Downlink Space-Time Processing for FDD systems

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ABSTRACT
The focus of this paper is downlink space-time processing for FDD (Frequency Division Duplex) radio-communication systems whose base stations are equipped with antenna arrays. It shows how space-time filters can be appropriately designed for downlink in order to both combat co-channel interferences and reduce fading effects. These filters, which can be estimated from uplink data only, are an extension of previous schemes which considered only space processing for downlink.

1 INTRODUCTION
Implementing antenna arrays at the base stations of radiomobile communication systems allows to reduce the CCI (Co Channel Interference) in both uplink (from mobile to base station) and downlink (from base station to mobile), and also gives rise to the concept of SDMA (Space Diversity Multiple Access), thus enhancing system capacity. In a recent paper [4] we have shown how space-time filters for downlink may be appropriately designed by optimizing a suitable criterion, in an FDD system like GSM, in order to both combat CCI and reduce fading effects: the proposed method is thus an extension of previous schemes [1, 2, 3] where spatial filters were only considered. It must be emphasized that these space-time filters can be determined from uplink data only, through the frequency transposition method recently developed in [3], without knowing downlink channels. The purpose of this paper is to detail the proof of the main result that was just stated in [4]: we refer to [4] for simulation results and practical implementation issues. However, to avoid systematic reference to [4], this paper is self-contained. In the sequel, we will assume a TDMA radiomobile system like GSM with a linear modulation (GMSK in GSM can be accurately modelled by a linear modulation); however, the proposed method could be applied to CDMA with minor modifications.

Modeling of propagation channels. Let us consider a multisensors base station consisting of M antennas and let $h_m(t, b)$ be the baseband impulse response between a mobile and the $m$-th antenna during burst number $b$ (time origin for each burst is taken at its start). The proposed method relies on the following classical baseband model of propagation channels:

$$h(t, b) = [h_1(t, b) \cdots h_M(t, b)]^T = \sum_n \beta_n(b, f_c) \delta(t - \tau_n(b)) \mathbf{d}(\theta_n(b), f_c),$$

where:
- $f_c$ is the carrier frequency ($f^{Rx}$ for downlink and $f^{Tx}$ for uplink);
- path $n$ is characterized by its delay $\tau_n(b)$, its complex amplitude $\beta_n(b, f_c)$, and its Direction Of Arrival (DOA) $\theta_n(b)$;
- $\mathbf{d}(\theta, f_c)$ denotes the steering vector of the array associated to the DOA $\theta$ at carrier frequency $f_c$.

Note that the channel is assumed stationary during one burst. Since the paths geometry varies slowly, the following assumptions will be done: $\tau_n(b) = \tau_n$ and $\theta_n(b) = \theta_n$ are constant over the processed bursts (typically 100 bursts for GSM). Mobile displacements of about one wavelength result in changes of the path complex amplitude $\beta_n(b, f_c)$ so that it varies from one burst to another. However, paths powers vary slowly. Therefore, $\beta_n(b, f_c)$ is modelled as a zero-mean circular random variable with the same constant variance (over the processed bursts) at $f^{Rx}$ and $f^{Tx}$, uncorrelated from one path to another:

$$E[|\beta_n(b, f_c)|^2] = \rho_n; \quad E[\beta_n(b, f_c)^* \beta_m(b, f_c)] = 0 \text{ for } m \neq n.$$  \hfill \(2\)

Let $I + 1$ be the total number of co-channel users during one time slot. In the sequel, quantities related to user number $u$ will be indexed by $u$. Thus, downlink channel impulse response for user $u$ is $h_u(t, b)$. The corresponding channel frequency response will be denoted by $H_u(f, b)$:

$$H_u(f, b) = \sum_n \beta_{un}(b, f_c) e^{-j2\pi\tau_n f} d(\theta_{un}, f_c).$$ \hfill \(3\)
In the case of flat fading, \( \mathbf{H}_u(f, b) \) is frequency independent and reduces to:
\[
\mathbf{H}_u(b) = \sum_n \beta_{u_n}(b, f_c) \mathbf{d}(\theta_{u_n}, f_c).
\] (4)

