SIMPLIFIED FLS ALGORITHM FOR LINEAR PHASE ADAPTIVE FILTERING

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ABSTRACT

The purpose of the present paper is to show that the linear phase property can be obtained with only a minor change in the regular FLS algorithm, with no additional multiplication, using the standard adaptation gain $\mathbf{g}(n)$. The proof is based on the linearly constrained filter called Generalized Sidelobe Canceller. Simulation results illustrate the efficacy of the simplified algorithm.

1. INTRODUCTION

In several applications of digital signal processing techniques, it is suitable to preserve a linear phase characteristic of FIR digital filters. This characteristic prevents phase distortions in the passband and can be obtained by constraining the *Z*-transfer function of the filter to a symmetrical or anti-symmetrical polynomial.

Channel equalization, system identification, frequency estimation and line enhancement are some of the typical applications where the linear phase property may be of great interest. In many cases, the parameters of the filter must be obtained by an adaptive procedure, in order to provide useful methods for real-time operations and non-stationary environments.

Fast Least-Square (FLS) algorithms have been proposed in order to accomplish this objective [1,2]. The computational complexity is proportional to the order of the adaptive filter, but when compared with the complexity of the classical FLS algorithm, a number of additional operations is required to update recursively a specific intermediate adaptation gain.

In this paper we show that the linear phase adaptive filter can be obtained with only a minor change in the regular FLS algorithm, using the standard adaptation gain $\mathbf{g}(n)$ without additional multiplication. The proof consists of imposing the symmetry or anti-symmetry of the filter impulse response by means of an appropriate set of linear constraints and using the GSC (Generalized Sidelobe Canceller) indirect implementation structure of a linearly-constrained filter. The effectiveness of the result is confirmed by simulation.

2. THE LINEAR PHASE CONSTRAINED FILTER

Let us consider the system identification scheme shown in Figure 1. All the parameters are supposed to be real. The linearly-constrained LMS (Least Mean-Square) problem is formulated as

$$\underset{\mathbf{h}}{\textit{minimize}} \quad E\left\{e^2(n)\right\} = E\left\{\left[d(n) - \mathbf{x}^t(n)\mathbf{h}\right]^2\right\} \qquad (1)$$

$$\textit{subject to} \quad \mathbf{C}^t\mathbf{h} = \mathbf{f}, \qquad (2)$$

where

$$\mathbf{x}(n) = \left[x(n),\ x(n-1),\ \ldots,\ x(n-N+1)\right]^t,$$

$$\mathbf{h} = \left[h_0,\ h_1,\ \ldots,\ h_{N-1}\right]^t$$

and the NxK constraint matrix $\mathbf{C}$ and the K_element response vector $\mathbf{f}$ establish the linear constraint equations.

The symmetrical (+) or anti-symmetrical (-) impulse response condition is described by

$$h_i = \pm h_{N-i-1}, \qquad (3)$$

for $i = 0, 1, \cdots, \frac{N}{2}-1$ (N even) or $i = 0, 1, \cdots, \frac{N-3}{2}$ (N odd). This condition can be easily reproduced by the constraints posing:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ \hdashline 0 & 0 & \cdots & 0 \\ \hdashline 0 & 0 & \cdots & \mp 1 \\ \vdots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots \\ 0 & \mp 1 & \cdots & 0 \\ \mp 1 & 0 & \cdots & 0 \end{bmatrix}_{N\mathrm{x}(N-1)/2} = \begin{bmatrix} \mathbf{I}_{(N-1)/2} \\ \hdashline \mathbf{0}^t \\ \hdashline \mp\mathbf{J}_{(N-1)/2} \end{bmatrix} \qquad (4)$$

for N odd or

$$\mathbf{C} = \begin{bmatrix} \mathbf{I}_{N/2} \\ \hdashline \mp\mathbf{J}_{N/2} \end{bmatrix} \qquad (5)$$

for N odd and

$$\mathbf{f} = \begin{bmatrix} 0 & \dots & 0 \end{bmatrix}^t = \mathbf{0} \tag{6}$$

in both cases. Then, on imposing $\mathbf{C}^t\mathbf{h} = \mathbf{f}$, (3) is satisfied and the linear phase characteristic is established in the minimization process. Now, the plus (+) or minus (-) sign corresponds to the anti-symmetry or symmetry condition of the coefficients, respectively.

