# BLIND AND INFORMED CYCLIC ARRAY PROCESSING FOR CYCLOSTATIONARY SIGNALS


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## ABSTRACT

*Limiting the analysis to the exploitation of the second order statistics of the complex data, the optimal Spatio-Temporal (ST) receivers in stationary contexts are Linear and Time Invariant (TI). However, for (quasi)-cyclostationary observations, it is now well-known that the optimal ST complex receivers become (poly)-periodic (PP) and, under some conditions of non circularity, Widely Linear (WL). Using these results and the fact that PP filtering is equivalent to FREquency SHifted (FRESH) filtering, the purpose of this paper is to present a new ST, PP and WL receiver structure, very useful for applications such as passive listening or source separation, taking into account the potential (quasi)-cyclostationarity and non circularity properties of the observations. This new cyclic receiver may be implemented either blindly or from the a priori knowledge or estimation of the useful signal steering vector. The performance computation of this new cyclic receiver shows off the great interest of the latter in cyclostationary contexts and its great capability of interferences rejection even from a one sensor reception.*


## 1. INTRODUCTION

Limiting the analysis to the exploitation of the second order statistics of the complex data, the optimal ST receivers in stationary contexts are Linear and TI. However, for (quasi)-cyclostationary observations, it is now well-known [1] that the optimal ST complex receivers become PP [2] and, under some conditions of non circularity [3], WL [4]. The interest of optimal PP and WL filters in the digital radiocommunication context has been presented in [2] for temporal filters and in [5] for spatial filters, assuming a training sequence is available for the useful signal. More recently, assuming the a priori knowledge or estimation of the useful signal steering vector (passive listening, source separation...), the interest of PP and WL ST filters has been analysed in [6], where a cyclic LCMV beamformer corresponding to a cyclic extension of the well-known Generalized Sidelobe Canceller [7] has been introduced.

The purpose of this paper is to present a new ST, PP and WL receiver structure, very useful for applications such as passive listening or source separation, taking into account the potential (quasi)-cyclostationarity and non circularity properties of the observations. This new cyclic receiver, belonging to the class of MMSE-based cyclic receiver and called Hybrid MMSE (HMMSE) Cyclic Receiver, may be implemented either blindly or from the a priori knowledge or estimation of the useful signal steering vector. The performance computation of this new cyclic receiver shows off the great interest of the latter in cyclostationary contexts and its great capability of interferences rejection even from a one sensor reception.

## 2. PROBLEM FORMULATION

Consider an array of $N$ Narrow-Band (NB) sensors and let us call $\boldsymbol{x}(t)$ the vector of the complex envelopes of the signals present at time $t$ at the output of the sensors. Each sensor is assumed to receive the contribution of a useful cyclostationary signal, a cyclostationary jammer and a background noise. Under these assumptions, the observation vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{x}(t) = s(t)\, e^{j(\Delta\omega_0 t+\phi_0)}\boldsymbol{s} + j(t)\, e^{j(\Delta\omega_i t+\phi_i)}\, \boldsymbol{J} + \boldsymbol{b}(t) \qquad (2.1)$$

where $\boldsymbol{b}(t)$ is the noise vector, assumed spatially white and stationary, $s(t)$, $\Delta\omega_0$, $\phi_0$ and $\boldsymbol{s}$ are the complex envelope, assumed zero-mean and cyclostationary, the carrier residue, the phase and the steering vector of the useful signal respectively, whereas $j(t)$, $\Delta\omega_i$, $\phi_i$ and $\boldsymbol{J}$ are the complex envelope, assumed zero-mean and cyclostationary, the carrier residue, the phase and the steering vector of the jammer respectively.

Under the previous assumptions, for a given ST observation ($NL$ x 1) vector $\boldsymbol{x}_{ST}(t) \triangleq (\boldsymbol{x}(t)^T, \boldsymbol{x}(t - T_e)^T, ..., \boldsymbol{x}(t - (L-1)T_e)^T)^T$, where $T_e$ is the sample period, using the fact that PP filtering is equivalent to FREquency SHifted (FRESH) filtering [2], the general output of a $M$-th order PP filter is defined by [6]

$$y(t) = \boldsymbol{h}_1{}^{\dagger}\boldsymbol{x}_{ST}(t) + \sum_{m=2}^{M} \boldsymbol{h}_m{}^{\dagger}\boldsymbol{x}_{ST}(t-\Delta_m)^{\zeta_m}\, e^{j2\pi\alpha_m t} \tag{2.2}$$

