SOURCE LOCALIZATION IN SHALLOW WATER IN THE PRESENCE OF SENSOR DEPTH UNCERTAINTY

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ABSTRACT

This paper studies passive source localization performance in shallow water using a vertical array whose sensor depths are unknown. The performance degradation with respect to the case of known sensor depths is studied via the Cramer-Rao Bound for a single far field narrow band source. Examination of the bound indicates that there is no inherent singularity in the Fisher Information Matrix due to uncertainty in the sensor depths (as opposed to the case of localization in free space). Numerical examples show that the performance degradation in source localization due to the need to estimate the sensor depths in a typical scenario is approximately 3-5dB.

1 INTRODUCTION

The application of Matched Field Processing (MFP) techniques to the problem of source localization in shallow water has received considerable attention over the past several years e.g., [1] (and references therein). While it is known that the performance of MFP based algorithms is highly sensitive to prior knowledge of sensor locations e.g., [2], little work on source localization in shallow water with sensor location uncertainties has been reported (especially compared to the analogous problem in free space propagation conditions, [3]).

This paper deals with the localization performance degradation for the special case of a vertical array whose sensor depth locations are unknown. We consider the Cramer-Rao Bound (CRB) for source range and depth and compare the achievable localization performance for known and unknown sensor depths as key scenario parameters such as signal-to-noise-ratio and number of sensors are varied.

2 PROBLEM FORMULATION

We consider a point source located in a wave-guide at depth, z_o, and range, r_o. The source is radiating a monochromatic signal at angular frequency, ω, with complex amplitude, b. The wave-guide is deterministic and time invariant. The field radiated by the point source is sampled by a vertical array of N sensors, whose depth locations, $z_i, i \in \{1, \cdots, N\}$, are unknown. The problem geometry is described in Fig. 1. If the source is located in the far field of the array, the signal measured by sensor i at time t can be expressed by e.g., [4]:

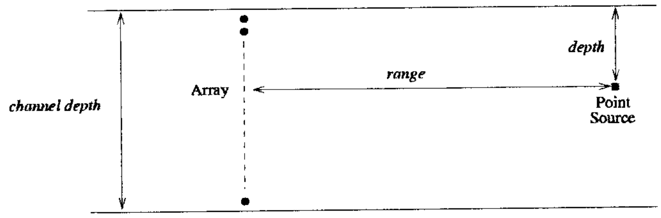

Figure 1: Problem geometry.

$$y_i(t) = b \sum_{m=1}^{M} \phi_m(z_o)\phi_m(z_i) \frac{e^{jk_m r_o}}{\sqrt{k_m r_o}} e^{j\omega t} + n_i e^{j\omega t} \qquad i \in \{1, \cdots, N\}, \quad (1)$$

where $\phi_m(\cdot)$, $m \in \{1, \cdots, M\}$, are the modal depth eigenfunctions, M is the number of propagating modes in the wave guide, and k_m and γ_m are respectively the horizontal and vertical wave numbers of the mth mode. The additive noise at the sensors is assumed to be zero-mean complex, circular Gaussian random vector with known covariance matrix, $\mathbf{\Lambda}$, where the $i-k$th element is given by:

$$\Lambda_{ik} = E\{n_i n_k^*\}, \quad (2)$$

where n_i is the additive Gaussian noise amplitude at sensor i, and $(\cdot)^*$ denotes complex conjugation.

The Fourier transform of (1) at the angular frequency ω is:

$$Y_i = b \sum_{m=1}^{M} \phi_m(z_o)\phi_m(z_i) \frac{e^{jk_m r_o}}{\sqrt{k_m r_o}} + n_i. \quad (3)$$

Letting $\boldsymbol{\alpha} \triangleq \left[r_o, z_o, \mathbf{b}^T\right]^T$, where $\mathbf{b} \triangleq [Re(b)\ Im(b)]^T$, $[\cdot]^T$ denotes the transpose and $\mathbf{z} \triangleq \left[z_1, \cdots, z_N\right]^T$, (3)