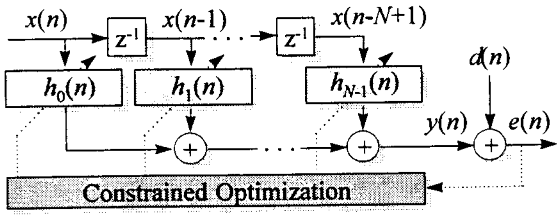

Figure 1: The constrained Wiener filter.

3. THE SIMPLIFIED FLS ALGORITHM

An alternative implementation of the linearly-constrained filtering is represented in block diagram form in Figure 2. This indirect structure is called the Generalized Sidelobe Canceller (GSC) [3,4]. Essentially, it consists of changing a constrained minimization problem into an unconstrained form.

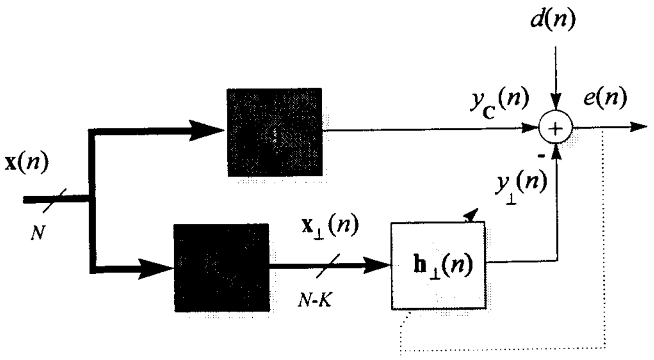

Figure 2: The GSC indirect structure.

The columns of the Nx$(N\text{-}K)$ matrix $\mathbf{C}_\perp$ represent a basis for the orthogonal complement of the space spanned by the columns of $\mathbf{C}$ ($\mathbf{C}^t\mathbf{C}_\perp = \mathbf{0}_{K\text{x}N-K}$). $\mathbf{C}_\perp$ is termed the signal blocking matrix. The $(N\text{-}K)$_element vector $\mathbf{h}_\perp$ represents an unconstrained adaptive filter and the fixed coefficients vector $\mathbf{q} = \mathbf{C}\left(\mathbf{C}^t\mathbf{C}\right)^{-1}\mathbf{f}$ a nonadaptive filter which satisfies the constraints ($\mathbf{C}^t\mathbf{q} = \mathbf{f}$).

Taking into account the linear phase constraints and a signal blocking matrix given by the projection operator $\mathbf{P} = \mathbf{I}_N - \mathbf{C}\left(\mathbf{C}^t\mathbf{C}\right)^{-1}\mathbf{C}^t$ (observe that $\mathbf{C}^t\mathbf{P} = \mathbf{0}_{K\text{x}N}$), the GSC implementation turns into the form represented by Figure 3, in that $\mathbf{f} = \mathbf{0}$.

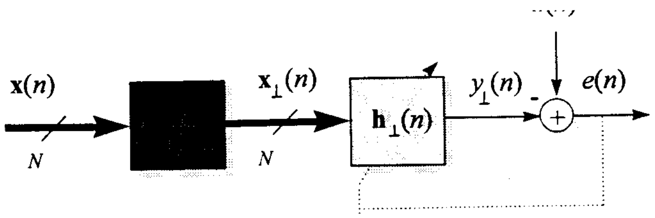

Figure 3: The GSC with linear phase constraint.

Nevertheless, by direct replacement of constraint matrix $\mathbf{C}$ the projection operator can be rewritten as

$$\mathbf{P} = \tfrac{1}{2}\left(\mathbf{I}_N \pm \mathbf{J}_N\right), \tag{7}$$

except in the anti-symmetry condition for N odd in which

$$\mathbf{P} = \tfrac{1}{2}\left(\mathbf{I}_N - \mathbf{J}_N + 2\mathbf{u}\mathbf{u}^t\right), \tag{8}$$

where

$$\mathbf{u} = \begin{bmatrix} \mathbf{0}_{(N-1)/2} \\ 1 \\ \mathbf{0}_{(N-1)/2} \end{bmatrix}.$$

In Eq. (7), the plus sign corresponds to the symmetry condition of the coefficients and the minus sign to anti-symmetry. Thus, the block diagram in Figure 3 can be represented in the simplified form illustrated in Figure 4.