where, for $1 \leq m \leq M$, $\boldsymbol{h}_m$ is a ($NL$ x 1) TI complex filter, $\zeta_m = \pm 1$ with $\boldsymbol{x}^{1} \triangleq \boldsymbol{x}$ and $\boldsymbol{x}^{-1} \triangleq \boldsymbol{x}^*$ (complex conjugate), $\Delta_m$ is a delay and $\alpha_m$ is a cyclic frequency. Note that the 1st-order PP filter, defined by (2.2) with $M = 1$, is the classical Linear and TI ST filter, whereas for $M > 1$, the Mth-order PP filter defined by (2.2) is Linear if all the $\zeta_m$ are equal to 1 and WL in the other cases, TI if all the $\alpha_m$ are zero and PP otherwise, Temporal if $N = 1$, Spatial if $L = 1$ and all the $\Delta_m$ are zero and ST in the other cases.

For given values of $L$ and $N$ and for given observations $\boldsymbol{x}_{ST}(t)$, the general problem of finding the optimal $M$-th order PP filter consists to find the quantities $\boldsymbol{h}_m$, $\Delta_m$, $\zeta_m$ and $\alpha_m$, $1 \leq m \leq M$, such that the output $y(t)$, defined by (2.2), gives the best restitution or estimation of the useful signal $x_s(t) \triangleq s(t)\, e^{j(\Delta\omega_0 t+\phi_0)}$, in a particular performance criterion sense, under the constraint of the knowledge of more or less a priori information on the signals. In this paper, no useful training sequence is assumed to be a priori avalaible and *at most* the steering vector, $\boldsymbol{s}$, of the useful signal is assumed to be a priori known.

## 3. PERFORMANCE CRITERION

Defining the ($MNL$ x 1) vectors $\boldsymbol{H} \triangleq [\boldsymbol{h}_1{}^T, \boldsymbol{h}_2{}^T, ..., \boldsymbol{h}_M{}^T]^T$ and $\boldsymbol{X}(t) \triangleq [\boldsymbol{x}_{ST}(t)^T, \exp(j2\pi\alpha_2 t)\, \boldsymbol{x}_{ST}(t-\Delta_2)^{\zeta_2\, T}, ..., \exp(j2\pi\alpha_M t)\, \boldsymbol{x}_{ST}(t-\Delta_M)^{\zeta_M\, T}]^T$, the expression (2.2) can be written as

$$y(t) = \boldsymbol{H}^{\dagger}\boldsymbol{X}(t) \tag{3.1}$$

The quality of the restitution of $x_s(t)$ by $y(t)$ could be evaluated by computing the Mean Square Error (MSE) between $y(t)$ and $x_s(t)$, as it is done for example in [2]. However while $\boldsymbol{H}$ and $\mu\boldsymbol{H}$, where $\mu$ is an arbitrary constant, are equivalent filters, since they give the same output contrast between the useful signal and the total noise (jammer plus noise), or, for digital useful signals, the same output Bit Error Rate (BER), they don't give the same MSE. This result means that generally, the MSE between $y(t)$ and $x_s(t)$ is not a good criterion to evaluate the performance of the filter $\boldsymbol{H}$.

To find a good alternative to the MSE criterion, let us introduce the ($MNL$ x 1) vectors $\boldsymbol{S}(t)$, $\boldsymbol{J}(t)$ and $\boldsymbol{B}(t)$ defined in a same manner as $\boldsymbol{X}(t)$ but with the vector $\boldsymbol{x}_{ST}(t)$ replaced by the vector containing only the useful signal, the jammer and the noise contribution in $\boldsymbol{x}_{ST}(t)$ respectively. With these definitions, the output $y(t)$ can be written as

$$y(t) = \boldsymbol{H}^{\dagger}\boldsymbol{S}(t) + \boldsymbol{H}^{\dagger}\boldsymbol{J}(t) + \boldsymbol{H}^{\dagger}\boldsymbol{B}(t) \tag{3.2}$$