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can be written in matrix notation:

$$\mathbf{y} = [Y_1, \cdots, Y_N]^T = b\mathbf{T}(\mathbf{z})\mathbf{q}(\boldsymbol{\alpha}) + \mathbf{n}. \quad (4)$$

The elements of the matrix $\mathbf{T}(\mathbf{z})$ and the vector $\mathbf{q}(\boldsymbol{\alpha})$ are given by:

$$T_{im}(\mathbf{z}) = \phi_m(z_i) \quad (5)$$
$$q_m(\boldsymbol{\alpha}) = \phi_m(z_o)\frac{e^{jk_m r_o}}{\sqrt{k_m r_o}}. \quad (6)$$

Our objective is to examine the performance degradation in the estimation of the source location parameters ,(r_o, z_o), when the sensors depths $\{z_i\}_{i=1}^N$ are unknown compared to the case in which they are known. We assume that the vector $\mathbf{b}$ is not known while the noise covariance and all the environmental parameters characterizing the wave guide are known.

3 THE CRAMER RAO BOUND

As a lower bound on the estimation error of the unknown parameters (r_o, z_o), we use the non-Bayesian CRB. For a deterministic source signal and Gaussian noise at the sensors outputs described by (4), the elements of the Fisher Information Matrix (FIM), $\mathbf{J}$, are given by e.g., [5]:

$$J_{kl} = 2Re\left[\frac{\partial(b\mathbf{Tq})^H}{\partial\beta_k}\mathbf{\Lambda}^{-1}\frac{\partial(b\mathbf{Tq})}{\partial\beta_l}\right], \quad (7)$$

where $\boldsymbol{\beta} \triangleq \left[\boldsymbol{\alpha}^T, \mathbf{z}^T\right]^T$ is the vector of all unknown source parameters and $[\cdot]^H$ denotes the conjugate transpose operation. If the distributions of the noise at the different sensors are independent and identically distributed, namely,

$$\mathbf{\Lambda} = \sigma^2\mathbf{I}_N, \quad (8)$$

where $\mathbf{I}_N$ is the N-dimensional identity matrix, then:

$$J_{kl} = \frac{2}{\sigma^2}Re\left[\frac{\partial(b\mathbf{Tq})^H}{\partial\beta_k}\frac{\partial(b\mathbf{Tq})}{\partial\beta_l}\right]. \quad (9)$$

The CRB for the vector $\boldsymbol{\alpha}$ with known sensors location is:

$$\mathbf{CRB}_o(\boldsymbol{\alpha}) = \mathbf{J}_{\alpha\alpha}^{-1}, \quad (10)$$

The FIM for the vector $\boldsymbol{\beta}$ is:

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{\alpha\alpha} & \mathbf{J}_{\alpha z} \\ \mathbf{J}_{z\alpha} & \mathbf{J}_{zz} \end{bmatrix}, \quad (11)$$

and the CRB for the vector $\boldsymbol{\alpha}$ with unknown sensors location is:

$$\mathbf{CRB}(\boldsymbol{\alpha}) = (\mathbf{J}_{\alpha\alpha} - \Delta\mathbf{J}_{\alpha\alpha})^{-1}, \quad (12)$$
$$\Delta\mathbf{J}_{\alpha\alpha} = \mathbf{J}_{\alpha z}\mathbf{J}_{zz}^{-1}\mathbf{J}_{z\alpha}.$$

$\mathbf{J}_{\alpha\alpha}$ was computed in [4]:

$$\mathbf{J}_{\alpha\alpha} = \frac{2}{\sigma^2}Re\left[\frac{\partial(b\mathbf{Tq})^H}{\partial\boldsymbol{\alpha}}\frac{\partial(b\mathbf{Tq})}{\partial\boldsymbol{\alpha}}\right]. \quad (13)$$

$\mathbf{J}_{zz}$ is defined as:

$$\mathbf{J}_{zz} = \frac{2}{\sigma^2}Re\left[\frac{\partial(b\mathbf{Tq})^H}{\partial\mathbf{z}}\frac{\partial(b\mathbf{Tq})}{\partial\mathbf{z}}\right]. \quad (14)$$

