A NOVEL ALGORITHM FOR DIGITAL IMAGE PROCESSING APPLICATIONS

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ABSTRACT.

This paper introduces an efficient algorithm for the calculation of 2-D convolution and correlation for image processing applications. The method combines the new 2-D Mersenne with the 2-D Fermat numbers transforms using the 2-D mixed radix conversion. The moduli of the transforms are selected to be close to one another, thus, the choice of the dynamic range and the constraint between the transform sizes and the world length becomes more flexible. The residue transforms are independent and can be calculated in parallel for high speed and high throughput. Hence the technique is suitable for image processing applications such as error free image filtering and enhancement.

I INTRODUCTION.

There is an ever-increasing interest in the use and the performance of discrete transforms in image processing applications[3]. This has revealed their ability to reduce considerably the number of arithmetic operations in the calculation of convolution/correlation, resulting in faster calculation.

Both convolution and correlation are fundamental for many image processing applications such as image filtering, enhancement and recognition etc.[4]

The aim of this paper is to introduce a novel algorithm for the calculation of 2-D convolution/correlation for digital image processing applications. The technique combines a recently developed 2-D transform based on Mersenne numbers(2-D NMNT) [2] with the 2-D Fermat number transform (2-D FNT)[6] using the 2-D mixed radix conversion (2-D MRC). The resulting combination uses fast 2-D residue transforms with small word lengths. These transforms are suitable for fast algorithms, have the 2-D Cyclic convolution property (2-D CCP) and can be implemented in parallel for high speed and high throughput rate. Hence the technique could be applied to the calculation of 2-D convolution/correlation for image processing applications without rounding or truncation errors.

1.1 Definition of the 2-D new Mersenne transform

The two dimensional new Mersenne number transform (2-D NMNT) is given below [2] :

$$X(k_1,k_2)=\left\langle (1/N)\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}x(n_1,n_2)\beta(n_1k_1,n_2k_2)\right\rangle_{M_p}$$

$$k_i=0,1,2,\ldots,N-1,\ i=1,2. \qquad (1)$$

and its inverse is exactly the same

$$x(n_1,n_2)=\left\langle (1/N)\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}X(k_1,k_2)\beta(n_1k_1,n_2k_2)\right\rangle_{M_p}$$

$$n_i=0,1,2,\ldots,N-1,\ i=1,2 \qquad (2)$$

The kernel $\beta(n_1k_1,n_2k_2)$ is as defined in [2]

1.2 Definition of the Fermat number transform.

The well known two dimensional Fermat number transform (2-D FNT) is defined as[6]:

$$X(k_1,k_2)=\left\langle \sum_{n_1=0}^{N-1}\ \sum_{n_2=0}^{N-1} x(n_1,n_{2,})\,\alpha_1^{n_1k_1}\,\alpha_1^{n_2k_{21}} \right\rangle_{F_t}$$

$$k_i = 0,1,2,\ldots,N-1, i=1,2. \qquad (3)$$

and its inverse (2-D IFNT) is given by:

$$x(n_1,n_2)=\left\langle (1/N^2)\sum_{k_1=0}^{N-1}\ \sum_{k_2=0}^{N-1} X(n_1,n_{2,})\,\alpha_1^{-n_1k_1}\,\alpha_1^{-n_2k_{21}} \right\rangle_{F_t}$$

$$n_i = 0,1,2,\ldots,N-1, i=1,2. \qquad (4)$$

where $\langle\ \rangle_{F_t}$ means mod $F_t = 2^{2^t}+1$, t=0,1,2,…; F_t is the tth Fermat number, N is power of two, α_1 and α_2 are Nth roots of unity (α_1^N , $\alpha_2^N = 1$ mod F_t and α_1^i , $\alpha_2^i \neq 1$, $1 \leq i < N$)

2 CALCULATION OF THE CYCLIC CONVOLUTION USING THIS METHOD

Both transforms defined in Eqs.1 and 3 have transform sizes which are a powers of two and also have the (2-D CCP) [2,6]. Hence they can be combined for greater dynamic range and parallelism. Therefore, the 2-D convolution modulo $M=m_1 x m_2 x\ldots x m_k$ can be calculated by first evaluating the 2-D residue convolutions modulo each m_i in parallel. Then using the 2-D MRC, the over all convolution y(n,m) is reconstructed, as shown in figure 1 for two moduli. In our case the m_i 's are the popular Fermat and Mersenne numbers where arithmetic operations are known to be much simpler than other moduli [1,2,5-7].

2.1 *Two dimensional mixed radix conversion (2-D MRC)*

The tow dimensional mixed radix conversion of $y(n,m)$ y is given by $z_L(n,m), z_{L-1}(n,m),\ldots, z_1(n,m)$ where $z_i(n,m)$'s and $y(n,m)$ are related by eq. 5

$$y(n,m)=z_L(n,m)\prod_{i=1}^{L-1} m_i + z_{L-1}(n,m)\prod_{i=1}^{L-2} m_i + \ldots + z_3(n,m)m_1m_2 + z_2(n,m) + z_1(n,m) \qquad (5)$$

In the 2-D mixed radix conversion, the $z_i(n,m)$'s are generated from the residue digits as follows

$$z_1(n,m)=\langle y(n,m)\rangle_{m_1} = y_1(n,m) \qquad (6)$$

$$z_2(n,m)=\left\langle m_1^{-1}(y_2(n,m)-y_1(n,m))\right\rangle_{m_2} \qquad (7)$$

$$z_3(n,m)=\left\langle m_2^{-1}(y_3(n,m)-y_1(n,m)-y_2(n,m))\right\rangle_{m_3} \qquad (8)$$

From these equations, it is clear that arithmetic operations are carried out in smaller moduli m_i's which are in our case the popular Fermat and Mersenne numbers.

