Solution of the Super-Resolution Problem Through Extrapolation of the Orthogonal Spectra Using Multi-Valued Neural Technique

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ABSTRACT

Two methods of the orthogonal spectra extrapolation problem are considered in this paper. Both solutions are based on the application of multi-valued neural element as a filter and as an extrapolator. The first method consist of approximation of the spectra in the higher frequency part using an iterative approximation and multi-valued non-linear filtering. The second method is reduced to the prediction of the spectral coefficients corresponding to the higher frequency part using possibility of MVN to predict time-series. Application of the proposed methods to solution of the super-resolution problem are presented.

1 INTRODUCTION

Multi-valued non-linear filters have been introduced in [1], and then have been applied in [2] to solution of the noise reduction and frequency correction problems. Multi-valued filters are closely related with multi-valued neurons introduced in [3].

We will consider here two approaches to extrapolation of the orthogonal spectra. This problem is very important, and some approaches to its solution exist, especially for 2-D spectrums and signals [4,5]. The importance of the problem is evident from fact that extrapolation of the spectrum always is better than interpolation of the signal in spatio-temporal domain.

One approach which we propose here is a significant improvement of methods proposed in [4,5]. They are based on the approximation of the higher frequency part of Fourier spectrum via special iterative procedure. We will derive here a similar technique for Cosine and Walsh spectrums which are much simpler from the computational point of view. To obtain the most precise approximation of spectrum we will use a non-linear multi-valued filtering.

Another approach is based on the possibility of multi-valued neuron (MVN) to implement a time-series prediction [6]. Supposing that each spectral coefficient is a function of some of the lowest ones, it is possible to train MVN to implement this function as a mapping between inputs and output of the neuron, and then to predict unknown spectral coefficients in the higher frequency part.

2 MULTI-VALUED FILTERS AND MULTI-VALUED NEURONS

Multi-valued neuron (MVN) [3] is the neural element which performs a mapping between n inputs and one output described by a multi-valued (k-valued) function $f(x_1, ..., x_n)$ by the following way:

$$f(x_1, ..., x_n) = P(w_0 + w_1 x_1 + ... + w_n x_n), \qquad (1)$$

where $x_1, ..., x_n$ are variables of the performed function (neuron inputs), and P is the following activation function:

$$P(z) = j, \text{ if } 2\pi(j+1)/k > arg(z) \geq 2\pi j/k, \qquad (2)$$

where $j = 0, 1, ..., k-1$ are values of the k-valued logic, $z = w_0 + w_1 x_1 + ... + w_n x_n$ is the weighted sum, $arg(z)$ is the argument of the complex number z.

Two-dimensional multi-valued filtering (MVF) in spatial domain is defined by following equation [1]:

$$\hat{B}_{ij} = P(w_0 + \sum_{i-n \leq s \leq i+n, \;\; j-m \leq l \leq j+m} w_{sl} Y_{sl}), \qquad (3)$$

where Y_{sl} are the signal values from a local window around ij-th pixel (in a complex form obtained by transformation $\epsilon^{B_{sl}} = exp(\mathrm{i}2\pi B/k) = Y_{sl}$, where B_{sl} is the integer signal value, k is the value of k-valued logic, it has to be equal for gray-scale images to number of the gray levels, i is an imaginary unit), i, j are the coordinates of the filtered pixel, $n \times m$ is a filter aperture, w_{sl} are the filter's coefficients (complex-valued in general), P is a non-linear function (2), which is an activation function of the multi-valued neuron.

3 APPROXIMATION OF THE SPECTRUM USING MULTI-VALUED FILTERING

The solutions of spectrum extrapolation problem which have been proposed in [4,5] are based on the two fundamental facts:

1) the two-dimensional Fourier-image of a spatial-limited function is an analytic function in frequency domain;

2) if an analytic function in frequency domain is defined exactly on the limited subdomain, it is defined on all domain, where it is analytical.

The iterative procedures which have been proposed in [4,5] are directed to the simultaneous restoring of unknown part of spectrum corresponding to the highest frequencies, and values of signal in spatial domain. Computer implementation of both algorithms is not difficult, but they have some disadvantages. The starting zero-values of spectral coefficients and signal values can not be recognized as a good solution. Such a method supposes in advance that the restored part of a spectrum will be rather smooth, and the signal values in the restored domain will not be so close to the ideal values.

We would like to propose here a solution of the same problem, which will be based on the same background, but will be free from the disadvantages of previous solutions. So, we will provide the similar iterative procedure, but with the following significant differences:

1) the starting values of the restoring spectrum and signal will not be zero-valued, or constant;

2) a final correction of the spectrum and signal will be realized by non-linear multi-valued filtering in spatial domain, which will be implemented using cellular neural networks;

3) not only Fourier, but Cosine and Walsh transforms will be used, moreover they are preferable because of their computational efficiency.