Since the uncertainties in the vector $\mathbf{z}$ are independent of each other, it is straightforward to show that:

$$\frac{\partial(b\mathbf{Tq})^H}{\partial\mathbf{z}} = diag\left[\varepsilon_1, \cdots, \varepsilon_N\right], \quad (15)$$
$$\varepsilon_i = b\boldsymbol{\phi}'^T(z_i)\mathbf{q}, \qquad \phi_m'(z) = \frac{\partial\phi_m(z)}{\partial z},$$
$$\boldsymbol{\phi}'(z_i) = [\phi_1'(z_i), \cdots, \phi_M'(z_i)]^T,$$

where $diag[\cdot]$ denotes "the diagonal matrix formed by the elements of,".
Substitution of (15) in (14) yields:

$$\mathbf{J}_{zz} = \frac{2}{\sigma^2}diag\left[|\varepsilon_1|^2, \cdots, |\varepsilon_N|^2\right]. \quad (16)$$

$\mathbf{J}_{\alpha z}$ is defined as:

$$\mathbf{J}_{\alpha z} = \mathbf{J}_{z\alpha}^T = \frac{2}{\sigma^2}Re\left[\frac{\partial(b\mathbf{Tq})^H}{\partial\boldsymbol{\alpha}}\frac{\partial(b\mathbf{Tq})}{\partial\mathbf{z}}\right]. \quad (17)$$

Substituting (15) in (17) yields:

$$\mathbf{J}_{\alpha z} = \mathbf{J}_{z\alpha}^T = \frac{2}{\sigma^2}Re\left[\frac{\partial(b\mathbf{Tq})^H}{\partial\boldsymbol{\alpha}}diag\left[\varepsilon_1, \cdots, \varepsilon_N\right]\right]. \quad (18)$$

Substituting (13) (16) and (18) in (12) and using the equality:

$$Im\{\mathbf{A}\}Im\{\mathbf{A}^T\} = Re\{\mathbf{A}\mathbf{A}^H\} - Re\{\mathbf{A}\}Re\{\mathbf{A}^T\}, \quad (19)$$

yields the following expression for the CRB:

$$\mathbf{CRB}(\boldsymbol{\alpha}) = \frac{\sigma^2}{2}\left[Im\{\mathbf{\Psi}\}Im\{\mathbf{\Psi}^T\}\right]^{-1}, \quad (20)$$
$$\mathbf{\Psi} = \frac{\partial(b\mathbf{Tq})^H}{\partial\boldsymbol{\alpha}}diag\left[\frac{\varepsilon_1}{|\varepsilon_1|}, \cdots, \frac{\varepsilon_N}{|\varepsilon_N|}\right].$$

This CRB matrix is positive definite, thus estimation of the source parameters, $\boldsymbol{\alpha}$, with sensor depth uncertainties is possible, unlike the case of source localization with sensors location uncertainties in free space (see [3]).

4 NUMERICAL EXAMPLES

In this section the achievable localization performance as predicted by the CRB, is calculated for the two cases in which the sensor depth locations are known and unknown. Then we consider the performance degradation of the latter with respect to the former.

The scenario is based on an ideal homogeneous wave guide such as that considered in [4]. The field was generated by a point source located at a depth $z_o = 40m$ below the upper surface of a homogeneous wave guide with depth, $D = 100m$. The source radiates a monochromatic signal at frequency, $f = 100Hz$. The propagation velocity in the wave guide is $c = 1500m/s$. The eigenfunctions are: $\phi_m(z) = \sqrt{2/D}\sin\gamma_m z$. The generated field is sampled by a vertical array whose origin is located $10km$ from the source. The vertical array consists of 30 sensors with $2m$ spacing between them. The center of the array is located $50m$ below the upper surface. The additive noise at the array sensors is zero mean Gaussian. The signal-to-noise-ratio (SNR) was taken to be the average SNR per sensor over the entire vertical array:

$$SNR = \frac{\mathbf{q}^H\mathbf{T}^H|b|^2\mathbf{T}\mathbf{q}}{N\sigma^2}. \tag{21}$$

