the identification of the \mathbf{h}_{m_1, m_2} , via the sequence of computations:

$$\mathbf{R}_m \equiv \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}) \rightarrow \mathbf{n}_m \rightarrow \mathbf{h}_{m_1, m_2},$$

where \mathbf{n}_m denotes the minimal eigenvector of \mathbf{R}_m and $\mathbf{h}_{m_1, m_2} = \mathbf{T}_m \mathbf{n}_m$. If the true channel impulse response is \mathbf{h}_{m_1, m_2}^z , then it is easy to show that the autocorrelation matrix of \mathbf{x}_m remains \mathbf{R}_m , because

$$\mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z) = \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}),$$

meaning that we can identify the nonzero part of \mathbf{h}_{m_1, m_2}^z , namely \mathbf{h}_{m_1, m_2} . This is directly related to the blind nature of the algorithm, that is, the exploitation of solely the channel output statistics, and will be proved very useful in the sequel.

Now, let us consider what happens when the true impulse response is \mathbf{h}_M , with $\|\mathbf{h}_M\|_2 = 1$, under the assumption that \mathbf{d}_{m_1, m_2}^z is “small”, i.e.,

$$\|\mathbf{d}_{m_1, m_2}^z\|_2 = \epsilon, \quad \epsilon \ll 1. \quad (3)$$

In this case

$$\|\mathbf{h}_{m_1, m_2}^z\|_2 = \|\mathbf{h}_{m_1, m_2}\|_2 = \sqrt{1 - \epsilon^2} \equiv \gamma. \quad (4)$$

The autocorrelation matrix of \mathbf{x}_m is

$$\begin{aligned} \tilde{\mathbf{R}}_m &= \mathcal{H}_m(\mathbf{h}_M) \mathcal{H}_m^T(\mathbf{h}_M) \\ &= \mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z) \mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z) \\ &= \mathbf{R}_m + \mathbf{E}_m, \end{aligned}$$

where \mathbf{E}_m denotes the resulting perturbation. The m -th order LS/SS method will “identify” \mathbf{h}_{m_1, m_2} , through the sequence of computations:

$$\tilde{\mathbf{R}}_m \rightarrow \tilde{\mathbf{n}}_m \rightarrow \tilde{\mathbf{h}}_{m_1, m_2} = \mathbf{T}_m \tilde{\mathbf{n}}_m.$$

At first, we address how close $\tilde{\mathbf{n}}_m$ is to \mathbf{n}_m . For this purpose, we may consider $\tilde{\mathbf{R}}_m$ as a perturbation of \mathbf{R}_m and apply eigenvector perturbation results. However, since \mathbf{n}_m and $\tilde{\mathbf{n}}_m$ are the minimal right singular vectors of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$ and $\mathcal{H}_m^T(\mathbf{h}_M)$, respectively, it is preferable to use singular vector perturbation results. We thus consider $\mathcal{H}_m^T(\mathbf{h}_M)$ as a perturbation of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$, and we give an upper bound for $\|\mathbf{n}_m - \tilde{\mathbf{n}}_m\|_2$.

We recall that under the no common zero assumption, $\text{rank}(\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)) = 2m + 1$, yielding $\sigma_{2(m+1)}(\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)) = 0$, with associated right singular vector \mathbf{n}_m ; in this case, \mathbf{n}_m defines the null space of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$. We denote by δ the smallest nonzero singular value of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$, i.e.,

$$\delta \triangleq \sigma_{2m+1}(\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)). \quad (5)$$

Since δ measures the distance in the matrix 2-norm of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$ from the matrices with rank $2m$, violating thus our assumption concerning its rank, it is clearly a measure of *diversity* of the channel \mathbf{h}_{m_1, m_2} .

Using (2), we identify the perturbation on $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$ as

$$\Delta_m \triangleq \mathcal{H}_m^T(\mathbf{h}_M) - \mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z) = \mathcal{H}_m^T(\mathbf{d}_{m_1, m_2}^z),$$

which, using the matrix 2-norm/ F -norm inequality, the structure of $\mathcal{H}_m^T(\mathbf{d}_{m_1, m_2}^z)$ and (3), yields

$$\|\Delta_m\|_2 = \|\mathcal{H}_m^T(\mathbf{d}_{m_1, m_2}^z)\|_2 \leq \epsilon \sqrt{m+1} \equiv \mathcal{E}. \quad (6)$$

The next theorem, whose proof is found in [9], provides an upper bound for $\|\mathbf{n}_m - \tilde{\mathbf{n}}_m\|_2$.

