

# DIGITAL FILTER DESIGN FOR I/Q IMBALANCE COMPENSATION

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## ABSTRACT

All communication receiver structures utilizing I/Q signal processing share the problem of matching the amplitudes and phases of the I- and Q-branches. In practise, the imbalances are unavoidable and this results in finite and in many cases insufficient attenuation of the image frequency band. Without any additional image rejection, this causes interference and needs to be compensated.

We carry out the general signal and imbalance analysis of an I/Q processing based receiver structure and discuss the possibilities of digital imbalance compensation in cases where the dominating part of the imbalance is known in advance. A novel method for fixed imbalance compensation in a wideband approximative quadrature sampling receiver is derived and the performance of the proposed solution is illustrated through an example design.

## 1 INTRODUCTION

The well-known direct-conversion architecture [1] as well as a more recent low-IF architecture [2] use quadrature mixing which in theory provides infinite attenuation for the image band. In practise, however, there is always some imbalance in the analog front-end between the I- and Q-branches, mainly due to finite tolerances of capacitor and resistor values used to implement the analog components [1][2]. While a perfectly balanced quadrature downconversion corresponds to a pure frequency translation, the imbalances introduce a frequency translation also in the other direction. This results in a mixture of the image and the desired signal. With careful analog circuit design, phase imbalance of 1-2° and amplitude imbalance of 1-2% are realistic, resulting in 30-40 dB attenuation of the image signal [2]. As a consequence, the problem of images in a low-IF receiver (and also in any other receiver structure that is using a non-zero IF frequency) is a major concern since the image signal can be of considerably higher power than the desired one [2][3]. This is one of the fundamental problems in various wideband (multichannel) downconverting receivers.

Traditionally, the image signal problem in radio receivers is solved using analog RF filtering. However, if the

needed image attenuation can be achieved using digital methods, the number of analog front-end components can be reduced and the receiver integration becomes a more feasible task to carry out. Advanced digital techniques are also more likely to be able to provide the needed flexibility in multistandard wireless equipment in the future.

In general, all the analog components, such as mixers and filters, in the I- and Q-branches contribute to the imbalance effects, but there can also be other sources of imbalance. In the approximative quadrature sampling receiver [4][5], the needed 90° phase shift between the I- and Q-branch signals is approximated using a proper sampling time difference. If the bandwidth of the input signal is comparable to the center frequency, the approximation is poor. Clearly, this corresponds to a certain phase imbalance between the I- and Q-branch signals and some kind of compensation is needed. This could be one important application of fixed imbalance compensation scheme since the imbalances are here known a priori.

## 2 SIGNAL ANALYSIS

A generalized block-diagram of a receiver utilizing I/Q signal processing [5] is shown in Fig. 1. The presented structure is traditional in the sense that it generates the I- and Q-channel signals using analog quadrature downconversion. In general, the needed 90° phase difference can be implemented in two ways, either in the input signal path or in the local oscillator (LO). Also digital methods, such as quadrature sampling, to directly generate the I- and Q-channels have been presented in the literature [5].

To simplify the analysis, the whole imbalance between the I- and Q-branches is modelled as an imbalanced quadrature mixer [3] with a LO signal  $x_{LO}(t) = \cos(\omega_{LO}t) - jg\sin(\omega_{LO}t + \phi)$  where  $g$  and  $\phi$  represent the amplitude and phase imbalances, respectively. This can be written as

$$x_{LO}(t) = K_1 e^{-j\omega_{LO}t} + K_2 e^{j\omega_{LO}t} \quad (1)$$

with imbalance coefficients

$$K_1 = \frac{1 + ge^{-j\phi}}{2}, \quad K_2 = \frac{1 - ge^{j\phi}}{2}.$$

Notice that introducing the imbalances only in the Q-branch of the LO signal is a valid model since what actually matters is the difference between the I- and Q-channels, not the absolute values.

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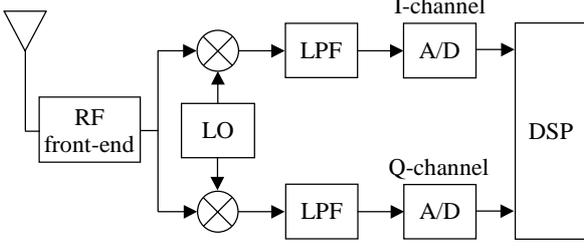


Figure 1: Generalized structure of an I/Q processing receiver.