In Fig. 2, the standard deviation of the source range and depth parameter estimates as predicted by the CRB are plotted as a function of SNR with and without sensor depth uncertainty (i.e., with and without the need to estimate the sensor depths). It can be seen that the relative degradation in range and depth estimation performance with respect to the case of known sensor depths, is constant and does not depend on SNR. The degradation in range, r_o, estimation is $3.06dB$ while the degradation in depth, z_o, estimation is $3.94dB$. The graphs indicate that asymptotically as the SNR tends to infinity, estimation errors associated with the source location parameters tend to zero both with and without a-priori knowledge of the sensor depths.

In Fig. 3, the standard deviation of the source range and depth parameter estimates as predicted by the CRB are plotted as a function of the number of sensors in the array with and without sensor depth uncertainty. The source complex amplitude, b is chosen such that the SNR is $10dB$ at the nominal array geometry (i.e, 30 sensors). The graphs indicate that the CRB with sensor depth uncertainity does not increase as the number of sensors increases, although the number of unknown parameters does increase. This result is consistent with the results obtained in [6].

Next, Fig. 4 shows the source location CRB's with and without knowledge of the sensor depths for 1000 random perturbations of the nominal array geometry (described at the beginning of this section). The sensor depth perturbations are Gaussian with zero mean and standard deviation of $1m$. The SNR is $10dB$. The lower left cluster of points correspond to the case of known sensor depths while the more diffuse collection in the center corresponds to the case of unknown sensor depths. The mean and standard deviation of the source range standard deviations with known sensor locations are respectively calculated to be: $0.47m$ and $0.02m$. The mean and standard deviation of the source depth standard deviation with known sensor locations are respectively: $0.11m$ and $0.01m$. For unknown sensor depths, the mean and standard deviation of the range standard deviations are $0.75m$ and $0.05m$ respectively. For source depth, the mean and standard deviation of the depth standard deviations are respectively $0.20m$ and $0.01m$ again for unknown sensor depth case. Thus, the average range and depth standard deviations are respectively $4.06dB$ and $5.19dB$ higher in the case of unknown sensor depths.

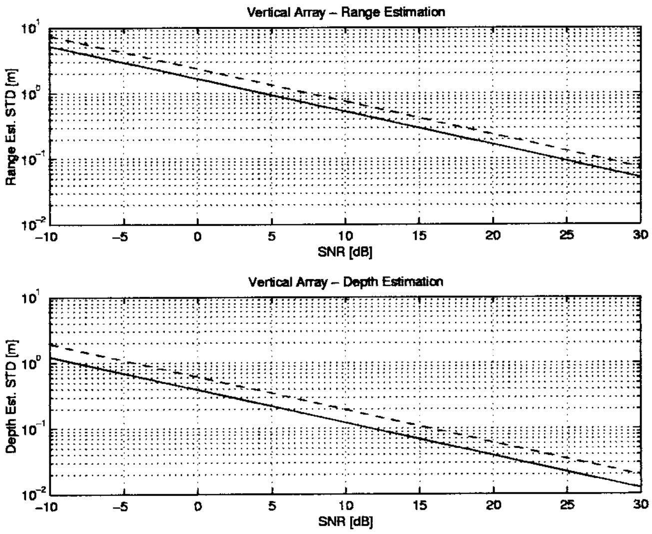

Figure 2: Source location parameter standard deviations as a function of SNR. (–) CRB with known sensor depths, (- -) CRB with unknown sensor depths.