If we call *signal of interest at the output of* $\boldsymbol{H}$ all the terms of $y(t)$ which are proportional to $x_s(t)$, it is obvious that the terms $\boldsymbol{H}^{\dagger}\boldsymbol{J}(t)$ and $\boldsymbol{H}^{\dagger}\boldsymbol{B}(t)$ belong to the total noise terms at the output of $\boldsymbol{H}$ whereas the term $\boldsymbol{H}^{\dagger}\boldsymbol{S}(t)$ contains both useful and noise components. Applying the projection theorem on the useful complex signal $x_s(t)$ at each component of $\boldsymbol{S}(t)$, with the inner product $(u(t), v(t)) \triangleq$ <E[$u(t)v(t)^*$]>, where the symbol <.> corresponds to the time average operation, the vector $\boldsymbol{S}(t)$ can be written as

$$\boldsymbol{S}(t) = x_s(t)\, \boldsymbol{S} + \boldsymbol{I}(t) \tag{3.3}$$

where <E[$\boldsymbol{I}(t)\, x_s(t)^*$]> = 0 and $\boldsymbol{S}$ is a time-independent vector which components are the coefficients of $x_s(t)$ appearing in the orthogonal projection of the components of $\boldsymbol{S}(t)$ on $x_s(t)$. Inserting (3.3) into (3.2) we obtain

$$y(t) = x_s(t)\, \boldsymbol{H}^{\dagger}\boldsymbol{S} + \boldsymbol{H}^{\dagger}\boldsymbol{I}(t) + \boldsymbol{H}^{\dagger}\boldsymbol{J}(t) + \boldsymbol{H}^{\dagger}\boldsymbol{B}(t) \tag{3.4}$$

$$\triangleq x_s(t)\, \boldsymbol{H}^{\dagger}\boldsymbol{S} + \boldsymbol{H}^{\dagger}\boldsymbol{B}_T(t) \tag{3.5}$$

where $\boldsymbol{B}_T(t) \triangleq \boldsymbol{I}(t) + \boldsymbol{J}(t) + \boldsymbol{B}(t)$. It then becomes obvious that the *signal of interest at the output of* $\boldsymbol{H}$ corresponds to the term $x_s(t)\boldsymbol{H}^{\dagger}\boldsymbol{S}$ and the term $\boldsymbol{H}^{\dagger}\boldsymbol{B}_T(t)$ is a noise term. From these results the Signal to Total Noise Ratio (Signal to Interference plus Noise Ratio or SINR) at the output of $\boldsymbol{H}$, SINR[$\boldsymbol{H}$], can be defined and is taken in the paper as the performance criterion

$$\text{SINR}[\boldsymbol{H}] \triangleq \pi_s\, |\boldsymbol{H}^{\dagger}\boldsymbol{S}|^2 / \boldsymbol{H}^{\dagger}R_{BT}\boldsymbol{H} \tag{3.6}$$

where $\pi_s \triangleq$ <E[$|s(t)|^2$]> and $R_{BT} \triangleq$ <E[$\boldsymbol{B}_T(t)\boldsymbol{B}_T(t)^{\dagger}$]>. Note that $\boldsymbol{H}$ and $\mu\boldsymbol{H}$ have the same output performance and it is easy to verify that

$$\text{SINR}[\boldsymbol{H}] = \pi_s\, / \,\text{MSE}[\boldsymbol{H}/\boldsymbol{S}^{\dagger}\boldsymbol{H}] \tag{3.7}$$

which unifys the SINR and the MSE, where MSE[$\boldsymbol{H}$] corresponds to the MSE between $x_s(t)$ and $\boldsymbol{H}^{\dagger}\boldsymbol{X}(t)$. Finally, introducing $R_X \triangleq$ <E[$\boldsymbol{X}(t)\boldsymbol{X}(t)^{\dagger}$]> and $\boldsymbol{r}_{Xs} \triangleq$ <E[$\boldsymbol{X}(t)\, x_s(t)^*$]>, the SINR[$\boldsymbol{H}$] can also be written as

$$\text{SINR}[\boldsymbol{H}] = (|\boldsymbol{H}^{\dagger}\boldsymbol{r}_{Xs}|^2/\pi_s) \,/\, [\boldsymbol{H}^{\dagger}R_X\boldsymbol{H} - (|\boldsymbol{H}^{\dagger}\boldsymbol{r}_{Xs}|^2/\pi_s)] \tag{3.8}$$