3 CONCLUSION

In this paper, the 2-D new Mersenne and the 2-D Fermat numbers transforms are combined using the 2-D mixed radix conversion leading to an efficient method for the calculation of 2-D convolution/correlation for digital image processing purposes.

The advantage of this method is that the 2-D convolution andcorrelation are calculated modulo a large integer number (M) which provides sufficient dynamic range but arithmetic operations are carried out modulo smaller Fermat and Mersenne numbers which are known to lead to much simpler arithmetic than other moduli. Also the use of small fast transforms in parallel leads to high speed and high throughput rate.

4 REFERENCES

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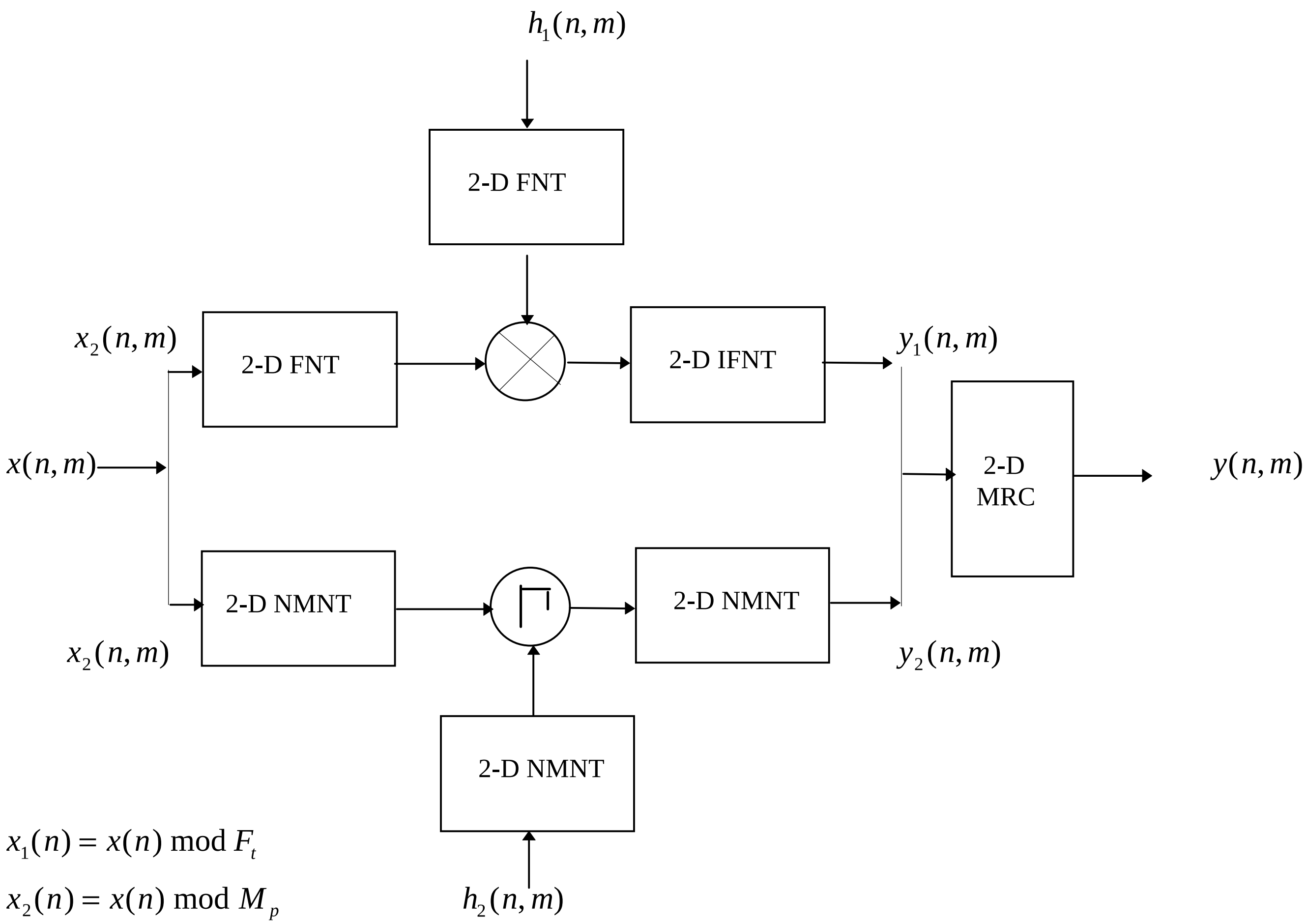

$$x_1(n) = x(n) \bmod F_t$$
$$x_2(n) = x(n) \bmod M_p$$

Figure 1 The calculation of 2-D convolution using this method