Theorem 1: Let $\text{rank}(\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)) = 2m + 1$. Let \mathbf{n}_m be the minimum right singular vector of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$, δ the minimum nonzero singular value of $\mathcal{H}_m^T(\mathbf{h}_{m_1, m_2}^z)$ and $\tilde{\mathbf{n}}_m$ the minimum right singular vector of $\mathcal{H}_m^T(\mathbf{h}_M)$. If $\mathcal{E} < \frac{\delta}{2}$, then

$$\|\mathbf{n}_m - \tilde{\mathbf{n}}_m\|_2 \leq 2\sqrt{2} \frac{\mathcal{E}}{\delta} \equiv \mathcal{D}. \quad (7)$$

Since $\mathbf{h}_{m_1, m_2} = \gamma \mathbf{T}_m \mathbf{n}_m$, $\tilde{\mathbf{h}}_{m_1, m_2} = \mathbf{T}_m \tilde{\mathbf{n}}_m$, and \mathbf{T}_m is orthogonal, we obtain from (4) and (7) [9]

$$\|\mathbf{h}_{m_1, m_2} - \tilde{\mathbf{h}}_{m_1, m_2}\|_2 \leq (1 - \gamma) + \mathcal{D}. \quad (8)$$

Consequently, if the diversity of the m -th order significant part is sufficiently large, and the size of the tails is sufficiently small, then the m -th order LS/SS method computes an impulse response, which is “close” to the m -th order significant part of the true channel. The delay m_1 is unknown, but this fact does not change dramatically the situation during the equalization step [9].

In the sequel, we consider the case $m = m'_2 - m'_1$, with $m'_1 < m_1$ and/or $m'_2 > m_2$, which means that we try to model not only the significant part of the channel but some “small” terms as well, and we derive an expression of the diversity of the channel we try to model, in terms of the size of the small terms. In this case, the factor which determines the accuracy of the estimation of $\mathbf{h}_{m'_1, m'_2}$ is

$$\delta' \triangleq \sigma_{2m+1}(\mathcal{H}_m^T(\mathbf{h}_{m'_1, m'_2})), \quad (9)$$

that is, the minimal nonzero singular value of $\mathcal{H}_m^T(\mathbf{h}_{m'_1, m'_2})$. In the next theorem [9], we give a relationship between δ' and $h_{m'_1}^{(j)}$, $h_{m'_2}^{(j)}$, for $j=1,2$, which provides much insight into the behavior of the algorithm, if $h_{m'_1}^{(j)}$ and/or $h_{m'_2}^{(j)}$, for $j=1,2$, are “small”.

Theorem 2: If δ' denotes the minimal nonzero singular value of $\mathcal{H}_m^T(\mathbf{h}_{m'_1, m'_2})$, then

$$\delta' \leq \min \left(\left| h_{m'_1}^{(1)} \right| + \left| h_{m'_1}^{(2)} \right|, \left| h_{m'_2}^{(1)} \right| + \left| h_{m'_2}^{(2)} \right| \right). \quad (10)$$

Since δ' decreases dramatically when we try to model “small” terms, it is clear that modeling tails, may lead to big estimation errors. This situation is analogous to “channel overmodeling”, where the subspace methods cannot identify uniquely the true channel.

4 SIMULATIONS

We process output data from the oversampled, by a factor of 2, microwave radio channel *chan4.mat*, found at <http://spib.rice.edu/spib/microwave.html>, with input 100 samples of an i.i.d. 4-QAM signal. The channel possesses long tails of small leading and trailing terms; the “small” terms are about two orders of magnitude smaller than the significant terms. In order to estimate the effective channel length, we compute (the “overmodeled”) $\tilde{\mathbf{R}}_{30}$, and we apply the AIC and MDL criteria, which, unfortunately, do not provide realistic estimates of the significant part of the channel. We found very useful the criterion:

$$\text{rank}(\mathbf{R}_L) = \arg \min_i \frac{\lambda_{i+1}(\tilde{\mathbf{R}}_L)}{\lambda_i(\tilde{\mathbf{R}}_L)}, \quad i = 1, \dots, 2L + 1,$$

which gives that the effective channel length is 2 (three taps). We apply the second order SS method. Then, we compute the corresponding first-order zero-forcing equalizers. In Fig. 3, we see that the eye is open.

