Identification of Nonlinear Satellite Mobile Channels using Volterra Filters

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ABSTRACT

This paper addresses the problem of nonlinear Satellite Universal Mobile Telecommunication System (S-UMTS) identification. The satellite channel is structured as a time-invaryring (TIV) filter, followed by a zero-memory nonlinearity (ZMNL) and a TIV linear filter. The nonlinearity distortion is due to the on-board satellite amplifier. The mobile channel is modeled by a Time-varying (TV) structure which describes the multipath effects. This paper shows that a specific class of TV Volterra series (with odd kernels) can model this specific satellite mobile bandpass channel. A least squares estimate of the TV Volterra kernels with finite memory is derived for any arbitrary channel input.

1 Introduction

Volterra filters are attractive nonlinear structures because 1) they are a straightforward generalization of the linear system modeling, 2) the unknown parameter are linearly related to the input and output samples. Moreover, many physical systems have been shown to be well modelled by Volterra filters [4]. The present work studies S-UMTS identification using Volterra series. The S-UMTS is defined by a TV nonlinear channel including a TIV filter cascaded with a TIV zero-memory nonlinearity (ZMNL), a TI filter and a multipath filter. The identification of S-UMTS using Volterra Series was for instance proposed in [5]. Here, the original contribution is to approximate the ZMNL by a specific finite-order polynomial. Such approximation proposed in [1] for TIV systems enables to model the nonlinear bandpass channel by a TV Volterra system with odd kernels. A Least Squares (LS) estimate of the TV Volterra kernels is then derived. This LS method does not require statistical assumptions on the input signal except certain persistence assumption. Section II derives the channel input-output relationships. Section III studies the S-UMTS identification problem. Conclusions are reported in section IV.

2 S-UMTS Model

The S-UMTS is depicted in fig. 1. The input samples denoted $a(n)$ are the information symbols. The nonlinear satellite transponder, which operates at or near saturation, is represented by a ZMNL. The nonlinearity is characterized by amplitude distortion (AM/AM conversion) or/and phase distortion (AM/PM conversion). The transponder is modeled, following Saleh model [8], as a frequency independent memoryless bandpass nonlinearity defined by:

$$\begin{aligned} A(r) &= \alpha_a/(1+\beta_a r^2) \quad \text{(AM/AM)}, \\ \Phi(r) &= \alpha_\Phi r^2/(1+\beta_\Phi r^2) \quad \text{(AM/PM)}, \end{aligned}$$

where r is the amplitude of the input waveform, $\alpha_a = 2, \beta_a = 1, \alpha_\Phi = 2.5$ and $\beta_\Phi = 2.8$. The nonlinear channel output can then be described as follows :

$$v(t) = A(\varphi(t))e^{j\Phi(\varphi(t))}u(t) = f(\varphi(t))u(t),$$

where $\varphi(t) = r^2(t) = |u(t)|^2$ and

$$u(t) = \sum_{k=-\infty}^{\infty} a(k)h_1(t-k). \tag{1}$$

According to Weierstrass theorem ([6], p. 399), the nonlinearity f can be approximated by an I-th order polynomial function such that:

$$f(\varphi(t)) = \sum_{i=1}^{I} \alpha_{2i-1}\varphi^{i-1}(t) = \sum_{i=1}^{I} \alpha_{2i-1}\left|u(t)\right|^{2(i-1)}, \tag{2}$$

where the α_{2i-1}'s are complex coefficients. Thus, the output of the nonlinear channel is

$$\begin{aligned} v(t) \;=\; & \sum_{i=1}^{I} \alpha_{2i-1} \sum_{k_1,\ldots k_i=-\infty}^{\infty} \prod_{r=1}^{i} a(k_r) \prod_{r=1}^{i} h_1(t-k_r) \\ & \sum_{k_{i+1},\ldots k_{2i-1}=-\infty}^{\infty} \prod_{r=1}^{i-1} a^*(k_{i+r}) \prod_{r=1}^{i-1} h_1^*(t-k_{i+r}). \end{aligned}$$

Owing to the multipath, several delayed copies of $r(t)$ arrive to the receiver, i.e.