Covariance of \( \mathbf{H}_u(f, b) \) will be required in the processing. It is easily seen to be frequency independent and will be denoted by \( \mathbf{R}_u \):
\[
E[\mathbf{H}_u(f, b) \mathbf{H}_u(f, b)^H] = \sum_n p_{u_n} \mathbf{d}(\theta_{u_n}, f_c) \mathbf{d}(\theta_{u_n}, f_c)^H = \mathbf{R}_u.
\] (5)

2 SPACE-TIME FILTERS DETERMINATION

Our purpose is to design for each user \( u \) a downlink space-time filter \( \mathbf{w}_u(f) = [w_{u_1}(f), \ldots, w_{u_M}(f)]^T \) in such a way that it combats flat fading and reduces CCI towards the other \( I \) co-channel mobiles. In order to be estimated from uplink data, these downlink filters must depend only on common features between uplink and downlink channels: delays \( \tau_m \), DOA’s \( \theta_m \), and path powers \( p_m \). This is why \( \mathbf{w}_u(f) \) must be burst independent. Let \( \tilde{s}_u(f, b) \) denote the Fourier Transform (FT) of the baseband signal \( s_u(t, b) \) dedicated to user \( u \) during burst \( b \). The FT of the signal emitted by the \( m \)-th sensor towards mobile \( u \) is \( w_{u,m}(f)^* \tilde{s}_u(f, b) \), and the FT of the total signal received at mobile \( u \) is equal to:
\[
\tilde{s}_u(f, b) \mathbf{H}_u(f, b) + \sum_{i \neq u} \tilde{s}_i(f, b) \mathbf{H}_u(f, b).
\]

In the above expression, the first term is the desired signal and the second one is the interference resulting from emission towards co-channel users: our goal is to reduce power fluctuations of the first term, and make the second one as small as possible.

Let \( T \) be the symbol duration, \( C(f) \) the pulse shaping filter frequency response and \( c(t) \) the associated impulse response. Downlink filters \( w_{u}(f) \) \( (u = 1, \ldots, I + 1) \) will be taken to be of the form:
\[
\mathbf{w}_u(f) = \sum_{k=1}^K w_{u_k} e^{2\pi i T_k f},
\] (6)

and the determination of the \( K \) spatial filters \( w_{u_k} \) in the above expression is the focus of this paper. As justified later, \( K \) is less or equal to the number \( M \) of sensors. Expression (6) shows that each spatial filter \( w_{u_k} \) transports a delayed version \( s_{u_k}(t - T_k, b) \) of the signal for user \( u \). Thus the resulting transfer function between the transmitted linearly modulated signal \( s_{u_k}(t, b) \) at the base station and mobile \( u \) is:
\[
\mathbf{w}_u(f)^H \mathbf{H}_u(f, b) = \sum_{k=1}^K w_{u_k}^H \mathbf{H}_u(f, b) e^{-2\pi i T_k f}.
\] (7)

Delays \( T_k \) in (6) will be chosen in such a way that the delayed messages \( s(t - T_k) \) are uncorrelated. Together with a unit power convention, this is achieved when:
\[
\frac{1}{T} \int |C(f)|^2 e^{2\pi i (T_m - T_n) f} df = \begin{cases} 
1 & \text{if } m = n; \\
0 & \text{if } m \neq n.
\end{cases}
\] (8)

In particular, when \( |C(f)|^2 \) is a Nyquist filter, delays \( T_k \) may be equal to \((k - 1)T\).