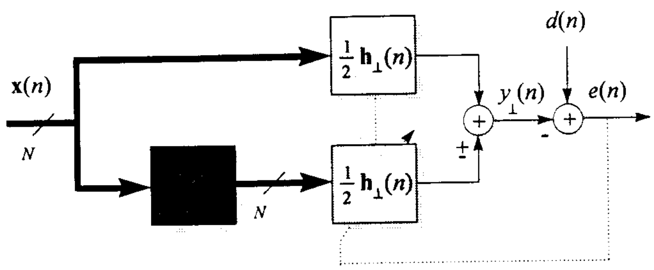

Figure 4: The GSC of figure 3 employing (7).

The error signal is given by

$$\begin{aligned} e(n) &= d(n) - \mathbf{h}_\perp^t\mathbf{P}\mathbf{x}(n) \\ &= d(n) - \tfrac{1}{2}\mathbf{h}_\perp^t\mathbf{x}(n) \mp \tfrac{1}{2}\mathbf{h}_\perp^t\mathbf{J}\mathbf{x}(n) \\ &= d(n) - \mathbf{h}_{\mathrm{LP}}^t\mathbf{x}(n), \end{aligned} \tag{9}$$

where

$$\mathbf{h}_{\mathrm{LP}} = \tfrac{1}{2}\left[\mathbf{I} \pm \mathbf{J}\right]\mathbf{h}_\perp \tag{10}$$

is the coefficient vector of the linear phase filter. Now, the LMS problem is expressed in terms of the GSC as the following unconstrained minimization problem:

$$\begin{aligned} \underset{\mathbf{h}_\perp}{minimize}\ E\left\{e^2(n)\right\} = \sigma_d^2 &+ \\ &-\tfrac{1}{2}\mathbf{p}_{\mathbf{x}d}^t\mathbf{h}_\perp \mp \tfrac{1}{2}\mathbf{p}_{\mathbf{x}d}^t\mathbf{J}\mathbf{h}_\perp + \\ &-\tfrac{1}{2}\mathbf{h}_\perp^t\mathbf{p}_{\mathbf{x}d} \mp \tfrac{1}{2}\mathbf{h}_\perp^t\mathbf{J}\mathbf{p}_{\mathbf{x}d} + \\ &+\tfrac{1}{4}\mathbf{h}_\perp^t\mathbf{R}_{\mathbf{xx}}\mathbf{h}_\perp \pm \tfrac{1}{4}\mathbf{h}_\perp^t\mathbf{R}_{\mathbf{xx}}\mathbf{J}\mathbf{h}_\perp + \\ &\pm\tfrac{1}{4}\mathbf{h}_\perp^t\mathbf{J}\mathbf{R}_{\mathbf{xx}}\mathbf{h}_\perp + \tfrac{1}{4}\mathbf{h}_\perp^t\mathbf{J}\mathbf{R}_{\mathbf{xx}}\mathbf{J}\mathbf{h}_\perp, \end{aligned} \tag{11}$$

where $\mathbf{R}_{\mathbf{xx}}$ is the input signal autocorrelation matrix and $\mathbf{p}_{xd}$ input-desired signal cross-correlation vector.

Taking the gradient of (11) with respect to $\mathbf{h}_{\perp}$ and setting it to zero yields:

$$\tfrac{1}{2}\mathbf{R}_{\mathbf{xx}}\mathbf{h}_{\perp} \pm \tfrac{1}{2}\mathbf{R}_{\mathbf{xx}}\mathbf{J}\mathbf{h}_{\perp} \pm \tfrac{1}{2}\mathbf{J}\mathbf{R}_{\mathbf{xx}}\mathbf{h}_{\perp} + \tfrac{1}{2}\mathbf{J}\mathbf{R}_{\mathbf{xx}}\mathbf{J}\mathbf{h}_{\perp} = \mathbf{p}_{xd} \pm \mathbf{J}\mathbf{p}_{xd} \quad (12)$$

Now, since $\mathbf{JJ} = \mathbf{I}$ and $\mathbf{J}\mathbf{R}_{\mathbf{xx}}\mathbf{J} = \mathbf{R}_{\mathbf{xx}}$ ($\mathbf{R}_{\mathbf{xx}}$ is a symmetric and persymmetric matrix), it follows from (12):

$$\mathbf{R}_{\mathbf{xx}}\mathbf{h}_{\perp} \pm \mathbf{R}_{\mathbf{xx}}\mathbf{J}\mathbf{h}_{\perp} = \mathbf{p}_{xd} \pm \mathbf{J}\mathbf{p}_{xd}$$

or

$$\begin{aligned}\mathbf{h}_{\perp} \pm \mathbf{J}\mathbf{h}_{\perp} &= \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{p}_{xd} \pm \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{J}\mathbf{p}_{xd} \\ &= \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{p}_{xd} \pm \mathbf{J}\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{p}_{xd}, \end{aligned} \quad (13)$$

in that $\mathbf{J}\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{J} = \mathbf{R}_{\mathbf{xx}}^{-1}$ (the inverse of a symmetric and persymmetric matrix is also symmetric and persymmetric).