$$s(t) = \sum_{l=1}^{L} \rho_l(t) r(t-\tau_l(t)), \tag{3}$$

where $\rho_l(t)$ and $\tau_l(t)$ are the attenuation factor and the propagation delay for the lth path and

$$r(t) = \sum_{\tau=-\infty}^{\infty} h_2(\tau)v(t-\tau).$$

Consider as approximation that the path delay changes linearly with time, i.e. $\tau_l(t) = \tau_l t + \tau_l^0$. The noisy received discrete time sequence can then be written as

$$\begin{aligned} y(n) &= \sum_{i=1}^{I} \sum_{k_1,\ldots,k_{2i-1}=-\infty}^{\infty} b_{2i-1}(n;\underline{k}_{2i-1}) \\ &\prod_{r=1}^{i} a(n-k_r) \prod_{r=1}^{i-1} a^*(n-k_{i+r}) + \varepsilon(n), \quad (4) \end{aligned}$$

with $\underline{k}_{2i-1} = (k_1, ..., k_{2i-1})$ and

$$b_{2i-1}(n;\underline{k}_{2i-1}) = \sum_{l=1}^{L} \theta_{2i-1,l}(\underline{k}_{2i-1}) e^{j2\pi f_l n}, \qquad (5)$$

where the frequency $f_l = \tau_l f_c$ is known as the Doppler frequency associated with the lth path [5] (f_c is the carrier frequency) and

$$\begin{aligned} \theta_{2i-1,l}(\underline{k}_{2i-1}) &= \rho_l(n)\alpha_{2i-1} e^{j2\pi f_c \tau_l^0} \\ & \sum_{k=-\infty}^{\infty} h_2(k) \underline{h}_1(k, \underline{k}_{2i-1}), \end{aligned}$$

where

$$\underline{h}_1(k,\underline{k}_{2i-1}) = \prod_{r=1}^{i} h_1(k_r - \tau_l - k) \prod_{r=1}^{i-1} h_1^*(k_{i+r} - \tau_{l_n} - k).$$

The variations of the parameters $\theta_{2i-1,l}(\underline{k}_{2i-1})$ with respect to n are small compared to that of the exponential $e^{j\omega_l n}$. The $\theta_{2i-1,l}(\underline{k}_{2i-1})$'s are then assumed complex constants in the following.

The input-output relationship (4) belongs to the class of TV Volterra filters with factorizable odd kernels. This equation represents a low-pass equivalent discret-time channel model. In absence of multipath, the whole system described by eq. (4) reduces to the TI nonlinear system studied in [1] or [2]. It is also worth noting that model (4) is an extension of the linear model studied in [7].

3 Identification problem

Assume that the kernels $b_{2i-1}(n;\underline{k}_{2i-1})$ have finite memory length, say q. The discrete-time received signal can then be written as

$$\begin{aligned} y(n) &= \sum_{i=1}^{I} \sum_{k_1^1,\ldots,k_i^1=0}^{q} \sum_{k_1^2,\ldots,k_{i-1}^2=0}^{q} b_{2i-1}(n;\underline{k}_{2i-1}) \\ &\prod_{r=1}^{i} a(n-k_r^1) \prod_{r=1}^{i-1} a^*(n-k_r^2) + \varepsilon(n), \quad (6) \end{aligned}$$

where $\underline{k}_{2i-1} = (\underline{k}_i^1, \underline{k}_{i-1}^2)$, $\underline{k}_i^1 = (k_1^1, ..., k_i^1)$ and $\underline{k}_{i-1}^2 = (k_1^2, ..., k_{i-1}^2)$ denote the i first and $i-1$ last components of $\underline{k}_{2i-1}$ respectively. The Volterra kernels can be assumed symmetric without loss of generality, i.e. $b_{2i-1}(n;\underline{k}_i^1;\underline{k}_{i-1}^2)$ is unchanged for all $i!$ indice permutations of $k_1^1, ..., k_i^1$ and for all $(i-1)!$ indice permutations of $k_1^2, ..., k_{i-1}^2$. In this case, the non-redundant region of the Volterra kernels is

$$\mathcal{F}_{2i-1} = \left\{ \begin{array}{c} \underline{k}_{2i-1} \,|\, 0 \le k_1^1 \le ... \le k_i^1 \le q, \\ 0 \le k_1^2 \le ... \le k_{i-1}^2 \le q \end{array} \right\}$$

where $i = 1, ..., I$. Let $P(\underline{k}_{2i-1})$ denote the number of distinguishable permutations of $(k_1^1, .., k_i^1)$ and of distinguishable permutations of $(k_1^2, .., k_{i-1}^2)$. The signal model (6) can be rewritten as

$$\begin{aligned} y(n) &= \sum_{i=1}^{I} \sum_{\underline{k}_{2i-1} \in \mathcal{F}_{2i-1}} b_{2i-1}(n;\underline{k}_{2i-1}) P(\underline{k}_{2i-1}) \\ &\prod_{r=1}^{i} a(n-k_r^1) \prod_{r=1}^{i-1} a^*(n-k_r^2) + \varepsilon(n). \quad (7) \end{aligned}$$