## 4. OPTIMAL M-TH ORDER PP FILTER

From (3.6), it becomes obvious that the optimal M-th order PP filter, $\boldsymbol{H}_o$, is non unique and defined by

$$\boldsymbol{H}_o = \mu_1\, R_{BT}^{-1}\, \boldsymbol{S} = \mu_2\, R_X^{-1}\, \boldsymbol{r}_{Xs} \tag{4.1}$$

where $\mu_1$ and $\mu_2$ are arbitrary constants, while the maximal output SINR is given by

$$\mathrm{SINR_o} \triangleq \pi_s\, \boldsymbol{S}^{\dagger} R_{BT}^{-1}\, \boldsymbol{S} \qquad (4.2)$$

The expression (4.1) shows that the optimal M-th order PP filters $\boldsymbol{H}_o$ correspond to the filters proportional to the M-th order PP Wiener filter, obtained for $\mu_2 = 1$. The implementation of this optimal filter $\boldsymbol{H}_o$ requires the a priori knowledge or estimation of the vectors $\boldsymbol{S}$ or $\boldsymbol{r}_{Xs}$, which seems itself to require the a priori knowledge of a useful training sequence as soon as $ML > 1$. However, for applications such as the passive listening, training sequences are not a priori available and, *at most*, the steering vectors of the signals may be a priori estimated. In such situations, the M-th order PP Wiener filter seems difficult to be implemented directly and alternative M-th order PP filters have to be found. A first alternative, corresponding to the M-th order PP GSLC (cyclic GSLC), has been proposed recently in [6] and has been shown to give relatively weak gain in performance with respect to the well-known Linear and TI Spatial Matched Filter, due to the strong constraints imposed to the useful signal. In the next section we propose a second alternative, the M-th order PP HMMSE filter (HMMSE Cyclic Receiver), which seems to be much more powerful than the Cyclic GSLC.

## 5. M-TH ORDER PP HMMSE FILTER

### 5.1 Problem description

The alternative PP filter proposed in this section aims at trying to implement the M-th order PP Wiener filter from, *at most*, the a priori knowledge of the useful signal steering vector $\boldsymbol{s}$. More precisely, as it has already been done in [8] for Linear and TI filters, the idea of this section consists, firstly, to try to estimate a *good* useful training sequence, $d_0(t)$, from the data and, *at most*, the a priori knowledge of $\boldsymbol{s}$, and, secondly, to implement the M-th order PP filter which minimizes the MSE between $d_0(t)$ and the filter output. In other words, the problem we address in this section is to find the M-th order PP filter $\boldsymbol{H}$ such that the output $y(t)$, defined by (3.1), minimizes the MSE, MSE[$d_0(t)$, $\boldsymbol{H}$], defined by

$$\mathrm{MSE}[d_0(t), \boldsymbol{H}] \triangleq \langle \mathrm{E}[|d_0(t) - \boldsymbol{H}^{\dagger}\boldsymbol{X}(t)|^2]\rangle \qquad (5.1)$$

where the training sequence $d_0(t)$, built from the data, has the form

$$d_0(t) = \boldsymbol{h}_0^{\dagger}\boldsymbol{x}_{\mathrm{ST}}(t-\Delta_0)^{\zeta_0}\, e^{j2\pi\alpha_0 t} \qquad (5.2)$$

and where the choice of the ($NL$ x 1) vector $\boldsymbol{h}_0$ and the quantities $(\Delta_m,\zeta_m,\alpha_m)$, $0 \le m \le M$, must be optimized. The filter $\boldsymbol{H}$ solution of this problem is said hybrid since it is the result of two cascaded optimization problems, the first one corresponding to the choice of the best reference signal $d_0(t)$, from the data and *at most* the vector $\boldsymbol{s}$, and the second one being the optimization of the MSE (5.1). Note that the problem which is addressed in [8] is the one which is addressed in this section but with $M = 1$ and $L = 1$, i.e. only with classical Linear, TI and Spatial complex filters.

