RLS-BASED INITIALIZATION FOR PER TONE EQUALIZERS IN DMT-RECEIVERS

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ABSTRACT

Per tone equalization has been proposed in [6] [7] as an alternative to time domain equalization for DMT-based systems. In this paper, an iterative initialization scheme based on so-called RLS with inverse updating is presented for these equalizers. Simulation results show convergence with an acceptably small number of training symbols. Complexity calculations are made for per tone equalization and for the case where tones are grouped. It is demonstrated with an example that in the latter case, initialization complexity becomes sufficiently low and comparable to complexity during data transmission.

1 INTRODUCTION

Discrete Multitone (DMT) has become an important transmission method, for instance for Asymmetric Digital Subscriber Line (ADSL) [1]. A conventional equalization scheme for such a DMT-modem consists firstly of a time domain equalizer (TEQ) which shortens the channel impulse response such that the total impulse response is shorter than the cyclic prefix, and secondly of a 1-taps frequency domain equalizer for each tone [3] [5] [9].

As an alternative to time domain equalization, *per tone equalization* has been proposed in [6] [7]. There is no TEQ involved anymore in the system, but a T-taps per tone equalizer (PTEQ) is inserted for each tone separately. This scheme enables us to do true SNR-optimization per tone, in contrast with the TEQ-based scheme, while complexity during data transmission is kept at the same level. Moreover, the per tone equalization has been shown to have a reduced sensitivity to the synchronization delay.

An initialization formula has been derived in [6] which is applicable when a channel model is available, as well as signal and noise covariance matrices. This direct initialization results in a high computational cost. Hence, there is need of a cheaper initialization algorithm.

In this paper, an iterative initialization scheme based on so-called *Recursive Least Squares (RLS) with inverse updating* is presented. It is demonstrated that this scheme achieves initialization with an acceptably small number of training symbols.

The paper is organized as follows. Section 2 gives a short overview of per tone equalization. In section 3 RLS with inverse updating is explained and adapted to PTEQ. Complexity calculations are presented in section 4. Section 5 shows simulations results. Finally conclusions are drawn in section 6.

2 PER TONE EQUALIZATION IN A DMT-MODEM

The concepts of per tone equalization are stated shortly. For more details, we refer to [6] [7].

The per tone approach is based on transferring the TEQ-operations to the frequency domain (i.e. after the FFT-demodulation) which results in a T-taps PTEQ for each tone separately. At first sight, multiple FFT's then seem to be needed per symbol. However, it is demonstrated in [6] that, for every T-taps PTEQ , there exists a modified T-taps PTEQ which has as its inputs the corresponding output of only one FFT and $T-1$ real difference terms. The modified equalizers then have 2 functions: they equalize the channel impulse response and at the same time incorporate the sliding Fourier transform computations, whereas the original equalizers only equalize the channel impulse response. The optimal modified equalizers, i.e. which give rise to maximum SNR on each tone i, are found by minimizing the cost function:

$$\min_{\mathbf{v}_i} J(\mathbf{v}_i) = \min_{\mathbf{v}_i} \mathcal{E}\left\{\left|\bar{\mathbf{v}}_i^T \cdot \underbrace{\left[\begin{array}{c|c|c} \mathbf{I}_{T-1} & \mathbf{O} & -\mathbf{I}_{T-1} \\ \hline \mathbf{O} & \multicolumn{2}{c}{\mathcal{F}_N(i,:)} \end{array}\right]}_{\mathbf{F}_i} \cdot \mathbf{y}^{(k)} - X_i^{(k)}\right|^2\right\} \quad (1)$$

where $\bar{\mathbf{v}}_i$ denotes the coefficient vector $\mathbf{v}_i$ (modified PTEQ for tone i) with its elements in reverse order, $\mathcal{F}_N(i,:)$ the ith row of the FFT-matrix $\mathcal{F}_N$, $\mathbf{y}^{(k)}$ a vector of received samples at symbol period k and $X_i^{(k)}$ the subsymbol on tone i at time k.

3 RLS-BASED INITIALIZATION

Direct equalizer coefficient computation, based on the knowledge of the channel impulse response as well as the signal and noise characteristics, has an excessively high computational cost. However formula (1) may also be used for a training sequence based initialization of the per tone equalizers. If a training sequence $X_{1:N}^{(k)}$, $k = 1 \ldots K$, is transmitted, the optimal $\mathbf{v}_i$ is computed as (compare to formula (1))

$$\min_{\mathbf{v}_i} J(\mathbf{v}_i) = \min_{\mathbf{v}_i} \sum_{k=1}^{K} \left| \bar{\mathbf{v}}_i^{\mathrm{T}} \cdot \mathbf{z}_i^{(k)} - X_i^{(k)} \right|^2 \qquad (2)$$

with $\mathbf{z}_i^{(k)} = \mathbf{F}_i \cdot \mathbf{y}^{(k)}$. This least squares estimation problem may be solved recursively.