Constraints on \( w_{u_k} \). The purpose of the \( K \) delays \( T_k \) associated with the spatial filters (6) is to increase diversity thus combating flat fading. In order to derive the needed properties of the spatial filters \( w_{u_k} \) to achieve that goal, we will assume temporarily flat fading channels (4) with complex gaussian paths amplitudes \( \beta_{u_n}(b, f_c) \). Let \( p_u(b) \) be the received signal power at mobile \( u \) during burst \( b \):
\[
\begin{align*}
 p_u(b) &= \frac{1}{T} \int |C(f)|^2 |\mathbf{w}_u^H(f) \mathbf{H}_u(b)|^2 df \\
 &= \sum_m (\mathbf{w}_u^H \mathbf{H}_u^H(b)) (\mathbf{H}_u(b) \mathbf{w}_{u_m})
\end{align*}
\]

where the last equality follows straightforwardly from expression (7) of \( \mathbf{w}_u(f) \) and relations (8). For a given mean power
\[
E[p_u(b)] = \sum_m w_{u_m}^H \mathbf{R}_u w_{u_m} = P_u, \quad \text{(9)}
\]
we will choose the \( w_{u_m} \)’s so as to minimize the power fluctuations over successive bursts, that is \( p_u(b)’s \) variance. A simple computation yields:
\[
\text{var}(p_u(b)) = \sum_m (w_{u_m}^H \mathbf{R}_u w_{u_m})^2 + \sum_{m \neq n} \sum_{r \neq m} |w_{u_m}^H \mathbf{R}_u w_{u_n}|^2. \quad \text{(10)}
\]

Under mean power constraint (9) and assuming \( K \leq M \), the second term of the right hand side member in (10) is obviously minimized when
\[
\begin{align*}
 w_{u_m}^H \mathbf{R}_u w_{u_n} &= 0 \quad \text{for } m \neq n, \quad \text{(11)}
\end{align*}
\]
while, as shown easily, the first one has its smallest value \( P_u^2/K \) when:
\[
\begin{align*}
 w_{u_m}^H \mathbf{R}_u w_{u_m} &= P_u/K \quad \text{for } m = 1, \ldots, K. \quad \text{(12)}
\end{align*}
\]

It is easily checked that values of \( K \) greater than \( M \) are useless since they do not allow any further power variance reduction than \( K = M \). Constraints (11) and (12) are therefore fully justified in the case of flat fading. In the sequel, they are also kept for frequency selective fading although there is no claim for optimality in that case: they just allow the \( K \) downlink subchannels \( w_{u_k}^H \mathbf{H}_u(f, b) \) in (7) to be uncorrelated and
to transport the same fraction $P_u/K$ of the total mean power $P_u$.

**Remark.** Constraints (11) were set a priori in [1] in the computation of downlink spatial filters.

**Remark.** It can be checked easily that, even in the case of selective fading, the above constraints (11) and (12) together with (8) imply that the mean received signal power at mobile $u$ is indeed equal to $P_u$.

**Criterion for space-time filters determination.** Subject to the space-time filters structure (6) together with constraints (11) and (12), we use the same criterion as the one proposed in [2] for spatial filters only. Our filters $w_u(f)$ for $u = 1, \ldots, I + 1$ must minimize the sum of the interferences to signal power ratios $P_u$, where $I_u$ is the interference mean power at mobile $u$:

$$\sum_{u=1}^{I+1} \sum_{u \neq \mu} \mathbb{E} \left[ \frac{1}{P_u} \left| \int |C(f)|^2 |w(f)|^2 |H_u(f,b)|^2 df \right| \right]$$

under constraints (11) and (12).

