A NEW LINEAR MODEL FOR IMAGE REPRESENTATION FOR USE WITH KALMAN FILTER RESTORATION

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ABSTRACT
When a Kalman filter is applied to an image for restoration purposes, the model of the original image affects the accuracy of the restoration. The model for effective restoration depends on the correlation of the original image and the variance of the noise. If these parameters are unknown, they have to be estimated from the observed image. In this paper, a method to estimate the unknown parameters in the image restoration process is proposed along with a method that identifies the region of support (number of pixels used for estimation and their positions). Three schemes were developed and used to represent the image and to identify the parameters for the filter. The approach has been tested on numerous images. Results show superior performance compared to methods and implementations previously reported in the literature both in terms of computational complexity and signal to noise ratio.

1. INTRODUCTION

The aim of image restoration is to remove blur and noise incurred in recording an image. Blur removal is a deconvolution type problem, which is often ill posed owing to the presence of observation noise as well as any singularity of the inverse system [1]. Hence, direct inversion (i.e., pseudo-inverse filtering) may cause large oscillations in the solution due to noise amplification. More recently, recursive estimation techniques have been applied to the area of image restoration in hopes of obtaining an optimal restoration technique. It is possible, however, to formulate a well-posed stochastic extension of the ill-posed restoration problem. Hence, many researchers have attempted to solve the image restoration problem using Kalman filtering techniques, where a statistical model of the image can be utilized to effectively regularize the ill-posed deconvolution problem. The better the model fits the image the better the regularization that can be achieved [2]. Kalman filtering for the restoration of degraded images has some advantages when compared to other stochastic filtering techniques. First, since it is recursive in the spatial domain, it does not require too much storage. Therefore, it is suitable for cheap hardware implementations. Second, it can be easily extended to optimal adaptive filtering of images through the use of space-variant image and blur models to improve its performance [3].

Aside from this introductory section, this paper is composed of 6 more sections. In section 2, we will briefly describe image modeling. Section 3 presents the degradation model. Section 4 summarizes the Kalman filter algorithm. Implementation, results and analysis are presented in section 5 and 6 respectively. We finally conclude in section 7 with observations and recommendations for continuation of the work presented here.

2. MODELING

Any image can be modeled by specifying its pixel value, $b(n_1,n_2)$, at every point $(n_1,n_2)$ in the image. An ensemble of such images could be classified as a two-dimensional random field, $x(n_1,n_2)$. If a class of images has a particular correlation function, then all the images could equivalently be characterized by a random field possessing the same correlation properties. Experimental evidence suggests that a large number of images can be characterized by an at least near wide-sense stationary autocorrelation function of exponential nature.

In this work, each pixel will be updated according to a region of support defined by to one of the following representation methods (Fig. 1).

2.1. Causal Representation

The pixel will be updated according to the following equation

$$u(m,n) = a_1 u(m-1,n) + a_2 u(m,n-1) + a_3 u(m-1,n-1) + a_4 u(m-1,n-1) + e(m,n)$$

(2.1)

Then (2.1) can be written as

$$L_4 X_n = L_2 X_{n-1} + e_n$$

(2.2)

$$\text{Cov}(e_n) = Q_u = \beta^2 I$$

where

$$L_1 = \{l_1(m,n)\} = \{\delta(m-n) - a_1 \delta(n+1-m)\}$$

$$L_2 = \{l_2(m,n)\} = \{a_2 \delta(m-n) + a_3 \delta(n+1-m) + a_4 \delta(m+1-n)\}$$

Now (2.2) can be written as a vector AR process

$$X_n = AX_{n-1} + Be_n$$

(2.3)

where $A = L_4^{-1} L_2$ and $B = L_4^{-1}$. The model parameters $a_1, a_2, a_3, a_4$ and $\beta^2$ can be obtained by multiplying (2.1) by $u(m-k,n-l)$ and taking the expectation where $k = 0, -1, 1$ and $l = 0, 1$. \hfill
Note that
\[
E[\{(m,n)u(m-k,n-l)\}]=0
\]
\[
E[\{u(m,n)u(m-k,n-l)\}]=\eta_{jl}
\]  \hspace{1cm} (2.4)