The identification problem consists of estimating the parameters $b_{2i-1}(n;\underline{k}_{2i-1})$, from the known system input and output samples $a(n)$ and $y(n)$. Note that the input sequence is assumed to satisfy certain persistence of excitation conditions [4] which guaranty identifiability of the Volterra kernels.

3.1 LS estimation of Volterra Kernels

Define

$$\begin{aligned} \boldsymbol{\theta}_{2i-1}(.) &\triangleq [\theta_{2i-1,1}(.), .., \theta_{2i-1,L}(.)], \\ \boldsymbol{\theta}_{2i-1} &\triangleq \left[\boldsymbol{\theta}_{2i-1}(0, .., 0), ..., \boldsymbol{\theta}_{2i-1}(q, q, ..., q)\right], \\ \boldsymbol{\theta} &\triangleq \left[\boldsymbol{\theta}_1, \boldsymbol{\theta}_3, ..., \boldsymbol{\theta}_{2I-1}\right]^T, \end{aligned}$$

and

$$\begin{aligned} a_{2i-1}(n;\underline{k}_{2i-1}) &\triangleq P(\underline{k}_{2i-1}) \prod_{r=1}^{i} a(n-k_r^1) \prod_{r=1}^{i-1} a^*(n-k_r^2), \\ \underline{\mathbf{a}}_{2i-1}(n) &\triangleq [a_{2i-1}(n;0,..,0), ..., a_{2i-1}(n;q,q,...,q)], \\ \underline{\mathbf{a}}(n) &\triangleq \left[\underline{\mathbf{a}}_1(n), \underline{\mathbf{a}}_3(n), ..., \underline{\mathbf{a}}_{2I-1}(n)\right]. \end{aligned}$$

The channel output in (7) can then be expressed as

$$y(n) = (\underline{\mathbf{a}}(n) \otimes \boldsymbol{\phi}(\mathbf{f};n))\,\boldsymbol{\theta} + \varepsilon(n),$$

where $\otimes$ denotes the Kronecker product operator, $\mathbf{f} = (f_1, ..., f_L)$ is the doppler frequency vector and

$$\boldsymbol{\phi}(\mathbf{f};n) \triangleq [e^{j2\pi f_1 n}, ..., e^{j2\pi f_L n}]^T.$$

Collecting N measurements $\mathbf{y} = (y(0), ..., y(N-1))^T$ of the received signal yields the following matrix formulation

$$\mathbf{y} = \mathbf{A}(\mathbf{f})\boldsymbol{\theta} + \boldsymbol{\varepsilon}, \qquad (8)$$

where

$$\mathbf{A}(\mathbf{f}) = \begin{bmatrix} \underline{\mathbf{a}}(0) \otimes \boldsymbol{\phi}(\mathbf{f}; 0) \\ \underline{\mathbf{a}}(1) \otimes \boldsymbol{\phi}(\mathbf{f}; 1) \\ \vdots \\ \underline{\mathbf{a}}(N-1) \otimes \boldsymbol{\phi}(\mathbf{f}; N-1) \end{bmatrix}$$

and $\boldsymbol{\varepsilon} = (\varepsilon(0), ..., \varepsilon(N-1))^T$. Eq. (8) shows that the S-UMTS identification problem reduces to estimating the unknown parameter vectors $\boldsymbol{\theta}$ and $\mathbf{f}$, given $\underline{\mathbf{a}}(n)$ and $\mathbf{y}$. The LS estimates of $\boldsymbol{\theta}$ and $\mathbf{f}$ are defined by

$$(\widehat{\boldsymbol{\theta}}, \widehat{\mathbf{f}}) = \arg\min_{\boldsymbol{\theta}, \mathbf{f}} J_1(\mathbf{y}; \boldsymbol{\theta}, \mathbf{f}), \tag{9}$$

where

$$J_1(\mathbf{y}; \boldsymbol{\theta}, \mathbf{f}) = (\mathbf{y} - \mathbf{A}(\mathbf{f})\boldsymbol{\theta})^H (\mathbf{y} - \mathbf{A}(\mathbf{f})\boldsymbol{\theta}).$$