**Problem solution.** Denote by $T_u$ and $Q_u$ the following matrices:

$$T_u = P_u^{-1} R_u$$

(14)

$$Q_u = \sum_{u \neq \mu} P_u^{-1} R_u.$$  

(15)

$Q_u$ will be referred to as the interference covariance matrix for mobile $u$. Let $v_{u1}, \ldots, v_{uk}$ be the $K$ greatest generalized eigenvectors of $(T_u, Q_u)$:

$$T_u v_{uk} = \lambda_{uk} Q_u v_{uk}$$

(16)

normalized according to:

$$v_{uk}^H Q_u v_{uk} = 1 \ \forall k.$$  

(17)

Then, the components $w_{u1}, \ldots, w_{uk}$ of $w_u(f)$ (6) minimizing criterion (13) under constraints (11) and (12) are given by:

$$w_u = K^{-1/2} V_u A_u^{-1/2} S_u$$

(18)

where:

$$W_u = [w_{u1}, \ldots, w_{uk}] ;$$

(19)

$$V_u = [v_{u1}, \ldots, v_{uk}] ;$$

(20)

$$A_u = \text{diag} \{ \lambda_{u1}, \ldots, \lambda_{uk} \} ;$$

(21)

$$S_u = \text{any} \ K \times K \ \text{unitary matrix}.$$  

**Remark:** equivalently, one can also use in equation (18) the eigenvectors and eigenvalues of $(T_u, T_u + Q_u)$ instead of $(T_u, Q_u)$, which simplifies implementation.

**Remark:** the particular case $K = 1$ yields the former solution of purely spatial filters for downlink proposed in [2].

**Proof:** From equations (5), (6), (8) and (15), criterion (13) to be minimized can be expressed as:

$$\sum_{u=1}^{I+1} \sum_{u \neq \mu} \mathbb{E} \left[ \frac{1}{P_u} \left| \int |C(f)|^2 |w(f)|^2 |H_u(f,b)|^2 df \right| \right]$$

under constraints (11) and (12).

**Criterion:**

$$\mathcal{F} = \sum_{u=1}^{I+1} \sum_{u \neq \mu} w_{uk}^H Q_u w_{uk}$$

$$- \sum_{u=1}^{I+1} \sum_{u \neq \mu} \lambda_{uk}(n,m) (w_{uk}^H T_u w_{uk} - K^{-1}\delta(m,n)).$$

(24)

Since constraints (23) are real for $m = n$ and complex otherwise, Lagrange multipliers $\lambda_{uk}(n,m)$ are real, while the $\lambda_{uk}(n,m)$’s for $(m \neq n)$ are complex and satisfy:

$$\lambda_{uk}(n,m) = \lambda_{uk}(m,n)^*.$$  

(25)

Cancelling the gradient of (24) with respect to $w_{uk}$ yields:

$$Q_u w_{uk} = \sum_m \lambda_{uk}(m,n) T_u w_{um}.$$  

(26)

The $\lambda_{uk}(n,m)$’s are easily derived by multiplying both sides of (26) by $w_{uk}^H$ and by taking into account constraints (23):

$$\lambda_{uk}(m,n) = K w_{uk}^H Q_u w_{uk}.$$  

(27)

By substituting for $\lambda_{uk}(m,n)$ from expression (27) into (26), our optimization problem amounts to solve with respect to matrix $W_u = [w_{u1}, \ldots, w_{uk}]$

$$Q_u W_u = K T_u W_u W_u^H Q_u W_u,$$  

(28)

under constraint:

$$W_u^H T_u W_u = K^{-1} I.$$  

(29)
In order to solve equations (28) and (29), let us introduce the generalized eigenvectors \( v_{u1}, \ldots, v_{uM} \) of the matrix pencil \( (T_u, Q_u) \) whose eigenvalues are assumed distinct and non-zero: \( \lambda_{u1} > \cdots > \lambda_{uM} \). Let

\[
X_u = [v_{u1}, \cdots, v_{uM}]
\]  

(30)

and

\[
Y_u = \text{diag} \{ \lambda_{u1}, \cdots, \lambda_{uM} \}
\]  

(31)

be the corresponding eigenvectors and eigenvalues matrices which satisfy:

\[
T_u X_u = Q_u X_u Y_u,
\]  

(32)

where eigenvectors are scaled so that:

\[
X_u^H Q_u X_u = I.
\]  

