TRACKING OF SPECTRAL LINES IN AN ARCAP TIME-FREQUENCY REPRESENTATION

M. Davy, B. Leprettre**, C. Doncarli*, N. Martin***
*IRCyN UMR 6597 BP 92101, 1, rue de la Noë 44321 NANTES Cedex 03, FRANCE
**LIS BP 46, 38402 Saint MARTIN d'HERES, FRANCE
e-mail : davy@lan.ec-nantes.fr

ABSTRACT

ARCAP time-frequency representations of narrow-band signals are made of instantaneous characteristics (frequencies and amplitudes), without any time links. In order to extract the frequency modulations (or sprectral trajectories), we propose to re-create them on the basis of ballistic integrator models. The analytic expression of the corresponding asymptotic Kalman filter gains allows a very simple implementation of association procedures including trajectory birth or death. The points being associated, a Fraser filtering leads to the smoothed spectral trajectories.

1 INTRODUCTION

Our purpose is to track spectral lines directly from the time-frequency plane. The proposed method applies to non-stationary signals consisting of narrow spectral bands signals embedded in additive noise, with unknown signal-to-noise ratio. In that context, recognizing and tracking spectral lines may be difficult and often needs a good knowledge of the signal generation process.

We put forward a method that manages with these constraints. From a gliding ARCAP time-frequency representation (TFR), the method tracks each frequency component as a function of time. A frequency component is defined as a narrow spectral band such that its projection in the time frequency plane represents an instantaneous frequency characterized by its frequency modulation law. In the parametric case, the poles of the model lead directly to the projection in the plane. On the other hand, due to the gliding estimator, the links between the estimated poles at different times are entirely lost. These time links, essential for the extraction of each frequency modulation, are re-created from a model.

A continuous model of a ballistic trajectory in a plane has been studied in a previous work [1]. In this paper, we propose a discrete model - a m-order integrator - more suited to the discrete character of the TFR. The trajectories are estimated by a recursive algorithm, the Kalman filtering. Compared to the previous work [1], the association rules are improved by a non-hierarchic process. In a last step, an adapted trajectory smoothing based on a Fraser filtering is proposed. Finally some results on simulated signals and concluding comments are given.

2 ARCAP TIME - FREQUENCY REPRESENTATION

The ARCAP estimator is efficient for narrow spectral bands signals estimation. It mixes the good qualities of two different estimators and requires two steps [2] :
First step : The frequencies are estimated on a gliding window, by the phasis of AR model poles.
Second step : For each estimated frequency, a Capon filter is computed in order to estimate the power.
The AR estimator is efficient in the frequency estimation, but poor in the amplitude estimation. Contrarily, the Capon estimator is a good power estimator. The hybrid estimator, called ARCAP is then efficient in both frequency and power estimation, and has proven its interest in applications such as mechanics [2] and seismology [5].

3 SPECTRAL TRAJECTORIES MAKE UP

Suppose that spectral trajectories are found in the ARCAP representation from time 0 to time k. Each spectral trajectory is a set of associated points, and the problem is to associate it with a next point (at time $k+1$). Considering a single trajectory, a prediction of its next point with a likelihood domain would enable the use of association rules. These considerations have led us to use Kalman filtering (KF), the implementation of which requires the definition of a model.

3.1 Choice of a model

The frequency modulations, or spectral trajectories, are supposed ballistic ones, either in the time-frequency and time-power planes. Instead of considering a continuous model, we have straight-away directed towards a discrete-time m-order integrator model (see Figure 1).

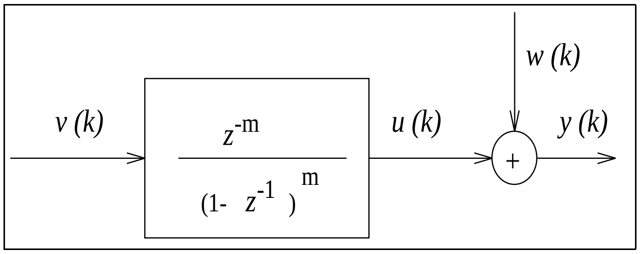

Figure 1: Trajectories model. z denotes the shift operator, v and w are independant white sequences of unknown variances q and r.

The state model writes :

$$\begin{cases} \underline{X}(k+1) & = & \underline{\underline{A}}\,\underline{X}(k) + \underline{B}\,v(k) \\ y(k) & = & \underline{C}\,\underline{X}(k) + w(k) \end{cases}$$

where $\underline{X}$ is the state vector, composed of the outputs of the m cascaded elementary integrators. Matrix $\underline{\underline{A}}$ and vectors $\underline{B}$ and $\underline{C}$ write :

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}_{m\times m} ,\underline{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_m$$

$$\underline{C} = [1\ 0\ \cdots\ 0]_m$$

3.2 Variances calculation

The use of a KF needs the knowledge of the signal model. The proposed one depends only on the values of the variances q and r, which can be estimated as summed up in Figure 2 [3] : as $y(k)$ is the sum of two linear filterings of $v(k)$ and $w(k)$, the variances of $n_1(k)$ and $n_2(k)$ are linear combinations of q and r. Choosing $T1 \gg T2$ avoids the linear system resolution, because q and r are directly given by each variance ergodic estimator (with an easily calculable muliplicative factor).