Iterative initialization, based on a simple LMS-scheme [2] for each tone, has a low computational complexity. Unfortunately *LMS-based schemes* for the per tone equalizers have poor convergence properties and have indeed been shown to require an excessively large number of training symbols.

Here an iterative initialization, based on a *Recursive Least Squares (RLS)-scheme* [2] for each tone, is presented. These schemes have optimal convergence properties and hence achieve initialization with an acceptably small number of training symbols. More specifically, we use RLS with so-called inverse updating [4]. This scheme is based on storing and updating a lower triangular matrix $\mathbf{L}_i^{(k)}$ which is such that $\mathbf{L}_i^{(k)\mathrm{H}} \cdot \mathbf{L}_i^{(k)} = \mathbb{X}_i^{(k)^{-1}}$ for all k, with $\mathbb{X}_i^{(k)}$ the covariance matrix of the filter input: $\mathbb{X}_i^{(k)} = \sum_{j=1}^{k} \mathbf{z}_i^{(j)*} \cdot \mathbf{z}_i^{(j)\mathrm{T}}$, together with the least squares estimate $\bar{\mathbf{v}}_i^{(k)}$ at iteration k. The following formulas, with the tone index i omitted for compact notation, describe the RLS algorithm:

Algorithm Inverse Updating

For $k = 1 \ldots K$

Given: $\mathbf{L}(k-1)$, new observation vector $\mathbf{z}^{(k)}$, desired response $X^{(k)}$

Step 1. Form the matrix-vector product

$$\mathbf{a} = -\mathbf{L}^{(k-1)} \cdot \mathbf{z}^{(k)^*}$$

Step 2. For $j = 1, \ldots, T$ determine unitary transformations $\mathbf{Q}_j$ so that

$$\left[\frac{\mathbf{0}}{\delta^*}\right] \Leftarrow \mathbf{Q}_T \mathbf{Q}_{T-1} \ldots \mathbf{Q}_1 \cdot \left[\frac{\mathbf{a}}{1}\right]$$

Step 3. Update $\mathbf{L}^{(k)}$

$$\left[\frac{\mathbf{L}^{(k)}}{-\delta^* \cdot \mathbf{k}^{(k)^{\mathrm{T}}}}\right] \Leftarrow \mathbf{Q}_T \mathbf{Q}_{T-1} \ldots \mathbf{Q}_1 \cdot \left[\frac{\mathbf{L}^{(k-1)}}{\mathbf{0}}\right]$$

Step 4. Update $\bar{\mathbf{v}}^{(k)}$

$$\bar{\mathbf{v}}^{(k)} = \bar{\mathbf{v}}^{(k-1)} + [-\frac{X^{(k)} - \mathbf{z}^{(k)^{\mathrm{T}}} \cdot \bar{\mathbf{v}}^{(k-1)}}{\delta}] \cdot [-\delta^* \cdot \mathbf{k}^{(k)^{\mathrm{T}}}]^{\mathrm{H}}$$

end

$\bar{\mathbf{v}} = \bar{\mathbf{v}}^{(K)}$

In *Step 2*, $\mathbf{Q}_j$ is an elementary unitary plane transformation[1] acting upon the jth and the last component of $\mathbf{Q}_{j-1} \ldots \mathbf{Q}_1 \cdot \left[\frac{\mathbf{a}}{1}\right]$, such that the jth component is zeroed. For further details, we refer to [4]. **Fig. 1** gives the signal flow graph (SFG) for the RLS algorithm with inverse updating. Note that $f[.]$ is a decision device. The output of this device may be used as the so-called desired response input, for further training in 'decision-directed mode', during data transmission.

By applying this algorithm to the per tone equalization, one obtains **Fig. 2**. Per received symbol, one FFT and $T-1$ difference terms are computed. Every used tone has a T-taps PTEQ with as inputs the FFT output for that tone and $T-1$ difference terms. An important aspect in view of the overall computational complexity, is that the $T-1$ real inputs (the difference terms) give rise to a real triangular part in the SFG which is common to all the used tones, *i.e.* matrix $\mathbf{L}_{(1:T-1,1:T-1)}^{(k)}$. The complex FFT output is taken as the Tth input to the adaptive filter. This input makes the row $\mathbf{L}_{(T,:)}^{(k)}$ complex, and different for different tones. The update of the filter coefficients is also executed separately for each tone. Remark that the filter coefficients are updated in the following order $[v_{i,1} \ldots v_{i,T-1}\, v_{i,0}]$ where $v_{i,0}$, the tap with complex input, is the right-most coefficient in the SFG.