(33)

Let us set for the expression of the unknown \( M \times K \) spatial filters matrix \( W_u \) (19)

\[
W_u = K^{-1/2} X_u Y_u^{-1/2} A_u,
\]  

(34)

where the determination of \( W_u \) now reduces to that of \( M \times K \) matrix \( A_u \). It is easy to check that equations (28) and (29) to be solved lead, in terms of \( A_u \) (34), to:

\[
A_u = Y_u A_u A_u^H Y_u^{-1} A_u
\]  

(35)

\[
A_u^H A_u = I,
\]  

(36)

where equation (36) shows that the columns of \( A_u \) are orthonormal. We note that matrix \( Y_u A_u A_u^H Y_u^{-1} \) in equation (35) is idempotent: it is therefore entirely determined by its range and its null space. From its expression \( Y_u A_u A_u^H Y_u^{-1} \), its null-space is orthogonal to the columns of \( Y_u^{-1} A_u \) and its range is spanned by the columns of \( Y_u A_u \). However, equation (35) shows that the range of \( Y_u A_u A_u^H Y_u^{-1} \) is also spanned by the columns of \( A_u \); moreover, (35) can be rewritten

\[
Y_u^{-1} A_u = A_u A_u^H Y_u^{-1} A_u
\]  

which implies that the null space (originally orthogonal to the columns of \( Y_u^{-1} A_u \)) is also orthogonal to the columns of \( A_u \). Thus, matrix \( Y_u A_u A_u^H Y_u^{-1} \) is also equal to:

\[
Y_u A_u A_u^H Y_u^{-1} = A_u A_u^H,
\]

or, equivalently:

\[
Y_u A_u A_u^H = A_u A_u^H Y_u.
\]  

(37)

Denoting by \( a_{ij} \) the elements of \( A_u A_u^H \), the above equation can be rewritten as

\[
\lambda_{uj} a_{ij} = \lambda_{uj} a_{ij} \quad \text{for} \quad i, j = 1, \cdots, M,
\]

which implies, since the eigenvalues are assumed distinct:

\[
a_{ij} = 0 \quad \text{for} \quad i \neq j.
\]

\( \text{Rank} \ K \ \text{projector} \ A_u A_u^H \) is therefore diagonal: its diagonal contains \( M - K \) zeros and \( K \) ones. Consequently, matrix \( A_u \) is equal to

\[
A_u = [e_{i_1}, \cdots, e_{i_K}] S_u
\]  

(38)

where \( 1 \leq i_1 < i_2 \cdots < i_K \leq M \), the \( k \)-th component of vector \( e_{i_k} \) is one and the remaining ones are zero, \( S_u \) is any \( K \times K \) unitary matrix. It follows from (34) that the unknown matrix \( W_u \) is given by:

\[
W_u = K^{-1/2} X_u Y_u^{-1/2} [e_{i_1}, \cdots, e_{i_K}] S_u.
\]  

(39)

From (22), the corresponding value of the criterion is easily shown to be:

\[
\sum_{u=1}^{I+1} \frac{I_u}{P_u} = \sum_{u=1}^{I+1} K \sum_{k=1}^{K} \frac{1}{\lambda_{uk}}
\]  

(40)

which takes its smallest value when \( i_1 = 1, \cdots, i_K = K \). Thus, expression (39) reduces to (37).

3 PRACTICAL IMPLEMENTATION

The practical computation of the downlink filters is presented in [4]: it is shown that these filters can be determined from uplink data only.

4 CONCLUSIONS

We have shown in this paper how space-time filters could be designed for downlink transmission in an FDD system so as to both combat CCI and reduce fading effects. This was achieved by setting appropriate constraints on the spatio temporal filters and by optimizing a criterion formerly proposed for spatially only downlink filters. Simulations in a GSM context with typical urban channels demonstrate the efficiency of the proposed method.

References