3.3 Optimal and Asymptotic Kalman Filter (AKF)

If q and r are constant, the optimal (non-stationary) KF converges towards asymptotic values. Usually, the Riccati equation has to be solved in a recursive numeric way. A very interesting property of the integrator model is that the asymptotic value of KF gain ($\underline{K}_\infty$) can be found analytically [4], and is a function of the only parameter q/r. Furthermore, an approximation of $\underline{K}_\infty$ writes :

$$\underline{K}_\infty \simeq [\mu\, t_1\ \mu^2\, t_2\ \ldots\ \mu^m\, t_m] \qquad (1)$$

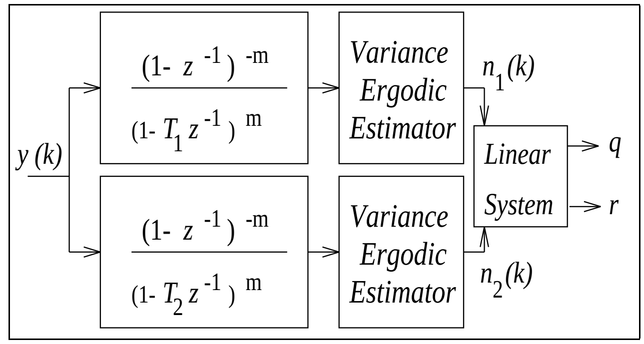

Figure 2: Variances estimation principle.

where :

$$\begin{cases} t_0 = 1 \\ t_i = \frac{1}{j} \sum_{j=1}^{i} \frac{\sin(j\ \pi\,/\,2)}{\sin(j\ \pi\,/\,2m)}\, t_{i-j}\ ,\ i \in [1\,,\,\ldots\,,\,m] \\ \mu = \sqrt[2m]{\frac{q}{r}} \end{cases}$$

Experiments have shown that the optimal KF gain and $\underline{K}_\infty$ lead to the same association results. Because of its simplicity, the latter is prefered.

3.4 Points association

As explained in part 2, ARCAP analysis gives a 3D representation of signals. A set of p points corresponds to each sampling time k. All these points are parts of a trajectory. Making up points association consists in finding what point at time $k+1$ is linked to a trajectory and detecting trajectory birth or death. In order to find the next point of a trajectory, a prediction is done in each plane (time-frequency and time-power planes) through two asymptotic Kalman filters (see §3.3). Then, we have a set of p predictions in the time-frequency-power space coming with their likelihood domains. In that space, time k, frequency f, and power $\mathcal{P}$ characterize points. Let $\alpha_{\text{estimated}}(k, f_{\text{estimated}}, \mathcal{P}_{\text{estimated}})$ and $\alpha_{\text{measured}}(k, f_{\text{measured}}, \mathcal{P}_{\text{measured}})$ be respectively an estimated point and a measured point. Let us define :

$$\begin{cases} U_f & = & f_{\text{estimated}} - f_{\text{measured}} \\ U_{\mathcal{P}} & = & \mathcal{P}_{\text{estimated}} - \mathcal{P}_{\text{measured}} \end{cases}$$

The Euclidean distance writes :

$$d = \sqrt{U_f^2 + U_{\mathcal{P}}^2}$$

We then face several cases :

1. If the likelihood domains do not overlap :
    - one measure falls in the likelihood domains : the trajectory goes on associating this measure,

- no measure falls in the likelihood domains : the trajectory dies,
- two or more measures fall in the likelihood domains : one trajectory or more starts.

2. If the likelihood domains overlap : The optimal association (minimizing the sum of Euclidean distances and the number of death or birth) is computed.

4 SMOOTHING

The rough associated measures define trajectories, which can be smoothed using a forward-backward procedure (Fraser smoothing) based on KF.

4.1 About reversibility of discrete-time m-order integrator model

The discrete-time m-order integrator model has a reversibility property. Let v_{reverse} represent the white sequence v read in the reverse sense (that is to say with decreasing k). Then one filters v_{reverse} by the integrator model (the same way as Figure 1). The output becomes u_{reverse}, which is the sequence u read in the reverse sense. Finally backward filtering is made of the same algorithm, with state $\underline{X}_{\text{backward}}$, and vector $\underline{C}$ is replaced by

$$\underline{C} = (-1)^m \left[C_{m-1}^0 \; C_{m-1}^1 \; \ldots \; C_{m-1}^{m-1}\right]$$

$\underline{X}_{\text{backward}}$ initial value is $\underline{X}_{\text{forward}}$ final value.