By this model we can obtain the set of the following simultaneous
equation in the following matrix form
\[
\begin{bmatrix}
\eta_{00} & \eta_{01} & \eta_{10} & \eta_{11} & 1
\end{bmatrix}
\begin{bmatrix}
\beta
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]
\[
\begin{bmatrix}
\eta_{00} & \eta_{01} & \eta_{10} & \eta_{11} & -a_1
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]
\[
\begin{bmatrix}
\eta_{00} & \eta_{01} & \eta_{10} & \eta_{11} & -a_2
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]
\[
\begin{bmatrix}
\eta_{00} & \eta_{01} & \eta_{10} & \eta_{11} & -a_3
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]
\[
\begin{bmatrix}
\eta_{00} & \eta_{01} & \eta_{10} & \eta_{11} & -a_4
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]  \hspace{1cm} (2.5)

The solution of this system of equations provide \( \beta \) and \( a_i \) where \( i=1,2,3,4 \).

### 2.2. Semicausal Representation

The pixel will be updated according to the following equation
\[
u(m,n) = a_2 u(m-1,n) + a_3 u(m+1,n) + a_4 u(m,n-1) + a_3 u(m,n-1) + a_4 u(m+1,n-1) + \delta(m,n)
\]  \hspace{1cm} (2.6)

The model parameters can be obtained by the same method as in the causal representation.

### 2.3. Noncausal Representation

The pixel will be updated according to the following equation
\[
u(m,n) = a_2 u(m-1,n) + a_3 u(m+1,n) + a_4 u(m,n-1) + a_2 u(m-1,n-1) + a_3 u(m,n-1) + a_4 u(m+1,n-1) + a_2 u(m,n-1) + a_3 u(m+1,n-1) + a_4 u(m, n-1) + \delta(m,n)
\]  \hspace{1cm} (2.7)

Denote \( X_a \) and \( \epsilon_a \) as \( N \times 1 \) columns of their respective arrays, then the previous equations can be written as
\[
L_1 X_k = L_2 X_{k-1} + \epsilon_k
\]  \hspace{1cm} (2.8)

where
\[
X_k = [X_n, X_{n+2}], \quad X_{k-1} = [X_{n-1}, X_{n+1}],
\]
\[
L_1 = \begin{bmatrix} A & 0 \\ O & B \end{bmatrix} \quad \text{and} \quad L_2 = \begin{bmatrix} X & Y \\ O & Z \end{bmatrix}
\]

where \( A = \{u(m,n)\} = \{\delta(m,n) - a_2 \delta(m+1,n-1) - a_3 \delta(n+1,m)\} \), \( B = \{b(m,n)\} = \{\delta(m,n) - b_2 \delta(m+1,n-1) - b_1 \delta(n+1,m)\} \), \( X = \{x(m,n)\} = \{a_2 \delta(m,n) + a_3 \delta(m+1,n-1) + a_4 \delta(n+1,m)\} \), \( Y = \{y(m,n)\} = \{a_2 \delta(m,n) + a_3 \delta(m+1,n-1) + a_4 \delta(n+1,m)\} \), \( Z = \{z(m,n)\} = \{b_2 \delta(m,n) + b_3 \delta(m+1,n-1) + b_4 \delta(n+1,m)\} \), \( O = \{0\} \) is the null matrix, and the size of \( L_1, L_2 \) and \( \epsilon_k \) is \( 2N \times 2N \).

