A Hybrid Transform Method for Image Denoising

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ABSTRACT
Image filtering in moving window in DCT domain has proven its capability in image edge preserving denoising. For the highest noise suppression capability, such a filtering requires, in principle, optimal selection of the size of the moving window for each particular image. In order to overcome this drawback, a combination of signal wavelet expansion (sub band decomposition) and moving window filtering in DCT domain is suggested and experimentally tested on a number on synthetic and real life images.

1. INTRODUCTION
Image denoising by means of nonlinear transformation of image representation in transform domain has attracted considerable attention of the researchers. Two advanced methods are moving window filtering in DCT domain ([1,2,7-11]) and wavelet shrinkage ([3-6]). While the former was demonstrated to have certain superiority to the wavelet shrinkage in terms of noise suppression capability and local adaptivity provided appropriate selection of the window size for each image ([12]), wavelet shrinkage is less sensitive to image properties and offers combination of global and local image representation.

One way to overcome this drawback of moving window filtering is parallel filtering in multiple moving windows of different size with selection of an appropriate window size for each image ([12]), wavelet shrinkage is less sensitive to image properties and offers combination of global and local image representation.

In the present paper, another, less computationally expensive alternative is suggested and evaluated by computer simulation - a hybrid wavelet-DCT representation based method aimed at making use of the advantages of both above methods.

2. MOVING WINDOW FILTERING IN DCT DOMAIN
Moving window filtering in DCT domain is a variety of space (time)- transform (frequency) signal representation and filtering. In this process, signal is analyzed in a moving window, and in each position of the window its spectrum on a certain basis is computed and then nonlinearly modified to generate, by means of inverse transform, improved estimate of the signal value in the central pixel of the window ([7-11]). Computationally efficient way of such a modification is scalar filtering described by the relationship:

\[ \hat{a}_r = \eta_r \beta_r, \]

where \( \beta_r \) - \( r \)-th spectral coefficient of the signal spectrum in the window, \( \hat{a}_r \) - its estimate, and \( \eta_r \) corresponding weight coefficient of the scalar filter. Two known modifications of such a scalar filtering are:

\[ \hat{a}_r = \left\{ \begin{array}{ll} \beta_r, & |\beta_r^2 - \text{Thr}| \leq \text{Thr} \\ \beta_r^2 - \text{Thr}, & |\beta_r^2 - \text{Thr}| > \text{Thr} \end{array} \right. \]

The first one implements a simplest empirical Wiener filter and is frequently referred to as “Soft” thresholding; the second one is frequently referred to as “Hard” thresholding. As it follows from the theory of empirical Wiener filtering, parameter \( \text{Thr} \) in these formulas has the order of magnitude of variance of the additive noise. Although moving window filtering can be implemented in any transform domain, using DCT has a number of advantages both in terms of noise suppression capability and computational efficiency ([7-11]). In such a filtering window size is an important filter parameter. As experiments show, optimal window size depends on the image and may vary even within the image. This motivates search for the ways to implement filtering in multiple windows with an appropriate combination of the filtering results.

3. WAVELET SHRINKAGE
In distinction to moving window filtering in transform domain, wavelet shrinkage methods ([3-6]) apply the above “Soft” and “Hard” thresholding to sub band decomposed image components as it is shown on block diagram of Fig. 1. Modifying low pass filtering and interpolation procedures one can implement different types of wavelet expansion for the optimization of the processing. Experimental experience shows that wavelet shrinkage, in its noise suppressing capability, underperforms DCT filtering in moving window ([12]). A possible reason is insufficient local adaptation of the wavelet shrinkage.
Fig. 1. Block-diagram of wavelet shrinkage

4. HYBRID FILTERING METHOD

A combination of multi resolution property of the wavelet shrinkage and local adaptivity of DCT filtering in the hybrid filtering method promises improvement of noise suppression capability of both.

Fig. 2 Hybrid filtering method: one stage

According to this method, image is first represented in multiple scales by its sub band decomposition as in the wavelet shrinkage. Each sub band component obtained is then subjected to DCT moving window filtering with size of the window being kept the same for all scales. As one can see, in the hybrid method simple “soft” or “Hard” thresholding of the signal in all scales is substituted by their DCT filtering in moving window. This substitution imitates parallel filtering of the initial image with a set of windows accordingly to the scales selected.

For high resolution images, DCT filtering in moving window requires window size 3x3, otherwise the filtering may result in loss of tiny image details. This requirement limits noise suppression capability of the filtering that increases with the window size. In hybrid filtering, when window 3x3 in the first scale is used, in the scale 2 effective window size is 6x6, in the scale 3 it is 9x9 and so on. This potentially may add to the filtering noise suppression capability.