5 CONCLUSIONS

This study has shown that using an array of sensors which is known to be vertical but whose depths are unknown, one can estimate the depth and the range of a narrow band source in a known shallow water channel. The incremental estimation error relative to the case where the sensor locations are known is SNR independent and in a typical example, is about 3-5dB. Note that if one assumes a certain array geometry while, due to uncertainties, the actual array geometry is different, the incremental estimation error can be much higher (e.g., [2]). Therefore, it has been proposed to measure the actual depth of each sensor via hardware, i.e., an array navigation system [7]. Our study suggests that estimating the sensor locations directly from sensors (instead of measuring them with an array navigation system) may cost by only about 3-5dB in source localization performance.

The main results of this work are:

1. Source localization in shallow water with a vertical array can be achieved even when the sensor depths are not known, as opposed to the free space

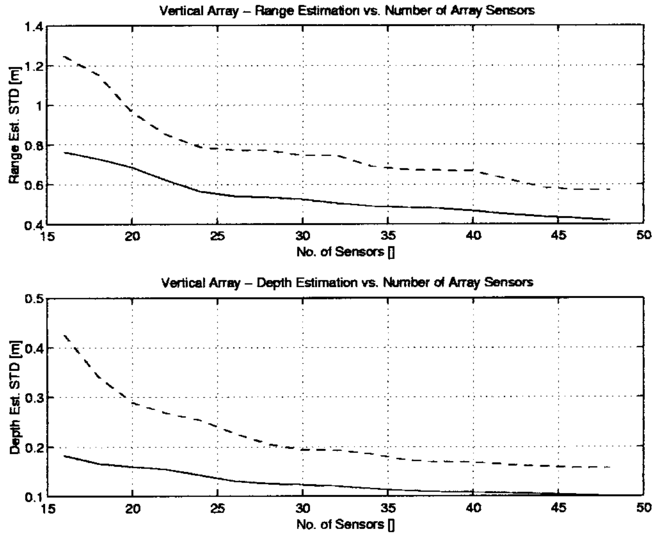

Figure 3: Source location parameter standard deviations as a function of number of sensors. (–) CRB with known sensor depths, (- -) CRB with unknown sensor depths

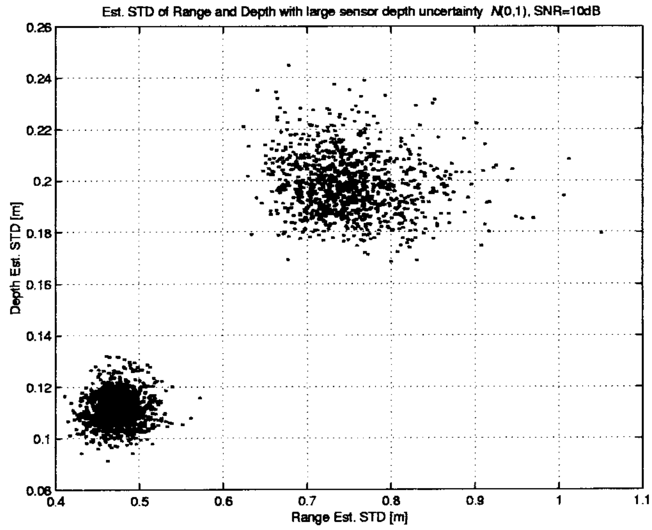

Figure 4: Source location parameter standard deviations with and without known sensor depths

case where an inherent singularity in the problem exists such that source localization with a linear array subject to uncertainties is impossible. This difference is due to the fact that the shallow water wave guide is vertically bounded. The resulting highly structured multipath enables joint estimation of source location and vertical sensor displacement.

2. For the scenarios considered, the degradation in the range and depth estimation performance with respect to the case of known sensor depths was observed to be approximately 3-5dB.

3. Increasing the number of sensors in a vertical array cannot increase the source range and depth estimation error, even though the number of unknown parameters needed to be estimated increases.

While the case of a strictly vertical array whose sensor depths are unknown is contrived since in general there will be sensor uncertainity in three dimensions, it has been shown that passive localization of a point source is feasible due to the fact that the channel is bounded in the vertical direction. While such boundedness is not present in the horizontal direction, future work will show that for piecewise linear sensor perturbation model, full 3D sensor position estimation is possible.

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