A careful observation of (13) reveals that $\mathbf{h}_{\perp}$ corresponds to the LMS optimal solution without constraint, described by the normal equation [5,6]:

$$\mathbf{h}_{\perp} = \mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{p}_{xd}. \quad (14)$$

Therefore, in the adaptive context, the FLS algorithm can be employed to update the unconstrained coefficients and (10) to obtain the linear phase filter:

$$\mathbf{h}_{\mathrm{LP}}(n+1) = \tfrac{1}{2}\left[\mathbf{h}_{\perp}(n+1) \pm \mathbf{J}\mathbf{h}_{\perp}(n+1)\right], \quad (15)$$

where

$$\mathbf{h}_{\perp}(n+1) = \mathbf{h}_{\perp}(n) + \mathbf{g}(n+1)e_{\perp}(n+1), \quad (16)$$

$\mathbf{g}(n)$ is the adaptation gain defined by

$$\mathbf{g}(n) = \mathbf{R}_{\mathbf{xx}}^{-1}(n)\mathbf{x}(n) \quad (17)$$

and

$$e_{\perp}(n+1) = d(n+1) - \mathbf{h}_{\perp}^{t}(n)\mathbf{x}(n+1). \quad (18)$$

In a more appropriate form, the coefficient vector of the linear phase filter can also be recursively computed through the set of equations:

$$e(n+1) = d(n+1) - \mathbf{h}_{\mathrm{LP}}^{t}(n)\mathbf{x}(n+1); \quad (19)$$

$$\mathbf{h}_{\mathrm{LP}}(n+1) = \mathbf{h}_{\mathrm{LP}}(n) + \tfrac{1}{2}\left[\mathbf{g}(n+1) \pm \mathbf{J}\mathbf{g}(n+1)\right]e(n+1). \quad (20)$$

It is worth pointing out that the same result also applies in the anti-symmetry condition for N odd [Eq. (8)]. Finally, the simplified FLS algorithm for linear phase adaptive filtering is summarized in Table I.

In the case of linear phase prediction the appropriate structure of the prediction-error filter is presented in Figure 5, in which both forward and backward prediction have been considered. For forward prediction, the sign of $x(n)$ in the sum is positive, while that the sign of $x(n-N+1)$ depends on the symmetry (+) or anti-symmetry (-) condition. The opposite combination [$\pm x(n)$ e $+x(n-N+1)$] corresponds to backward prediction. Since the problem is handled as a linear phase identification filtering, the algorithm in Table I can be directly applied. Now, the desired signal is composed of $x(n)$ and $x(n-N+1)$.

TABLE I
Simplified FLS Algorithm for Linear Phase Adaptive Filtering

➢ ***Initialization***: $\mathbf{h}_{\mathrm{LP}}(0) = \mathbf{0}$

➢ ***Updating***:

(1) New data at time $n+1$: $x(n+1)$ and $d(n+1)$;

(2) Adaptation gain: $\mathbf{g}(n+1)$: FLS algorithm [7];

(3) A priori error signal:

$$e(n+1) = d(n+1) - \mathbf{h}_{\mathrm{LP}}^{t}(n)\mathbf{x}(n+1);$$

(4) Adaptation of $\mathbf{h}_{\mathrm{LP}}(n+1)$:

i) Symmetry condition:

$$\begin{aligned} h_i^{(\mathrm{LP})}(n+1) &= h_{N-i-1}^{(\mathrm{LP})}(n+1) \\ &= h_i^{(\mathrm{LP})}(n) + \\ &\quad + \tfrac{1}{2}\left[g_i(n+1) + g_{N-i-1}(n+1)\right]e(n+1); \end{aligned}$$

ii) Anti-symmetry condition:

$$\begin{aligned} h_i^{(\mathrm{LP})}(n+1) &= -h_{N-i-1}^{(\mathrm{LP})}(n+1) \\ &= h_i^{(\mathrm{LP})}(n) + \\ &\quad + \tfrac{1}{2}\left[g_i(n+1) - g_{N-i-1}(n+1)\right]e(n+1); \end{aligned}$$