### 5.2 M-th order PP HMMSE filter expression

Defining $\boldsymbol{x}_0(t)$ by $\boldsymbol{x}_0(t) \triangleq \boldsymbol{x}_{\mathrm{ST}}(t-\Delta_0)^{\zeta_0}\, e^{j2\pi\alpha_0 t}$ and the ($MNL$ x $NL$) matrix $r_{Xx0}$ by $r_{Xx0} \triangleq \langle \mathrm{E}[\boldsymbol{X}(t)\, \boldsymbol{x}_0(t)^{\dagger}]\rangle$, the M-th order PP HMMSE filter, $\boldsymbol{H}_{hy}$, solution of the previous problem, can be written as

$$\boldsymbol{H}_{hy} = R_X^{-1}\, r_{Xx0}\, \boldsymbol{h}_0 \qquad (5.3)$$

In these conditions, for given values of $M$, $L$ and $N$, the problem is to choose the vector $\boldsymbol{h}_0$ and the quantities $(\Delta_m,\zeta_m,\alpha_m)$, $0 \le m \le M$, such that the SINR at the output of $\boldsymbol{H}_{hy}$ is as high as possible and, if possible, maximized over all the filters $\boldsymbol{H}$, in which case, $\boldsymbol{H}_{hy}$ and $\boldsymbol{H}_o$ would coïncide.

### 5.3 Parameters optimization

A necessary and sufficient condition for $\boldsymbol{H}_{hy}$ to coïncide with $\boldsymbol{H}_o$ is that the vector $r_{Xx0}\boldsymbol{h}_0$ becomes proportional to the vector $\boldsymbol{S}$. Using (2.1) into (5.2) and taking into account only ($N$ x 1) spatial vectors for $\boldsymbol{h}_0$, we deduce from (3.4) and (5.2) the expression of $r_{Xx0}\boldsymbol{h}_0$, given by

$$\begin{aligned} r_{Xx0}\boldsymbol{h}_0 = {} & \langle \mathrm{E}[x_s(t)x_s(t-\Delta_0)^{\zeta_0 *}]\, e^{-j2\pi\alpha_0 t}\rangle\, (\boldsymbol{s}^{\zeta_0 \dagger}\boldsymbol{h}_0)\, \boldsymbol{S} \\ & + \langle \mathrm{E}[\boldsymbol{I}(t)\, x_s(t-\Delta_0)^{\zeta_0 *}]\, e^{-j2\pi\alpha_0 t}\rangle\, (\boldsymbol{s}^{\zeta_0 \dagger}\boldsymbol{h}_0) \\ & + \langle \mathrm{E}[\boldsymbol{J}(t)\, x_j(t-\Delta_0)^{\zeta_0 *}]\, e^{-j2\pi\alpha_0 t}\rangle\, (\boldsymbol{J}^{\zeta_0 \dagger}\boldsymbol{h}_0) \\ & + \langle \mathrm{E}[\boldsymbol{B}(t)\, \boldsymbol{b}(t-\Delta_0)^{\zeta_0 \dagger}]\, e^{-j2\pi\alpha_0 t}\rangle\, \boldsymbol{h}_0 \end{aligned} \qquad (5.4)$$

From this expression, we deduce that a necessary and sufficient condition for the component proportional to $\boldsymbol{S}$ not to be nulled is that the term $(\boldsymbol{s}^{\zeta_0 \dagger}\boldsymbol{h}_0)$ is not zero and that the parameters $(\Delta_0,\zeta_0,\alpha_0)$ are such that the cyclic correlation function of $x_s(t)$ associated to these parameters is not zero. In other words, the parameters $(\Delta_0,\zeta_0,\alpha_0)$ must be associated to a cyclic frequency of the useful signal.

On the other hand, due to the stationarity of $\boldsymbol{b}(t)$, the term of expression (5.4) associated to the noise vector $\boldsymbol{B}(t)$, becomes zero provided that for each value of $m$ ($1 \le m \le M$), $(\Delta_m,\zeta_m,\alpha_m)$ is different from $(\Delta_0,\zeta_0,\alpha_0)$.

Besides, the term associated to the jammer in the expression (5.4) becomes nulled if $(\boldsymbol{J}^{\zeta_0 \dagger}\boldsymbol{h}_0)$ or if $\langle \mathrm{E}[\boldsymbol{J}(t)\, x_j(t-\Delta_0)^{\zeta_0 *}]\, e^{-j2\pi\alpha_0 t}\rangle$ is zero, the latter situation generally occuring when the signal and the jammer do not share any cyclic frequency.

Finally, under the previous conditions, the term associated to the vector $\boldsymbol{I}(t)$ in (5.4) becomes zero for some particular choices of parameters $(\Delta_m,\zeta_m,\alpha_m)$ $(0 \le m \le M)$ and for some useful signal modulations.