4.2 Smoothing

A forward-backward AKF is proceeded. Finally each smoothed trajectory is computed as the following weighted average (optimal Bayesian estimation) :

$$T_{\text{final}} = \frac{\Sigma_{\text{forward}}^{-1}(k) T_{\text{forward}}(k) + \Sigma_{\text{backward}}^{-1}(k) T_{\text{backward}}(k)}{\Sigma_{\text{forward}}^{-1}(k) + \Sigma_{\text{backward}}^{-1}(k)}$$

where $\Sigma_{\text{forward}}(k)$ is variance at time k (provided by AKF) of the current point of T_{forward} (respectively backward).

5 RESULTS

We apply these KF techniques on the ARCAP TFRs of synthetic signals and evaluate their respective interest for spectral line extraction. An example of this study is represented on Figure 3 (ARCAP analysis) and Figure 4 (tracking of spectral trajectories) . The simulated signal consists of four spectral alternating components at 0.2, 0.3, 0.25 and 0.2 Hz (normalized frequencies by the sampling frequency), switching at times 180, 210 and 240. An ARCAP TFR of this signal has been computed (see Figure 3). After AKF processing, four 1D spectral lines have been extracted from the TFR (see Figure 4). The onset moment of each curve has then been calculated to estimate the moments when the abrupt changes in the spectral content appear (see Table 1). Others parameters (for example mean frequency, length, tendency) can be derived from these 1D spectral lines for detection and/or classification purposes.

Trajectory	Exact Switching Times Begin	Exact Switching Times End	Estimated Switching Times Begin	Estimated Switching Times End
No. 1	0	180	16	179
No. 2	181	210	181	211
No. 3	211	240	210	242
No. 4	241	499	240	497

Table 1: Switching times

6 CONCLUSION

Estimation of instantaneous frequencies and amplitudes lines of narrow-band signals can be performed from ARCAP representations. The Kalman filters derived from ballistic integrator models and the association procedure are very simple and depend on only two easily tuned parameters. Furthermore the detection of birth or death of spectral lines allows the recognition of abrupt changes.

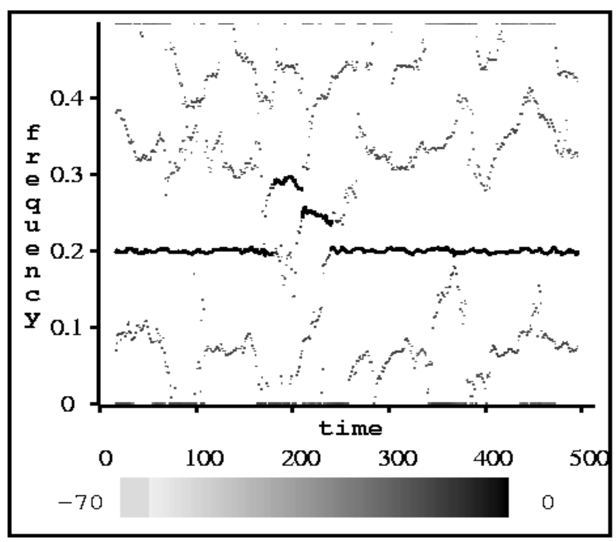

Figure 3: ARCAP Analysis - orders 8/15 - Window 31 - Gliding step 1

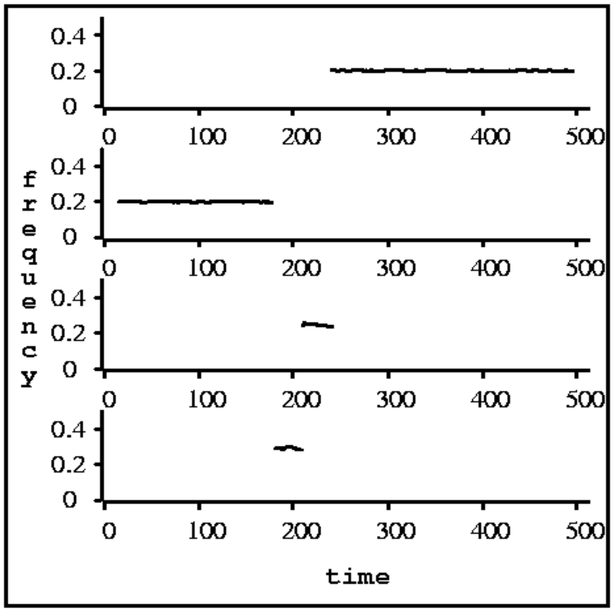

Figure 4: Extraction of spectral lines.

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