It is worth to note that the criterion $J_1(\mathbf{y}; \boldsymbol{\theta}, \mathbf{f})$ is linear with respect to $\boldsymbol{\theta}$ but non-linear with respect to $\mathbf{f}$, which yields a difficult non-linear LS problem. Instead, this paper proposes to estimate the unknown Doppler frequency vector $\mathbf{f}$ and to estimate $\boldsymbol{\theta}$ by minimizing $J_1(\mathbf{y}; \boldsymbol{\theta}, \widehat{\mathbf{f}})$ using a standard linear LS problem :

$$\widehat{\boldsymbol{\theta}} = \left(\mathbf{A}(\mathbf{f})^H \mathbf{A}(\mathbf{f})\right)^{-1} \mathbf{A}(\mathbf{f})^H \mathbf{y}. \tag{10}$$

3.2 Identification of Doppler Frequencies for Circularly Inputs

Suppose that the input $a(n)$ is a circular Gaussian independent sequence. This section shows that the doppler frequencies can be estimated from the cross-correlation between the output $y(n)$ and conjugated, lagged copies of the input $a(n)$

$$m_{ya}(n, \tau) \triangleq E\left[y(n)a^*(n-\tau)\right].$$

A straightforward computation shows that

$$m_{ya}(n, \tau) = \sum_{l=1}^{L} \Psi_l(\tau) e^{j2\pi f_l n}, \tag{11}$$

with

$$\Psi_l(\tau) = \sum_{i=1}^{I} \sum_{\underline{k}_{2i-1} \in \Gamma_{2i-1}} \theta_{2i-1,l}(\underline{k}_{2i-1}) m_{a,2i}(\underline{k}_{2i-1}, \tau),$$

where

$$m_{a,2i}(\underline{k}_{2i-1}, \tau) = E\left\{a_{2i-1}(n; \underline{k}_{2i-1}) a^*(n-\tau)\right\}.$$

The estimation of $m_{ya}(n, \tau)$ from a single realization of $y(n)$ is a difficult problem. However, eq.(11) shows that $m_{ya}(n, \tau)$ is almost periodically time-varying. Consequently, the generalized Fourier series coefficient can be defined [3]:

$$M_{ya}(f, \tau) \triangleq \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} m_{ya}(n, \tau) e^{-j2\pi f n}. \tag{12}$$

It can be shown that

$$M_{ya}(f, \tau) = \sum_{l=1}^{L} \Psi_l(\tau) \delta(f - f_l),$$

i.e. $M_{ya}(f, \tau)$ peaks at the Doppler frequencies provided that $\Psi_l(\tau) \neq 0$. Gardner and al [3] studied a consistent estimator of $M_{ya}(f, \tau)$ denoted $\widehat{M}_{ya}(f, \tau)$, defined as follows:

$$\widehat{M}_{ya}(f, \tau) = \frac{1}{N} \sum_{n=1}^{N} y(n) a^*(n-\tau) e^{-j2\pi f n}.$$

The Doppler frequencies f_l's can then be determined by selecting the L significant peaks in $\widehat{M}_{ya}(f, \tau)$. In order to ensure $\Psi_l(\tau) \neq 0$, the L significant peaks of $\sum_{\tau=0}^{q} M_{ya}(f, \tau)$ can also be considered.

4 Simulation results

Consider a linear third order Volterra system driven by a white Gaussian circular input sequence. The system memory is $q = 10$ and the Doppler frequency vector is $f = (0, 0.05)$.

Fig. 2 represents $\sum_{\tau=0}^{q} \widehat{M}_{ya}(f, \tau)$ from a single record of input-output data of length $N = 256$ with a signal-to-noise ratio $SNR = 10dB$. The Doppler frequencies can clearly be estimated from the 2 peaks of $\sum_{\tau=0}^{q} \widehat{M}_{ya}(f, \tau)$. When SNR decreases, the Doppler frequencies can be estimated provided that N increases. As an exemple, Fig. 3 shows $\sum_{\tau=0}^{q} \widehat{M}_{ya}(f, \tau)$ for $SNR = -5dB$ and $N = 1024$. The MSE's of the linear and third order kernel LS estimates are then computed using 50 Monte-Carlo runs. Fig. 4 displays the MSE of the LS estimate vector as a function of SNR. A good performance is clearly achieved for higth and moderate SNR's.