Note that in DCT filtering in the window 3x3 only the following four basis function are involved ([11]):

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
-1 & -2 & -1 \\
-1 & -2 & -1
\end{bmatrix}
\]

As one can see, second and third functions represent vertical and horizontal Laplacian operators and the last function can be decomposed into a sum of diagonal Laplacians. Therefore, DCT filtering in window 3x3 can be replaced by filtering in the domain of four directional Laplacians. Experimental experience proves that, for high resolution images, such an implementation is advantageous to simple DCT since it produces less filtering artifacts ([11]). Its use in hybrid filtering promises additional advantages because it is equivalent to the corresponding increase of number of effective basis functions. Examples of the effective set, for the input image plane, of these directional Laplacians in four scales are shown in Fig. 3. This implementation was used in the experiments described below.

Fig. 3 Effective basis functions in multi resolution DCT
5. DESCRIPTION OF EXPERIMENTS AND DISCUSSION

For testing the method, modeled images and a number of real life images were used. For all images, Gaussian white noise was added to produce noisy images for the filter input. Three types of filters were tested: moving window DCT filter, wavelet shrinkage filter and the suggested hybrid filter.

In the wavelet shrinkage filter, a code of image pyramid from University of Pennsylvania package [13] was used. For all filters, optimal parameters were experimentally found that minimize RMS difference between initial noise free image and the filtered one. For DCT moving window filtering, window size and parameter $\text{Thr}$ was optimized. For wavelet shrinkage and hybrid filter, parameters $\text{Thr}$ were optimized and the best of the following types of wavelets was chosen: Haar wavelet (“haar”), Binomial coefficient filter (“binom3”, “binom9” and “binom13”), Daubechies wavelet (“daub2” and “daub4”) and Symmetric Quadrature Mirror Filters (“qmf5”, “qmf9” and “qmf13”). Table summarizes results obtained for four of 12 test images of 256x256 pixels with 256 quantization level and standard deviation of additive Gaussian noise of 13: a test piece-wise constant image, “Lenna” image, a MRI image and a air photograph. For DCT moving window filtering, optimal window size is shown in brackets. For wavelet shrinkage, the best wavelet filter kernel is indicated in brackets.

Table. Standard deviation of the difference between initial noise free and filtered images for different filtering methods (Standard deviation of additive noise is 13).

<table>
<thead>
<tr>
<th>Filter</th>
<th>P-W const. image</th>
<th>Lenna image</th>
<th>MRI</th>
<th>Air photo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hard</td>
<td>8.1 (3x3)</td>
<td>9.4 (3x3)</td>
<td>6.7 (3x5)</td>
<td>8.2 (3x3)</td>
</tr>
<tr>
<td>Soft</td>
<td>7.5 (3x3)</td>
<td>8.6 (5x5)</td>
<td>6.3 (7x7)</td>
<td>7.7 (3x3)</td>
</tr>
<tr>
<td>WL-Shrinkage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hard</td>
<td>8.6 (binom 5)</td>
<td>10.1 (binom 5)</td>
<td>8.5 (binom 5)</td>
<td>9.3 (binom 5)</td>
</tr>
<tr>
<td>Soft</td>
<td>8.4 (binom 5)</td>
<td>9.0 (qmf13)</td>
<td>7.8 (binom 5)</td>
<td>8.1 (binom 5)</td>
</tr>
<tr>
<td>Hybrid</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Hard</td>
<td>8.7 (binom 5)</td>
<td>9.4 (binom 5)</td>
<td>6.6 (binom 5)</td>
<td>8.2 (binom 5)</td>
</tr>
<tr>
<td>Soft</td>
<td>7.9 (binom 5)</td>
<td>8.6 (binom 5)</td>
<td>6.2 (binom 5)</td>
<td>7.5 (binom 5)</td>
</tr>
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</table>

It was found in the experiments that

- All filters give substantial noise reduction provided optimal selection of the filter parameters.

- In terms of the variance of the residual filtering error, DCT filtering outperforms wavelet shrinkage while hybrid filtering outperforms both of them.

- Soft thresholding is always better than hard thresholding both in terms of minimal variance of the residual error and in terms of visual evaluation of image quality.

- There is quite a noticeable difference between filter performance in terms of image residual distortions. Hybrid filtering provides least visual distortions and artifacts; DCT moving window filtering is slightly though certainly worse and wavelet shrinkage appears to be the worst one.
Some illustrative examples for the different filters are given in Figs. 4-5.

6. REFERENCES