Now eqn. (2.8) can be written as a vector AR process
\[
X_k = AX_{k-1} + B \epsilon_k
\]  \hspace{1cm} (2.9)

where
\[
A = L_1^{-1} L_2 \quad \text{and} \quad B = L_1^{-1}
\]

The model parameters \( a_2, b_2, \epsilon_1, \epsilon_2 \) where \( i=1,2,3, \ldots, 8 \) and \( j=1,2,3, \ldots, 5 \) can be obtained by the previous method.

### 3. DEGRADATION MODEL

The degradation process is modeled as an operator \( H \), which together with an additive noise term, \( V(x,y) \), operates on an input image, \( X(x,y) \), to produce a degraded image, \( Z(x,y) \).

### 4. THE KALMAN FILTER

The following equations briefly describe the operation of the Kalman Filter [4].

**Prediction of states:**
\[
X_{k|k-1} = AX_{k-1|k-1} + Bu_k
\]  \hspace{1cm} (4.1)

**Prediction of the covariance matrix of states:**
\[
P_{k|k-1} = AP_{k-1|k-1}A^T + BQB^T
\]  \hspace{1cm} (4.2)

**Kalman gain matrix:**
\[
K(k) = P_{k|k-1}H^T \left[H P_{k|k-1}H^T + Q \right]^{-1}
\]  \hspace{1cm} (4.3)

**Update of the state estimation:**
\[
X_k = X_{k|k-1} + K(k) [Z(k) - HX_{k|k-1}]
\]  \hspace{1cm} (4.4)

**Update of the covariance matrix states:**
\[
P_{k|k} = [I - K(k)H]P_{k|k-1}
\]  \hspace{1cm} (4.5)

where \( P_1(k) = E[(X(k) - X_i(k))(X(k) - X_i(k))^T] \), \( i=a,b \)

The parameters for the model are defined as follows:
\( X_0(k), X_n(k) \) are the a priori estimate, the a posteriori estimate respectively, \( P_0(k), P_n(k) \) are the priori, posteriori error covariance matrix respectively, \( K(k) \) is the Kalman gain matrix, and \( Q_a \) and \( Q_b \) are the correlation matrices.
Table 1. Initialization for the parameters for the proposed technique

<table>
<thead>
<tr>
<th>Causal</th>
<th>Semi-Causal</th>
<th>Non-Causal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X(0))</td>
<td>(M^*O_N)</td>
<td>(M^*O_N)</td>
</tr>
<tr>
<td>(P_a(0))</td>
<td>(\delta^2_i*I_N)</td>
<td>(\delta^2_i*I_N)</td>
</tr>
<tr>
<td>(Q_a)</td>
<td>(\beta^2*I_N)</td>
<td>(\beta^2*I_N)</td>
</tr>
<tr>
<td>(Q_v)</td>
<td>(\delta^2_v*I_N)</td>
<td>(\delta^2_v*I_N)</td>
</tr>
</tbody>
</table>

Figure 2. Sample results

Table 2. Statistical data and results pertaining to image in Figure 2 where \(\delta^2_\eta\) is 0.005. The mean and variance for the original image are 0.5231 and 0.0479 respectively.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Noisy</th>
<th>FPKF</th>
<th>Proposed techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Causal</td>
</tr>
<tr>
<td>Mean</td>
<td>0.5235</td>
<td>0.5217</td>
<td>0.5224</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0517</td>
<td>0.0494</td>
<td>0.0463</td>
</tr>
<tr>
<td>S/N</td>
<td>22.938</td>
<td>26.187</td>
<td>34.405</td>
</tr>
</tbody>
</table>

Table 3. Average signal to noise ratio for 17 images, for different levels of additive noise (\(\delta^2_\eta\)), due to applying the proposed techniques and FPBK technique [5].