Thus, when all the previous conditions are verified, $\boldsymbol{H}_{hy}$ and $\boldsymbol{H}_o$coïncide. In this case, we can show [1] [5] that the gain in performance obtained in using $\boldsymbol{H}_{hy}$ instead of a classical Linear and TI filter increases as the cyclic correlation coefficient of the jammer associated to the parameters $(\Delta_m,\zeta_m,\alpha_m)$ $(1 \le m \le M)$ increases, which gives a new information for the choice of $(\Delta_m,\zeta_m,\alpha_m)$ $(1 \le m \le M)$. The filter $\boldsymbol{H}_{hy}$ will be blind if no useful signal information is used to construct $\boldsymbol{h}_0$ and informed in the other cases.

## 6. EXAMPLE

To illustrate the previous results, the figure 1 shows for $N = 1$ and $N = 2$, the variations of the SINR at the output of the classical Linear and TI Wiener filter ($M = 1$), noted (W), and at the output of the HMMSE filter, noted (H), as a function of the product $T_s$x$\Delta f_j$, where $T_s$ is the symbol duration of the useful signal and $\Delta f_j$ is the carrier residu of the jammer, for BPSK useful signal and jammer, which pulse functions are 1/2 Nyquist filters with a roll off equal to 1. The useful signal impinges on the array at 0° from broadside with $\Delta f_s = \phi_s = 0$, SNR (Signal to Noise Ratio) = 5 dB and $T_s = 6T_e$. The jammer impinges on the array at 5° from broadside with $\phi_j = 0$, JNR (Jammer to Noise Ratio) = 20 dB and a symbol duration $T_j = T_s$. The filter $\boldsymbol{h}_0$ has all its components nulled except the first one which is equal to one. For all the filters, $L = 6$. For the HMMSE filter, $M = 4$ with $(\Delta_0,\zeta_0,\alpha_0) = (0, -1, 0)$, $(\Delta_2,\zeta_2,\alpha_2) = (0, -1, 2\Delta f_j)$, $(\Delta_3,\zeta_3,\alpha_3) = (0, -1, 2\Delta f_j + 1/T_j)$, $(\Delta_4,\zeta_4,\alpha_4) = (0, -1, 2\Delta f_j - 1/T_j)$.

Under these assumptions, the M-th order PP HMMSE filter and the M-th order PP Wiener filter coïncide and the figure 1 shows their great capability of rejection with respect to that of the classical Wiener filter for both N = 1 and N = 2, which shows the great interest of the HMMSE cyclic filter for applications such as passive listening or blind source separation after the blind identification of the sources steering vectors.

## 7. CONCLUSION

In this paper, a new M-th order PP receiver which implementation requires *at most* the a priori knowledge of the useful signal steering vector, has been presented to improve the performance of the classical Linear and TI Wiener filter in (quasi)-cyclostationary contexts. This new receiver is called Hybrid MMSE cyclic receiver since its implementation is made in two stages. In a first time, a useful training sequence is constructed from the data and in a second time, the HMMSE is computed by minimizing the averaged MSE between the estimated training sequence and the filter output. The conditions under which the new filter implements the M-th order PP Wiener filter (optimal in a SINR maximisation sense), either blindly or from the a priori knowledge of $\boldsymbol{s}$, have been given. The performance illustration of this new filter shows the great interest of the latter for passive listening or blind source separation in cyclostationary contexts.

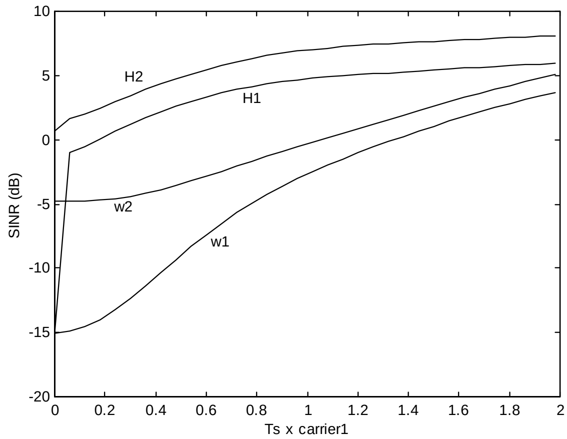


**Fig. 1** - *SINR at the output of the HMMSE Cyclic (H) and Classical Wiener (W) receiver as a function of* $T_s$x$\Delta f_j$*, for N = 1 and 2*

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