If N is even:

$i = 0, 1, \cdots, N/2 - 1$;

Otherwise:

$i = 0, 1, \cdots, (N-3)/2$

and

$$\begin{aligned} h_{(N-1)/2}^{(\mathrm{LP})}(n+1) &= h_{(N-1)/2}^{(\mathrm{LP})}(n) + \\ &\quad + g_{(N-1)/2}(n+1)e(n+1). \end{aligned}$$

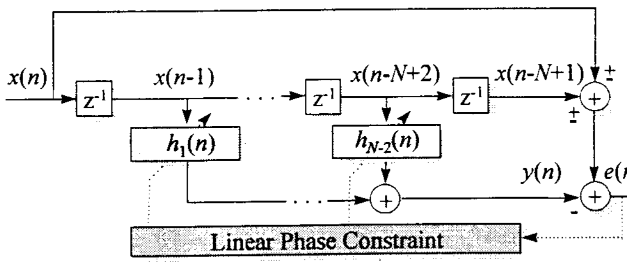

Figure 5: The linear phase prediction-error filter.

4. SIMULATION RESULTS

In order to verify the effectiveness of the simplified algorithm, let us consider the linear phase prediction of a signal composed of three sinusoids in white noise. The normalized frequencies of the sinusoids are 0.1, 0.15 and 0.4, and the sinusoid-to-noise ratio is 10 dB. A predictor of order 6 (N=7) is used with linear phase imposition by the symmetry of its coefficients. Concerning the FLS algorithm, a forgetting factor W=0.99 and an initial error energy E_0=0.1 are utilized.

Figure 6 shows the evolution of the prediction error power where we can remark the high performance of the LS technique in terms of convergence rate. The magnitude response of the prediction-error filter after convergence is presented in Figure 7. Three notches are observed near the input frequencies and the bias regarding their estimation occurs due to the low sinusoid-to-noise ratio. Finally, the phase response is plotted in Figure 8 where its linear characteristic is verified. It is also interesting to emphasize that the same results are obtained by the approach in [8] at the expense, however, of a significantly greater computational complexity.

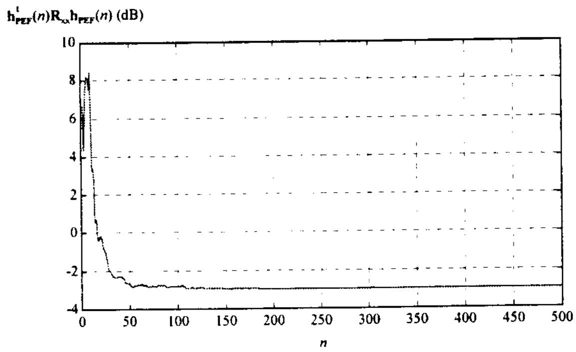

Figure 6: Evolution of prediction error power.

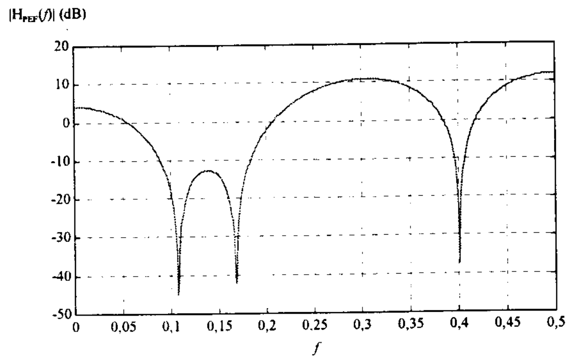

Figure 7: Magnitude response.

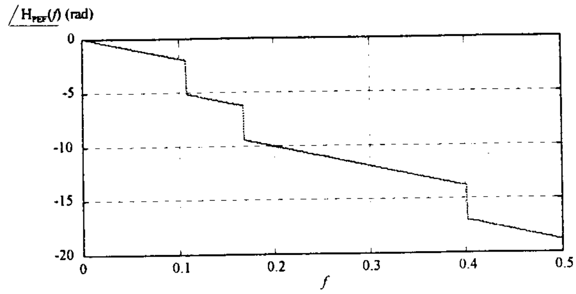

Figure 8: Phase response.

5. CONCLUSION

The fundamental aim of this work is to show that linear phase adaptive filtering can be achieved with a minor change in the classical FLS algorithm. The gain from the simplified algorithm is significant when its complexity is compared with other algorithms. It is can be used in a general adaptive context of system identification or linear prediction, where the linear phase property is desired.

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