Unfortunately, we did not manage to process reliably data obtained by some channels available at this site (for example, *chan3.mat*, *chan7.mat*). Finally, we found it difficult to process reliably more complicated input constellations, like, for example, 16-QAM. These are, probably, obstacles against general applicability of the method. In the cases in which we are able to open the eye, 100 input samples are enough. However, only channels whose significant part provides enough diversity and, at the same time, the unmodeled tails are sufficiently small can be approximated, and consequently equalized, sufficiently well. The development of an efficient method for the estimation of the effective channel length remains open. Results with a similar flavor concerning the LP method have been derived in [10].

References

[1] L. Tong, G. Xu and T. Kailath, “Blind identification based on second-order statistics: A time domain approach,” *IEEE Trans. Information Theory*, vol. 40, no. 2, pp. 340-349, 1994.

[2] G. Xu, H. Liu, L. Tong and T. Kailath, “A least-squares approach to blind channel identification,” *IEEE Trans. Signal Processing*, vol. 43, pp. 2982-2993, Dec. 1995.

[3] E. Moulines, P. Duhamel, J. F. Cardoso and S. Mayrargue, “Subspace methods for the blind identification of multichannel FIR filters,” *IEEE Trans. Signal Processing*, vol. 43, pp. 516-525, Feb. 1995.

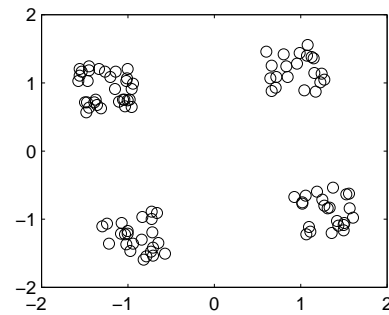


Fig. 3. Best case output of first-order zero-forcing equalizer corresponding to second-order SS.

[4] D. Slock, “Blind fractionally-spaced equalization, perfect reconstruction filterbanks, and multilinear prediction,” in *Proc. ICASSP'94*, Adelaide, Australia, April 1994.

[5] J. R. Treichler, I. Fijalkow and C. R. Johnson, Jr., “Fractionally spaced equalizers. How long should they really be?,” *IEEE Signal Processing Magazine*, pp. 65-81, May 1996.

[6] I. Fijalkow, A. Touzni and J. R. Treichler, “Fractionally spaced equalization using CMA: Robustness to channel noise and lack of disparity,” *IEEE Trans. Signal Processing*, vol. 45, pp. 56-66, Jan. 1997.

[7] H. Zeng and L. Tong, “Mean-Squared Error Performance of Constant Modulus Receiver for Singular Channels,” *Proc. ICASSP'97*, Munich, Germany, April 1997.

[8] T. J. Endres, B. D. O. Anderson, C. R. Johnson and M. Green, “On the robustness of the fractionally-spaced constant modulus criterion to channel order undermodeling: Part I,” *Proc. IEEE Workshop on Signal Proc. Advances in Wireless Comm.*, Paris, France, April, 1997.

[9] A. P. Liavas, P. A. Regalia and J. P. Delmas, “Robustness of least-squares and subspace methods for blind channel identification/equalization with respect to channel undermodeling,” submitted to the *IEEE Trans. Signal Processing*, December 1997.

[10] A. P. Liavas, P. A. Regalia and J. P. Delmas, “Robustness of the linear prediction method for blind channel identification with respect to effective channel undermodeling/overmodeling,” submitted to the *IEEE Trans. Signal Processing*, April 1998.

[11] H. Zeng and L. Tong, “Connections between the least-squares and subspace approaches to blind channel estimation,” *IEEE Trans. Signal Processing*, vol. 44, pp. 1593-1596, June 1996.