4 COMPLEXITY

In our complexity calculations, only the number of *real multiplications* is considered. A multiplication of a real number with a complex number is counted as 2 real multiplications and a multiplication of 2 complex numbers is counted as 4 real multiplications.

Initialization complexity of PTEQ based on an RLS-scheme for each tone, then becomes:

- Real part of $\mathbf{L}$ (common for all used tones):
  - Matrix vector product: $\frac{(T-1)T}{2}$
  - Real Rotations: $[\frac{(T-1)T}{2} + T - 1]4$ (4 multiplications per rotation)

[1] *i.e.* a transformation matrix that, whenever applied to a vector, modifies (only) two entries of the vector according to

$$\begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} \cos\theta & e^{j\psi}\sin\theta \\ -e^{-j\psi}\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}.$$

Such a transformation is represented by a hexagon in the SFG's.

- Complex part of $\mathbf{L}$ and filter update (per tone)
  - Matrix vector product: $(T-1)2+4$
  - Complex Rotations: $(T+1)9$
    (9 multiplications per rotation with 1 real and 1 complex input)
  - Filter update: $(T-1)2+4+4+T4$

The real part is quadratic in T but shared by all used tones, while the complex part is linear in T and executed for each tone separately. The latter part will mostly dominate the total complexity, e.g. for ADSL downstream transmission, where $N_u >> T$, with N_u the total number of used tones. By assuming $N_u = \frac{N}{2}$ and the symbol duration $\frac{N}{F_s}$, we have as total complexity: $8.5F_s \cdot (T+1) + \frac{F_s}{N}\frac{1}{2}(5T^2+3T-8)$.

While performance is almost the same for both per tone and per M tones equalization (e.g. for $M=11$) [8], it can be shown that the overall complexity drops significantly when tones are grouped: the first term reduces to $\frac{8.5}{M}F_s \cdot (T+1)$ when tones are combined into groups of M tones[2]. As an example, we take $N=512$ (ADSL downstream) and $M=11$ (see section 5), which leads to a complexity of roughly $0.8 \cdot F_s \cdot T + 0.005 \cdot F_s \cdot T^2$. It is seen that, in this case, initialization complexity is very comparable to complexity during data transmission, namely $F_s \cdot T$ [6].

5 SIMULATION RESULTS

Fig. 3 gives some ADSL simulation results for the downstream standard channel T1.601-#9 with additive white noise of -140dBm/Hz. The symbol size N is 512, the prefix length is 32 and the used tones are from tone 38 up to tone 256. The sample rate F_s is 2.208MHz. The equalizer size T is 16 and the initial values are set to $\mathbf{v}_i = [0\ldots0;1;0\ldots0]$ with '1' in the $(T/2)$th position, and $\mathbf{L}_i = 10^5 \cdot \mathbf{I}_T$. Per tone equalization is compared with per 11 tones equalization for the RLS algorithm with inverse updating. 'Per 11 tones equalization' means that the tones are combined into groups of 11 tones. The optimal per tone equalizer then is only computed for the center tone of each group, with direct or iterative initialization, and reused for the other tones in the same group [8]. **Fig. 3(a)** shows the bitrate as a function of the number of training symbols for per tone (in dashed-dotted line) and per 11 tones (in solid line) equalization. Also the optimum bitrate, computed with direct initialization, is plotted for both schemes. They are almost the same: 7.1882Mbit/s (for per tone equalization, in dotted line) and 7.1719Mbit/s (for per 11 tones equalization, in dashed line). In **Fig. 3(b)** SNR distributions

[2] Some extra computations have to be done when combining tones. Firstly, transforming the equalizer from the center tone to appropriate equalizers for the other tones in the same group which has to be performed once. Secondly, additional 1-taps frequency domain equalizers are needed for the non-center tones [8].

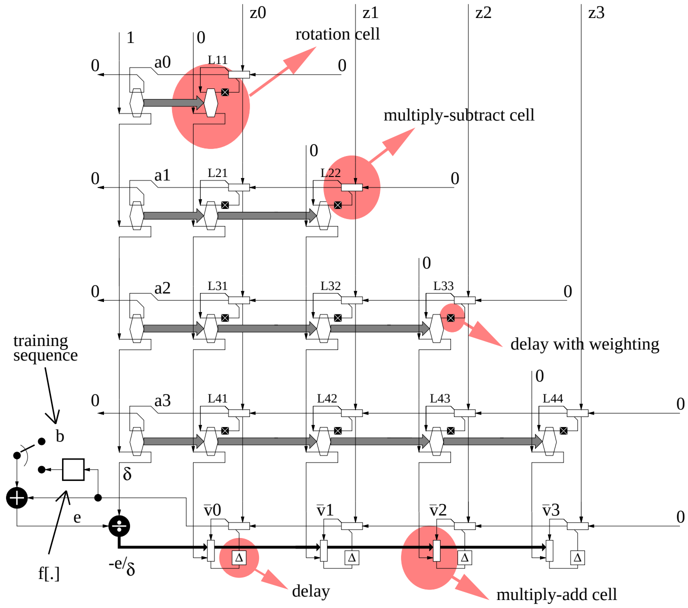

Figure 1: SFG for inverse updating RLS-based equalization.