Based on Eq. (1), the image attenuation provided by the quadrature downconversion is usually defined as

$$L_{QUAD} = |K_1|^2 / |K_2|^2. \quad (2)$$

As a model of finite matching between the I- and Q-branches, the received RF signal  $r(t)$  is quadrature mixed with  $x_{LO}(t)$ . Due to imbalances, two frequency translations instead of one will take place and this results in a mixer output signal  $r_{MIX}(t)$  where the desired signal is interfered by the image signal.

Considering only the desired and image band signals, the received signal  $r(t)$  can be formally written as

$$r(t) = \text{Re} \left( z(t) e^{j\omega_{LO}t} \right), \quad (3)$$

where  $z(t)$  is the combined interference-free low-pass equivalent of the desired and image channel signals. Based on Eqs. (1) and (3), the IF signal after low-pass filtering  $r_{IF}(t)$  can then be written as

$$r_{IF}(t) = K_1 z(t) + K_2 z^*(t). \quad (4)$$

The second term  $K_2 z^*(t)$  in Eq. (4) represents the image "aliasing" effect. The model of Eq. (4) can also be used as a starting point for deriving the needed compensation structures. To further illustrate this basic problematic, the spectra of the received signal and the mixer output signal are shown in Fig. 2.

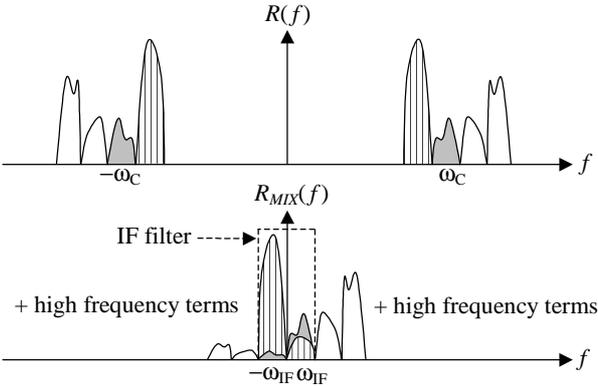


Figure 2: A frequency domain illustration of finite image rejection due to I/Q imbalances. This clearly shows how the imbalances result in a mixture of the desired signal (grey) and the image signal (cross-hatched).

### 3 IMBALANCE COMPENSATION

Both analog and digital techniques to compensate the effects of I/Q imbalance have been presented in the literature, see, e.g., [2]-[8]. One possible fixed imbalance compensation scheme utilizing baseband digital signal processing is here derived from Eq. (4). The idea is to estimate and cancel the effects of the interfering signal components  $K_2 z^*(t)$ . Adaptive solution based on the use of traditional interference canceller with novel observation generation was proposed in [3]. For fixed compensation, the same idea of simple interference cancellation can be used but with remarkably simpler reference signal generation. First, we note that

$$r_{IF}^*(t) = K_2 z^*(t) + K_1 z^*(t). \quad (5)$$

Then, using the signal of Eq. (5) as a reference signal, we can cancel the effects of interfering signal components as

$$\begin{aligned} y(t) &= r_{IF}(t) - C r_{IF}^*(t) \\ &= (K_1 - CK_2^*) z(t) + (K_2 - CK_1^*) z^*(t), \end{aligned} \quad (6)$$

where  $y(t)$  is the "cleaned" signal and  $C$  denotes the compensator gain. Then, the image attenuation  $L$  after compensation can be defined as

$$L = \frac{|K_1 - CK_2^*|^2}{|K_2 - CK_1^*|^2}. \quad (7)$$

Clearly, the optimum solution that cancels the image interference completely is given by

$$C_{OPT} = K_2 / K_1^*. \quad (8)$$

This is the same optimum solution as the one given in [3]. Here, however, we assume that the dominating part of the imbalance is known in advance and a simpler reference signal generation than in [3] can be used. This is indeed possible because no adaptive algorithm is included here and the imbalances are assumed to be known. The actual compensator output signal using the optimum solution of Eq. (8) is then given by

$$\begin{aligned} y_{OPT}(t) &= r_{IF}(t) - C_{OPT} r_{IF}^*(t) \\ &= \left( K_1 - \frac{K_2 K_2^*}{K_1^*} \right) z(t). \end{aligned} \quad (9)$$

Under the assumption that the receiver imbalance properties are realistic,  $K_2 \ll K_1$  in absolute value, and  $y_{OPT}(t) \approx z(t)$  as desired.