Let $f(x,y)$ be a discrete $n \times n$ image (without loss of generality), defined on the spatial subdomain $\tilde{A} \subset A$. The function $F(u,v) = \Phi[f(x,y)]$; $u,v \in \tilde{B} = \{0,1,...,n-1\}^2$, where Φ is the two-dimensional (separable) Fourier, or Cosine, or Walsh (ordered by Walsh) transform, is the spectrum of the signal f. The problem is an extrapolation of the function F to the domain $u,v \in B = \{0,1,...,n-1,n,...,2n-1\}^2$. In other words, it is evaluation of the values of the image $f(x,y)$ of a $2n \times 2n$ sizes on the whole domain A. So, it is always a dual problem: extrapolation of the spectrum, and interpolation of the image. Let us suppose that

$$g(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in \tilde{A}, \\ s(x,y), & \text{if } (x,y) \in A \setminus \tilde{A}, \end{cases} \qquad (4)$$

where $s(x,y)$ is uniform noise with the same mean value that $f(x,y)$, and a small dispersion. We will use the function g defined by (4) as a starting approximation in our iterative and recursive algorithm. So, $g(x,y)$ should be considered as $f(x,y)$, corrupted by the additive uniform noise, moreover, we know in such a case, that $f(x,y)$ is corrupted within the domain $A \setminus \tilde{A}$ only. It means that our problem may be formulated as a problem of a noise reduction and further correction of the highest frequencies. We will use the multi-valued filters described in previous Section to remove the noise, and then to amplify the highest frequencies. First of all we have to obtain a more precise approximation of our resulting signal than (4). We will obtain it from (4) taking into account that the exact values of the signal $f(x,y)$ on the subdomain $\tilde{A}$, and therefore, of its spectrum $F(u,v)$; $u,v \in \{0,1,...,n-1\}$ are known.

Let us build the following iterative and recursive process. Let $f_1(x,y) = g(x,y)$ be the starting approximation of the signal. Then

$$\tilde{F}(u,v) = \Phi[f_1(x,y)], \qquad (5)$$

and

$$F_1(u,v) = \begin{cases} F(u,v), & (u,v) \in \tilde{B}, \\ \tilde{F}(u,v), & (u,v) \in B \setminus \tilde{B}, \end{cases} \qquad (6)$$

will be the starting approximation of the spectrum. Evidently, for the n-th approximation we have:

$$F_{n-1}(u,v) = \begin{cases} F(u,v), & (u,v) \in \tilde{B}, \\ \Phi[f_{n-1}(x,y)], & (u,v) \in B \setminus \tilde{B}, \end{cases} \qquad (7)$$

$$g_{n-1}(x,y) = \Phi^{-1}[F_{n-1}(u,v)], \qquad (8)$$

$$f_n(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in \tilde{A}, \\ g_{n-1}(x,y), & \text{if } (x,y) \in A \setminus \tilde{A}. \end{cases} \qquad (9)$$

So, the equations (5)-(9) define the iterative process obtaining the best approximation for the signal $f(x,y)$ and its spectrum $F(u,v)$. Since the discrete functions are not analytical it is impossible to obtain a precise proof of the convergence for process (5)-(9). Despite this fact experiments have shown that such a process is stabilized after not more than 6-7 steps, and we will obtain the following:

$$f_n(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in \tilde{A}, \\ f(x,y) + \tilde{s}(x,y), & \text{if } (x,y) \in A \setminus \tilde{A}, \end{cases} \qquad (10)$$

where $\tilde{s}(x,y)$ is an additive noise. According to the equation (10) $\tilde{f}(x,y) = f_n(x,y)$ contains an additive noise in subdomain $A \setminus \tilde{A}$. The MVF will be the best way for de-noising since it removes noise with a maximal preservation of the useful signal (of image boundaries) [1, 2], and it is possible to use the MVF not only for the noise removal, but also for the high frequencies amplification [1, 2], which is very important for sharpening of the smallest image details and boundaries. We can use the following weighting template for the implementation of filter (1) [1]:

$$W_0 = C; \quad W = \begin{pmatrix} 1 & 1 & 1 \\ 1 & w_{22} & 1 \\ 1 & 1 & 1 \end{pmatrix}; \quad 1 \le w_{22} < 10.$$

Let $\tilde{\tilde{f}}(x,y)$ will be the result of the filtering the signal $\tilde{f}(x,y) = f_n(x,y)$. We can obtain more close approximation for f(x,y) by the following way:

$$\hat{f}(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in \tilde{A}, \\ \tilde{\tilde{f}}(x,y), & \text{if } (x,y) \in A \setminus \tilde{A}. \end{cases} \qquad (11)$$

Finally, we have to correct the high frequencies of the signal $\hat{f}(x,y)$, since the high frequency domain coefficients have been smoothed by filtering. The multi-valued filter (1) implemented by the following weighting template [2]

$$W = \begin{pmatrix} -0.5 & -0.5 & -0.5 \\ -0.5 & G & -0.5 \\ -0.5 & -0.5 & -0.5 \end{pmatrix}; \quad 6 \le G < 16,$$

will be used to make such a correction of the signal $\hat{f}(x,y)$ which has been obtained by (10). Let $\hat{\hat{f}}(x,y)$ is a result of such a correction. To complete the process we have to do the following:

$$f^*(x,y) = \begin{cases} f(x,y), & \text{if } (x,y) \in \tilde{A}, \\ \hat{\hat{f}}(x,y), & \text{if } (x,y) \in A \setminus \tilde{A}, \end{cases} \tag{12}$$

where $f^*(x,y)$ is the final approximation for the $2n \times 2n$ image $f(x,y)$ defined on the domain A. Evidently, it is possible to repeat this process, defined by the (4)-(12), and to obtain the approximation $f_2^*(x,y)$ for a $4n \times 4n$ image, and so on.