5 Conclusion

This paper studied the identification of a time-varying nonlinear Volterra system with odd kernels. This system was shown to model accurately Satellite Universal Mobile Telecommunication Systems. The TV Volterra identification problem was expressed as a non-linear least squares estimation problem. This problem was solved using a two step procedure: 1) Doppler frequency estimation using cross-correlations between input and ouput samples 2) Linear LS estimation.

Other S-UMTS identification procedures were studied in the literature. These procedures include neural network based strategies [9] or Volterra series based algorithms [5]. It is important to note that the proposed S-UMTS identification yields lower computational cost than the existing methods. The use of adaptive filtering for the S-UMTS identification problem (involving TV LMS or TV RLS algorithms) is also under study.

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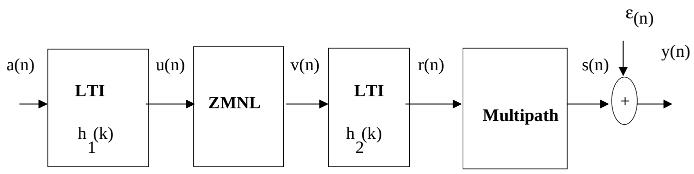

Fig. 1 : The S-UMTS Model.

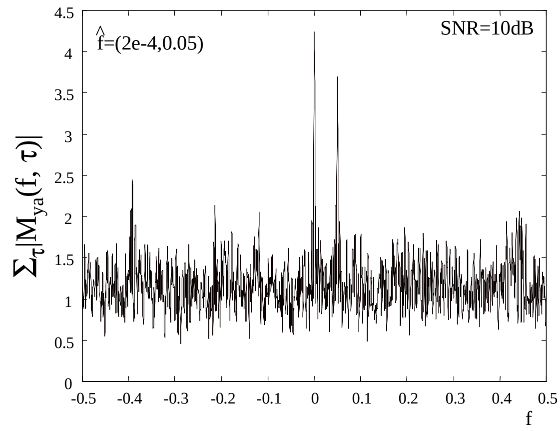

Fig.2 :Doppler frequency estimation ($N = 256$).

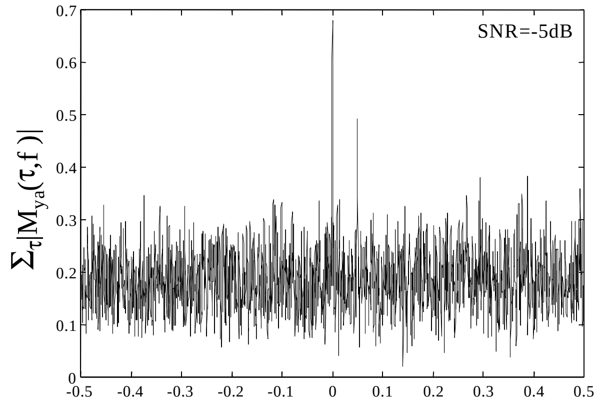

Fig.3 :Doppler frequency estimation ($N = 1024$).

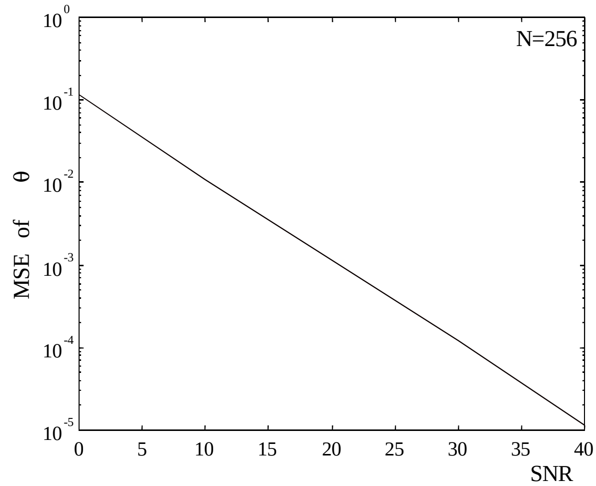

Fig. 4 : LS estimation MSE ($N = 256$).