In the case that the amplitude and phase imbalances depend on frequency, we get a frequency domain equivalent of Eq. (8). Formally, this can be written as

$$C_{OPT}(f) = \frac{K_2(f)}{K_1^*(f)}. \quad (10)$$

This describes the desired frequency response for the compensation filter. Practical implementation is then to approximate the optimum compensation filter of Eq. (10) with a discrete-time FIR/IIR filter. The block-diagram for the proposed compensator structure is presented in Fig. 3. Notice that the optimum filters given by Eq. (10) can be non-causal, so in practise they have to be delayed to be realizable. This being the case, also the same delay has to be introduced into the upper branch of the compensator.

One situation where we know the imbalances in advance is the case of approximative quadrature sampling [4][5]. The idea in this kind of technique is to implement the needed  $90^\circ$  phase difference between the I- and Q-branch signals using a proper sampling time difference of

$$\Delta T = 1/(4f_c), \quad (11)$$

where  $f_c$  is usually the center frequency of the useful signal band before sampling. A simplified block-diagram of such a system is presented in Fig. 4.

For a sinusoidal signal of frequency  $f_c + f_\Delta$ , a sampling time difference of  $\Delta T = 1/(4f_c)$  corresponds to a phase difference of

$$\Delta\phi = (1 + f_\Delta/f_c) \frac{\pi}{2}, \quad (12)$$

where  $f_\Delta$  is the frequency offset from  $f_c$ . Notice that the phase difference is the needed  $90^\circ$  only at  $f_c$ , so the actual phase error in light of imbalance analysis is now

$$\phi(f_\Delta) = -\frac{f_\Delta}{f_c} \frac{\pi}{2}. \quad (13)$$

For a more thorough discussion on the use of approximative quadrature sampling to generate the I- and Q-channel signals, see [4].

If the sampling rate is equal to  $f_c$ , or its subharmonic, frequencies around  $f_c$  will be aliased down to baseband. Then, if the desired channel is centered at  $f_c$ , the imbalances cause a self-image, for which the attenuation requirements are not very critical. The situation where several channels are sampled together is more critical. This is because in this case, the desired channel (or one of the desired channels) is translated by aliasing to a low IF frequency, and the image comes from another channel which may have much stronger power level. This corresponds to the situation illustrated in the lower part of Fig. 2.

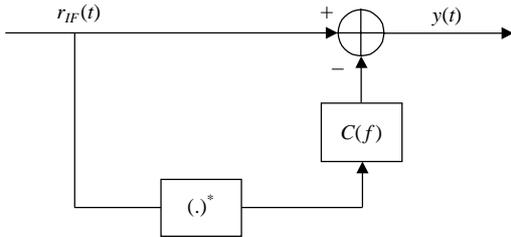


Figure 3: Imbalance compensation structure.

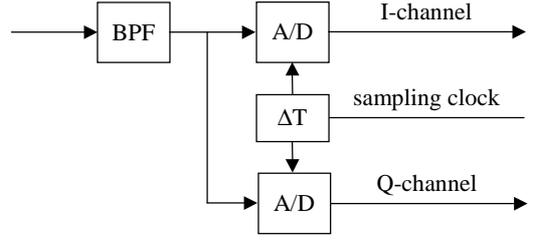


Figure 4: Approximative quadrature sampling to generate the I- and Q-channel signals.

## 4 EXAMPLE DESIGN

For further illustration of the proposed idea, we study the case of linearly frequency dependent phase error  $\phi$  and no amplitude imbalance, i.e.  $g = 1$ . The values of  $\phi$  ranges from  $10^\circ$  to  $-10^\circ$  within the useful signal band. We also assume that the low IF frequency is half of the channel bandwidth. This situation corresponds to an approximative quadrature sampling receiver described in Section 3, where several channels are sampled at once and the desired channel is located at low IF frequency after sampling.

The actual compensation filter  $C(z)$  to be used here is given by

$$C(z) = a_1(z - z^{-1}) + a_2(z^2 - z^{-2}) + a_3(z^3 - z^{-3}). \quad (14)$$

In practise, a delay of three samples needs to be introduced and the same delay has to be included also in the upper branch of the compensation structure of Fig. 3. The filter coefficients  $a_i$ ,  $i = 1, 2, 3$  are optimized in the least-squares (LS) sense to match the optimum compensator response given by Eq. (10). In designing the compensation filter, oversampling factors of 2 and 4 (with respect to the complete band of interest) are used. The optimization results are presented in the following Figs. 5 - 7. Clearly, the optimum compensator response can be accurately reproduced with the given filter of Eq. (14) and thus, the image band signal is successfully attenuated.