4 PREDICTION OF THE HIGHEST FREQUENCY COEFFICIENTS ON MULTI-VALUED NEURON

Let us take a signal f (its discrete values $f_1, f_2, ..., f_i, f_j, ..., f_N$) defined on the equal intervals (we will consider the one-dimensional description for simplicity). Our problem is to evaluate values

$$f_{i+s}, \quad i = 1, ..., N;$$

$i + s < j$; $s = 1/p, 2/p, ..., (p-1)/p$; $p \in \{2, 4, 8, 16, ...\}$. For example: $n = 4$, we have to obtain $f_{i+0.25}, f_{i+0.5}, f_{i+0.75}$ between f_i and f_j. We would like to propose here approach which is based on application of the MVN as time-series extrapolator (predictor) [6]. So, solution of the considered problem is reduced to the following steps:

1) Evaluation of the spectrum (Fourier, Cosine, Walsh) of the signal f : $S_f = (s_1, ..., s_N)$;

2) Supposing that each spectral coefficient is a function of the q lowest ones, it is possible to train MVN to implement mapping that is described by the function g;

3) Extrapolation of the spectrum: $\tilde{S}_f = (s_1, ..., s_N, \tilde{s}_{N+1}, ..., \tilde{s}_{pN})$ by prediction of the coefficients corresponding to the highest frequencies;

4) Evaluation of the pN - dimensional inverse transformation and obtaining of the needed values of the signal f.

5 SIMULATION RESULTS

Figures 1-4 present the examples which illustrate both solutions. Fig. 1 contains the original 256×256 image "Lenna". Fig. 2 contains the starting approximation of the further 512×512 image ($f_1(x,y) = g(x,y)$, see (4)). Fig. 3 contains 512×512 image obtained from original one by the iterative procedure (4)-(12) (four iterations), and approximation of the Cosine spectrum using multi-valued filtering (for reduction of the rest of noise and high frequency correction). Fig. 4 contains 512×512 image obtained from original one by prediction of the Walsh (ordered by Walsh) spectrum coefficients corresponding to the highest frequencies domain on multi-valued neuron.

6 CONCLUSIONS

Both approaches of the spectra extrapolation problem presented here are better than previous solutions proposed in [4,5]. Both are more effective for extrapolation of the spectra of a size 512×512 and higher. For example, standard deviation between the original 512×512 "Lenna" and images respectively from Figures 3 and 4 is equal to 2.7 and 3.5. For extrapolation of the smaller spectra the results are successful from visual point of view, but standard deviation is much higher.

Mutual comparison of both approaches (spectra approximation by multi-valued filtering, and by prediction of the highest-frequency domain coefficients on the MVN) shows that both solutions are effective. An implementation of prediction of the spectra coefficients is simpler for 1-D signals (learning process may be organized by simpler way than for 2-D case). An approximation of the spectra via multi-valued filtering is preferable for images.

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Figure 1: Original 256×256 Lenna image

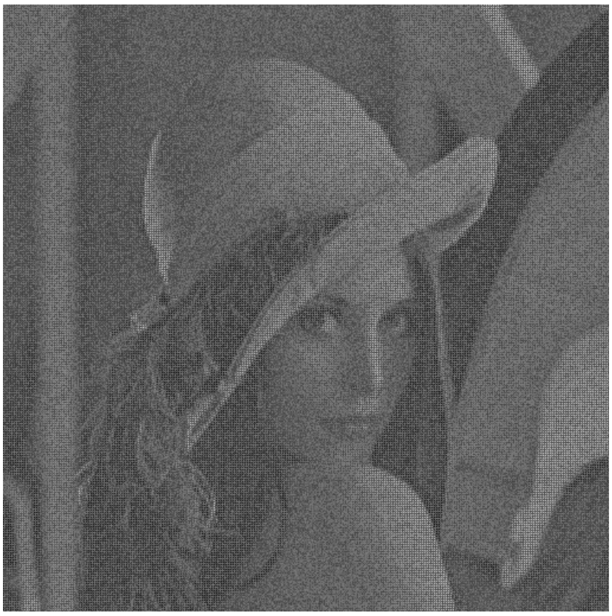

Figure 2: Starting interpolation of 512×512 image

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Figure 3: Interpolation by the cosine spectrum

Figure 4: Interpolation by the Walsh spectrum