<table>
<thead>
<tr>
<th>(\delta^2_\eta)</th>
<th>Noisy</th>
<th>FPKF</th>
<th>Proposed techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Causal</td>
</tr>
<tr>
<td>0.005</td>
<td>21.792</td>
<td>26.384</td>
<td>31.204</td>
</tr>
<tr>
<td>0.009</td>
<td>18.212</td>
<td>23.742</td>
<td>29.019</td>
</tr>
<tr>
<td>0.01</td>
<td>16.563</td>
<td>23.696</td>
<td>29.514</td>
</tr>
<tr>
<td>0.02</td>
<td>11.116</td>
<td>19.402</td>
<td>25.108</td>
</tr>
<tr>
<td>0.03</td>
<td>6.3274</td>
<td>17.332</td>
<td>22.149</td>
</tr>
</tbody>
</table>
5. IMPLEMENTATION

It is necessary to have appropriate initial conditions so that the overall estimate will be optimal. This is especially important for the first few data points.

Table 1 defines the initialization of the parameters where $m$ is the mean of the image, $Z(2)$ is the 2nd column of the observation image, $\delta_1^2$, $\delta_2^2$, are the variance of the image and the noise respectively, $I_N$ is the identity matrix and $O_N = [1 1 1 ...1]^T$ of size $(N^*1)$.

6. RESULTS

The effectiveness of the proposed techniques was examined on images of different nature (portraits, full-body, crowds and busy details). The corrupted image was obtained by adding a white Gaussian noise to the original image. Figure 2 shows samples of the filtered images due to applying the proposed technique for which the statistics are listed in Table 2.

As can be observed from Table 2, the variances of the filtered images are lower than the original image, implying some loss of information in these images, which can be largely attributed to smoothing of the edges. We note that the signal-to-noise ratio (S/N) was improved by applying the proposed technique especially for the noncausal representation technique.

For comparison purposes, the full plane block Kalman filter (FBPK) technique proposed in [5] was fully implemented. Table 3 summarizes the average results for 17 images, for different levels of additive noise, due to applying the proposed techniques and the FBPK. As can be observed the S/N improved by applying the proposed techniques specially for the causal and noncausal cases. Moreover, the processing time of the proposed techniques is less than the processing time for the FPKF.

It was noted in filtering the images with crowded details that the edges were not restored to their original sharpness. The smoothing of the edges is attributed to the following reasons:

- Choice for the previous state vector for the pixel $(i, j)$ need not be the one associated with the pixel $(i, j-1)$. This is apparent at the boundary between two regions. Pixel $(i, j-1)$ may be in a region whose autocorrelation function differs widely from that of the region to which pixel $(i, j)$ belongs. Thus, utilizing the state vector at $(i, j-1)$ as the previous one to the state vector at $(i, j)$ may give poor results.
- Choice of the nearest neighbors for the previous state information may not be the optimum. The criterion function used for the nearest neighbor decision is based on the spatial activity information of the pixel's neighbors as well as the intensity levels.
- The existence of a large number of regions with no a priori information about the initial states.

7. CONCLUSION

The advantage of spatial domain approaches over frequency domain approaches is that the estimation algorithm can be recursive and hence have well-known implementation efficiencies. However, the filters resulting from these approaches are essentially low-pass filters. Therefore, resulting images can be smoothed by these filters, and this is especially apparent at object edges.

A new recursive technique has been presented for the restoration of images degraded by additive Gaussian noise. Three models have been used to represent and model system degradation. Among the prediction models considered, the causal and noncausal models seem to offer good results against the semicausal model. The cumulative mean square error calculated for the images restored by the proposed techniques showed improvement over the images restored by the full plane block Kalman filter [5]. It is apparent from the results that the proposed method for image restoration provides encouraging results. It is also provides a general formation so that it is easily adapted to a large variety of images.

The novel features for the proposed technique are:

- A new linear dynamic system model is developed for representing two-dimensional image data, which accounts for correlation present between adjacent pixels. The resulting system equations are directly applicable to Kalman filter algorithms.
- The required a priori information to model the system is minimal.
- The method updates a vector at each iteration. Thus, the required processing time to filter the degraded image is much less than previous methods reported in the literature.

References