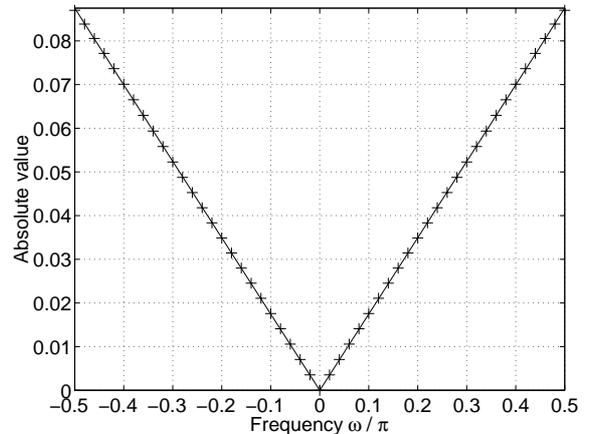


Figure 5: Ideal (solid) and least-squares optimized (+) compensation responses for oversampling factor of 2.

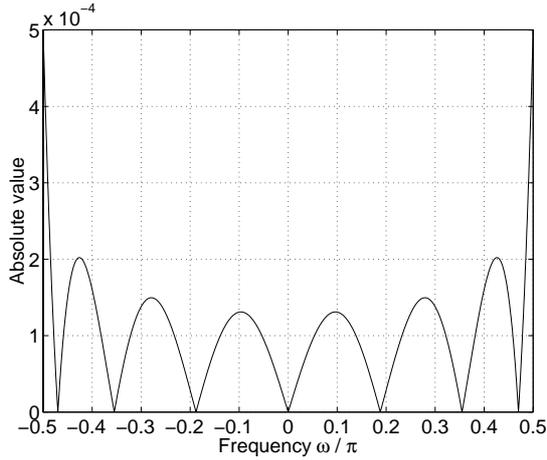
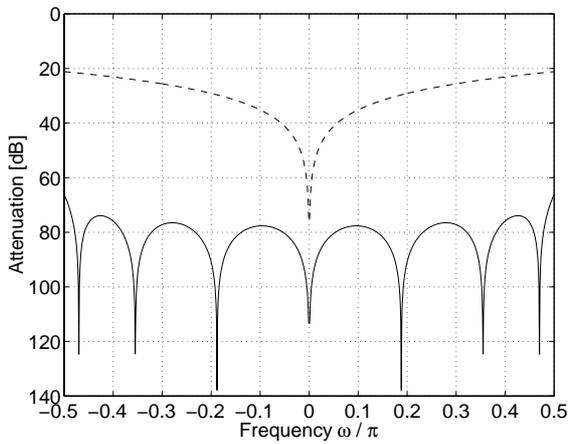
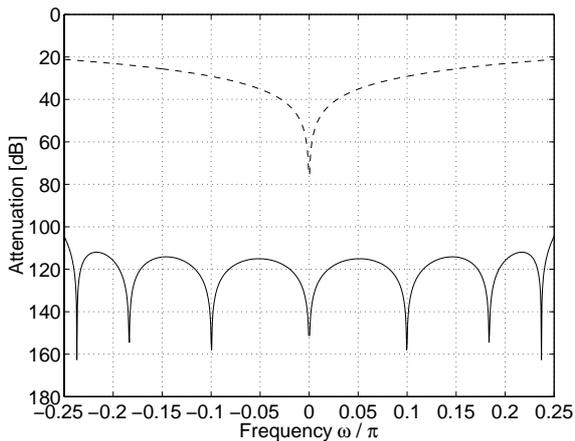


Figure 6: Error in the least-squares optimized compensation filter for oversampling factor of 2.



(a)



(b)

Figure 7: Equivalent image attenuation with (solid) and without (dashed) imbalance compensation for oversampling factors of (a) 2 and (b) 4.

## 5 CONCLUSIONS

The problem of achieving sufficient image signal rejection in the presence of amplitude and phase mismatches in I/Q processing based communication receivers was considered. Signal analysis of an imbalanced I/Q processing receiver was carried out and a possible digital signal processing based solution for fixed imbalance compensation was derived. To effectively attenuate the image band signal, the idea was to subtract a properly weighted image signal estimate from the desired signal observation.

One practical situation with known frequency dependent imbalance, the approximative quadrature sampling receiver, was used to illustrate the possibilities of the proposed compensation method. For this application, the ideal performance was analyzed through a design example. Based on the obtained results, the known part of the I/Q imbalance can be efficiently compensated. Image signal attenuation in the order of 100 dB was shown to be possible, even with a very short compensation filter. This makes the proposed approach attractive also from the implementation point of view. However, non-idealities of the receiver analog components, such as the A/D converters, can turn out to reduce the performance of the proposed compensator. Thus in practical systems, the use of the proposed method should be combined with proper design of the analog front-end circuitry.

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