are plotted for per tone equalization for different lengths of the training sequence. It is seen that convergence to optimal performance may be achieved with an acceptably short training sequence.

6 CONCLUSIONS

An iterative initialization procedure based on RLS with inverse updating is presented for per tone equalization. It is demonstrated that part of the signal flow graph is common to all used tones, resulting in a significant complexity reduction. Complexity calculations are presented and simulations results show convergence with an acceptable number of training symbols.

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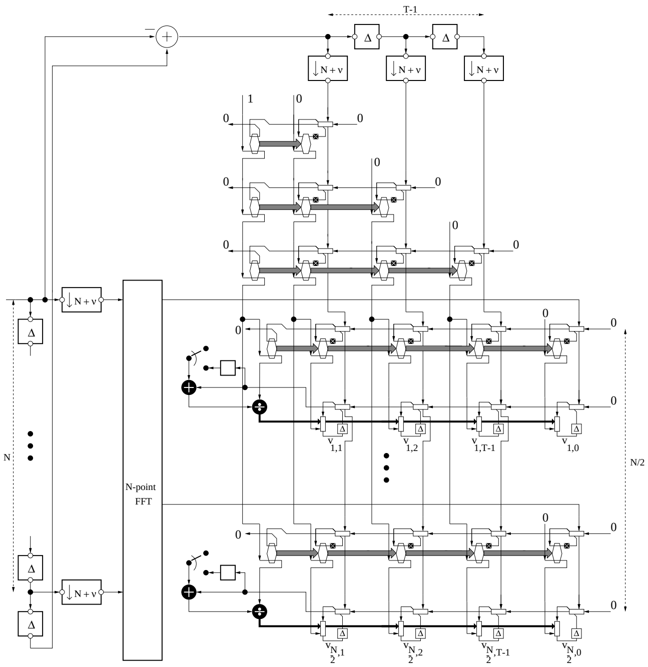

Figure 2: SFG for inverse updating RLS-based per tone equalization

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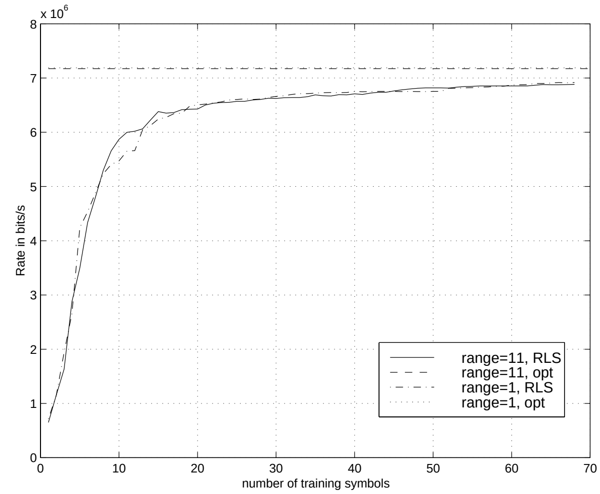
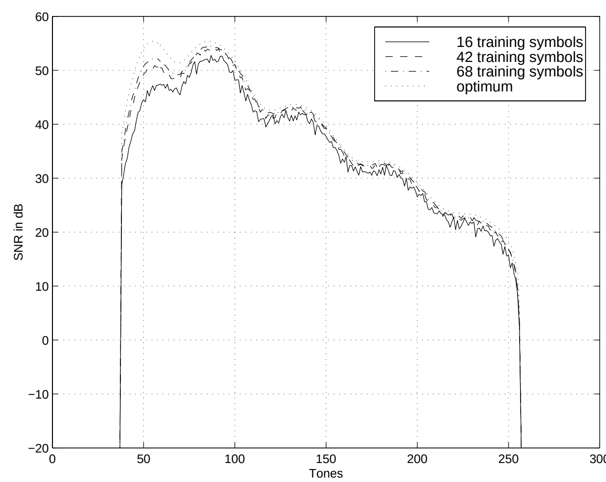


(a) Bit rate for per tone and per 11 tones equalization.



(b) SNR for per tone equalization.

Figure 3: Simulation results for RLS with inverse updating.